Towards a theory of dissipative Dark Matter

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Assoc.-Prof. Josef Pradler[1,2]

JCAP03(2026)034 [arXiv:2509.12317]
arXiv:26XX.XXXX

November 2026

Supported by:

European Research Council
Established by the European Commission

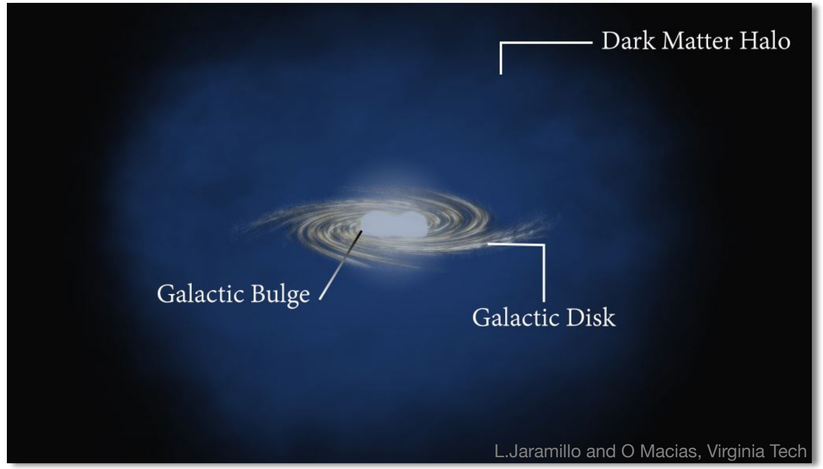
NLO-DM

1. University of Vienna, Faculty of Physics 2.MBI - Marietta-Blau-Institut für Teilchenphysik der Österreichischen Akademie der Wissenschaften
3. University of Vienna, Vienna Doctoral School in Physics

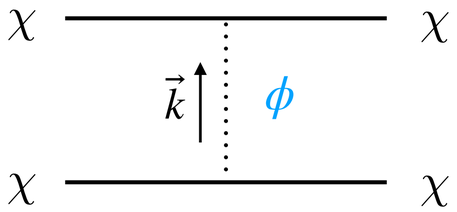
Self-Interacting Dark Matter (SIDM)

- ΛCDM: successful on scales $\gtrsim$ Mpc
- Galactic/subgalactic scales:

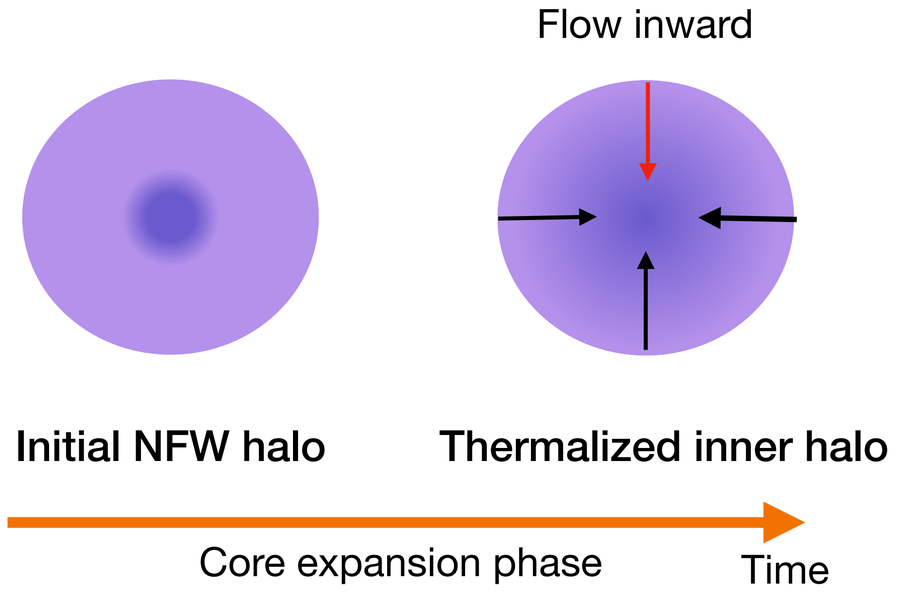
  Possible[1] deviations from collisionless DM



SIDM[2]



Heat transport



[1] Review: Rev. Mod. Phys. **97**, 045004 [2] Phys.Rev.Lett. 84 (2000) 3760-3763

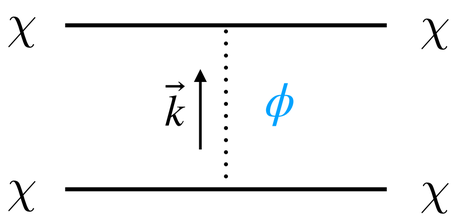
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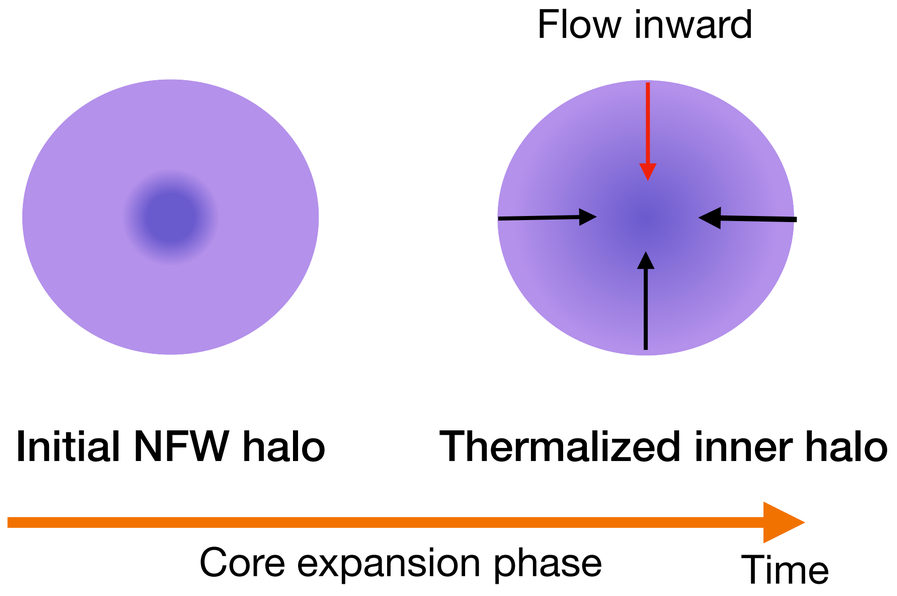
  Possible[1] deviations from collisionless DM

A diversity of DM density profiles!

