Reflection-enhanced gain in traveling-wave parametric amplifiers and plasma oscillation phase matching

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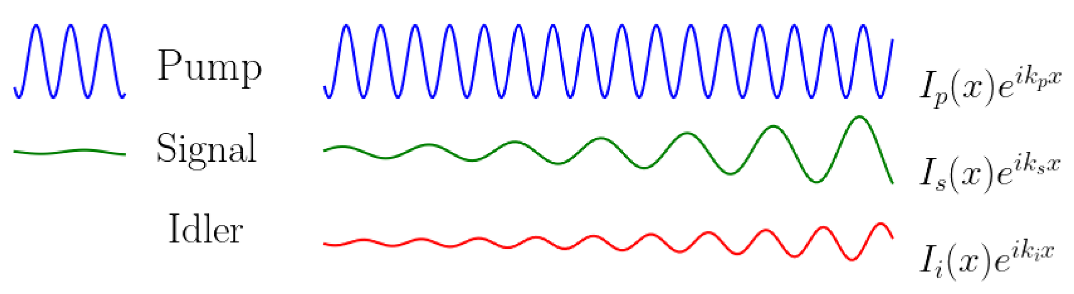
Abstract

Traveling-Wave Parametric Amplifiers (TWPAs) are essential tools for ultra-sensitive measurements, particularly in quantum systems. Their performance, however, is often limited by challenges in impedance and phase matching. Conventional phase-matching techniques, such as periodic impedance modulation or the use of resonators and phase shifters, complicate circuit design and increase impedance mismatch. We show that reflections caused by such mismatches significantly alter both gain and phase-matching conditions in TWPAs [1]. To capture this behavior, we extend standard coupled-mode theory, which typically assumes only forward-propagating waves, by including reflected waves. This leads to a corrected gain formula that more accurately reflects real device performance. To overcome these limitations, we propose a simpler and more integrated solution: leveraging tunable dispersion in a Josephson junction array waveguide [2]. The desired dispersion is engineered through a metamaterial structure, achieved by periodically adding a parallel capacitor to every n-th Josephson junction. This design enables efficient phase matching in the three-wave mixing regime. Our analytical framework is supported by numerical simulations that account for the complex nonlinear dynamics present in realistic devices.

TWPA – State of Art

- Coupled mode theory predicts high broadband gain for phase-matched waves

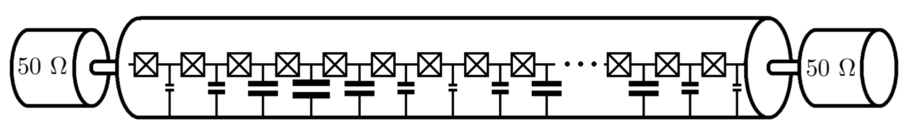
$$v_p^2\frac{\partial^2 I(x,t)}{\partial z^2}-\frac{\partial^2 I(x,t)}{\partial t^2}=\frac{\partial^2}{\partial t^2}\left(\frac{1}{2}\epsilon I(x,t)^2+\frac{1}{3}\xi I(x,t)^3\right)$$



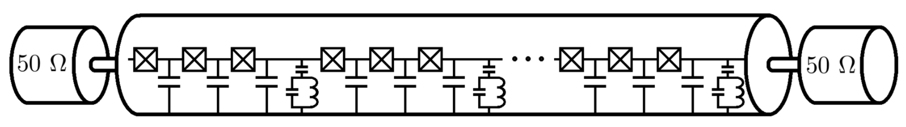
$$e^{i\Delta kx}\equiv e^{i(k_p-k_s-k_i)x}$$

$$\Delta k\approx k_p\frac{\xi\,|I_p|^2}{8}$$

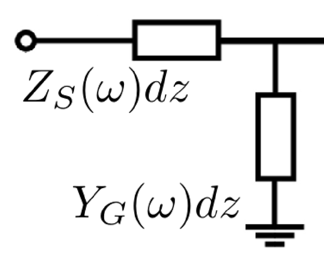
- Photonic crystal phase-matching



- Resonant phase-matching



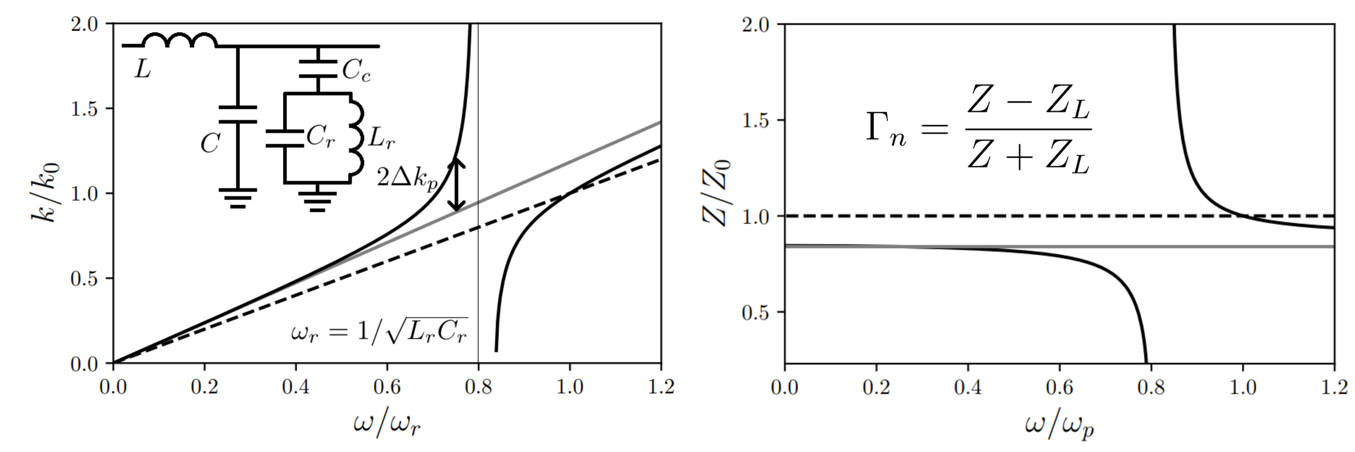
- Dispersion can by estimated



$$Z(\omega)=\sqrt{\frac{Z_S(\omega)}{Y_G(\omega)}}$$

$$v_p(\omega)=i\frac{\omega}{\sqrt{Z_S(\omega)Y_G(\omega)}}$$

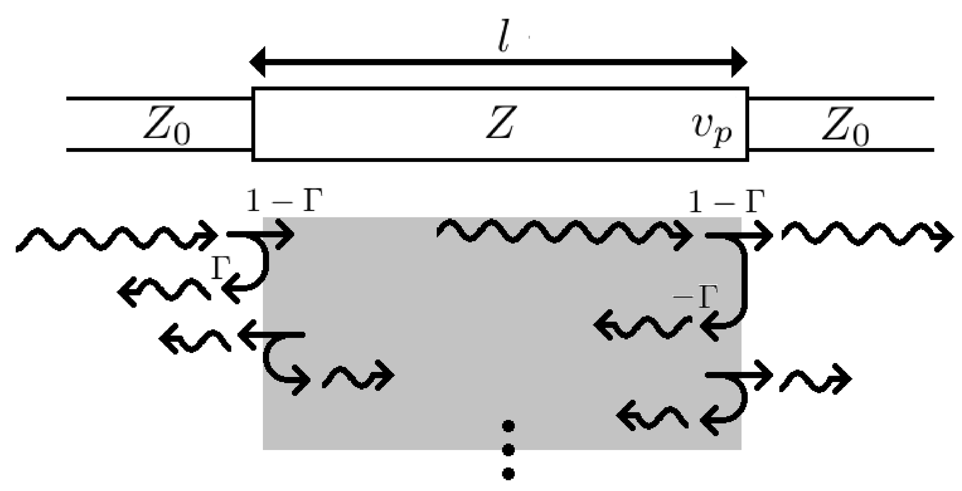
- impedance changes too!



Left: Dispersion of resonantly loaded transmission line. Pump near the resonance introduces mismatch Δk satisfying phase-matching condition. However, the dispersion also enables the propagation of unwanted sidebands, such as $\omega_p+\omega_s$, which degrades the amplifier's gain and bandwidth. **Right:** The impedance is also modified by the resonance, leading to an impedance mismatch and causing reflections.

Reflection inside TWPA

- Partial reflections and interference due to impedance mismatch



$$e^{ikz}+\Gamma e^{ik(l-z)}+\Gamma^2e^{ik(2l+z)}+\cdots=\frac{e^{ikz}+\Gamma e^{ik(l-z)}}{1-\Gamma e^{i2kl}}$$

- Modified CMT including reflections [2]

$$e^{i\Delta kz}\to\mathcal{F}_{si}^p\equiv\left(e^{ik_pz}+\Gamma e^{ik_p(l-z)}\right)\frac{\left(e^{-ik_iz}+\Gamma^*e^{-ik_i(l-z)}\right)}{\left(e^{ik_sz}-\Gamma e^{ik_s(l-z)}\right)}$$

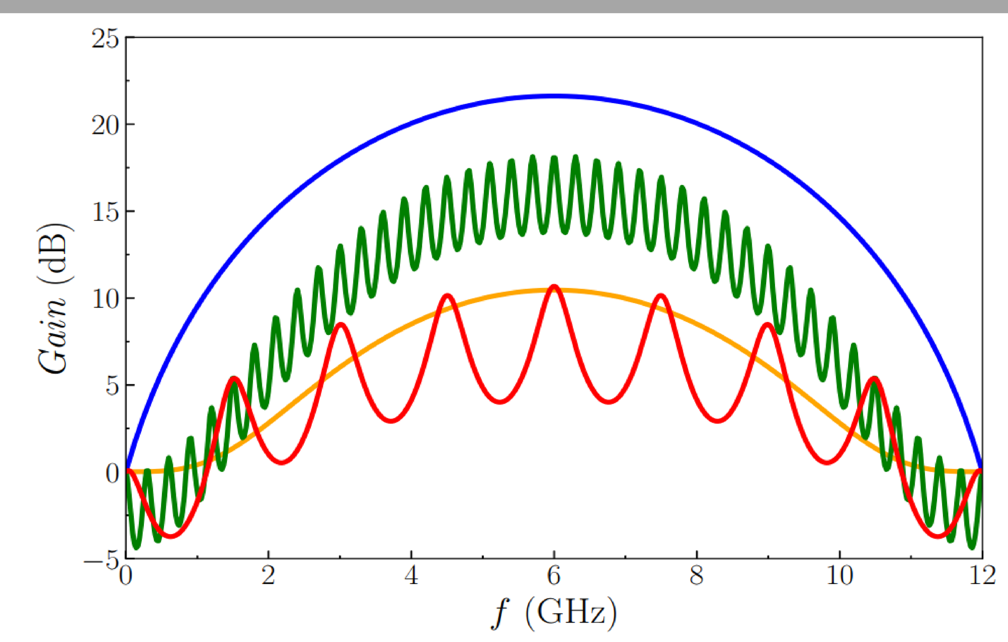
- Phase-matching condition

$$\Delta k\approx k_p\frac{\xi\,|I_p|^2}{8}\left(1-\Gamma^2\right)$$

- Gain factor

$$g\approx\sqrt{k_sk_i\epsilon^2\,|I_p|^2\left(1+\Gamma^2\right)}$$

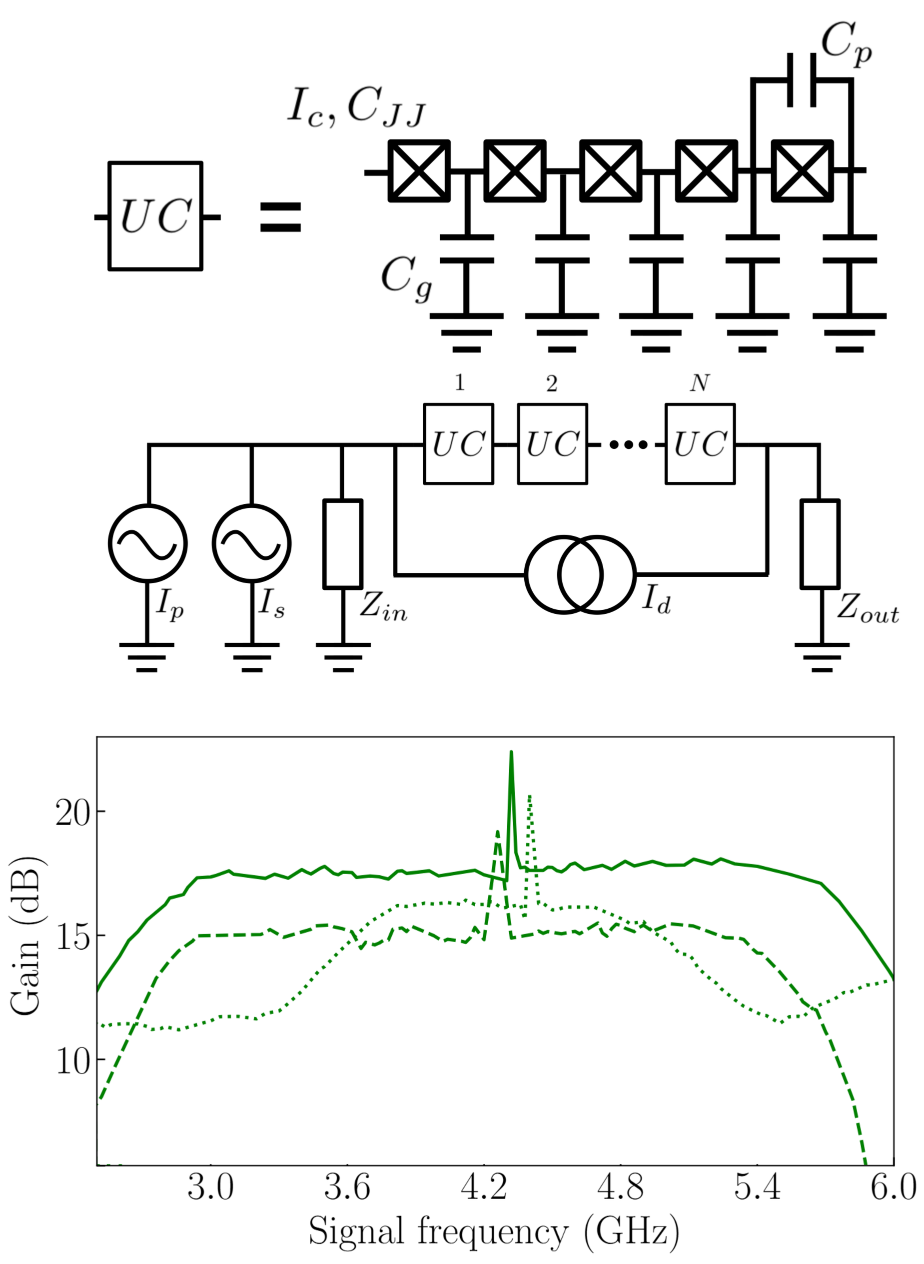
- Gain Ripples – rapid oscillations in gain vs frequency profile – reduces amplifiers bandwidth.



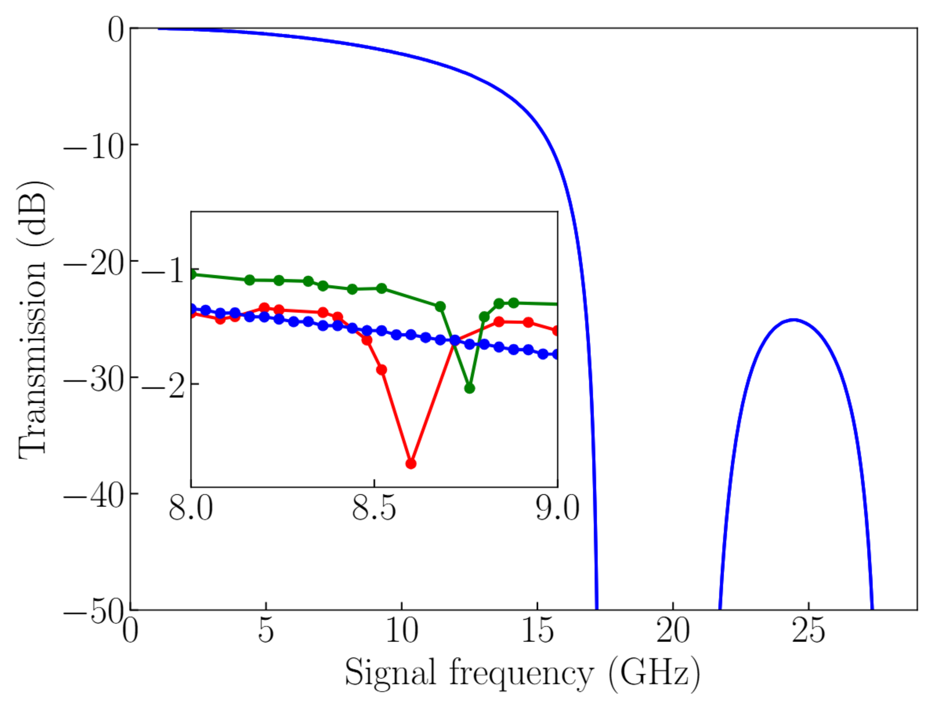
Gain profile for TWPA without reflections: orange – no phase matching, blue – perfect phase-matching. Moderate reflections ($\Gamma=0.6$) enhance both the gain and the bandwidth even without any phase-matching (green curve). Red curve is for 5-times shorter waveguide.

Plasma resonance phase-matching and Third Harmonic Generation (THG) mediated gain in TWPA

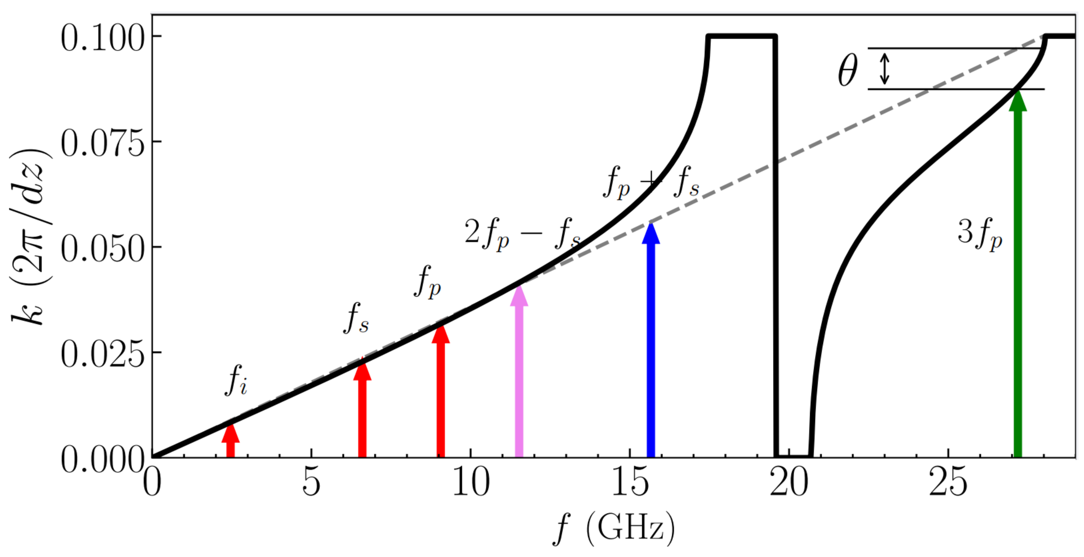
- The shunting capacitance C_p of JJ shifts its plasma frequency into GHz range.
- In a periodic arrangement, the unique dispersion enables phase matching in PTWPA [2] through THG [3].



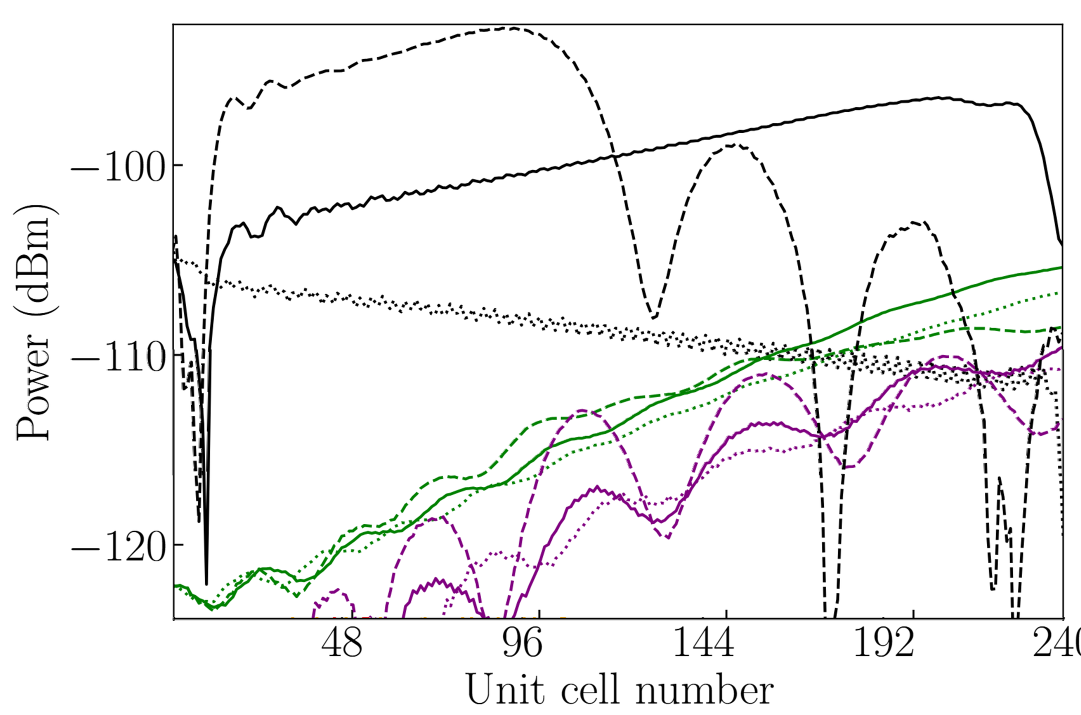
Top: Diagram of a waveguide unit cell composed of five Josephson junctions (JJs), with one junction shunted by an additional capacitor. **Middle:** Schematic of TWPA circuit consisting of $N=240$ unit cells, simulated using JoSim. **Bottom:** Gain of the TWPA near the sweet spot.



Transmission of the PTWPA probed by weak signal. The inset shows the transmission within the "sweet spot" frequency range, where amplification is enhanced, for two higher signal powers. The dip in the high-power transmission indicates efficient third-harmonic generation.



Black curve is dispersion of the PTWPA calculated by ABCD matrix method. Dashed line is the linear approximation with additional phase shift for third harmonic. Dispersion enables effective phase-matching of the THG. Indicated modes are signal, idler, pump, third harmonic and sidebands.



Evolution of the respective modes: third harmonic $3f_p$ (black), signal f_s (green), and f_p+f_s (purple) along the PTWPA for various pump frequencies near the sweet spot, where the efficiency of third-harmonic generation can be tuned. Highly efficient THG (solid lines) is accompanied by high signal gain. This also corresponds to the widest achieved amplification bandwidth.

Conclusions

- Besides phase matching, the bandwidth of TWPAs is also limited by gain ripples due to impedance mismatches.
- Additionally, the dominant factor reducing gain is sideband generation, which cannot be broadly suppressed. However, third-harmonic generation helps mitigate its negative impact.

References

[2] S. Kern, P. Neilinger, et al.:, *Physical Review B* **107**, 174520 (2023).

[1] E. Rizvanov, S. Kern, P. Neilinger, M. Grajcar, *Journal of Applied Physics* **136**, 17 (2024).

[3] E. Rizvanov, et al.r: *Third harmonic-mediated amplification in TWPA*, arXiv:2508.05295

Acknowledgements

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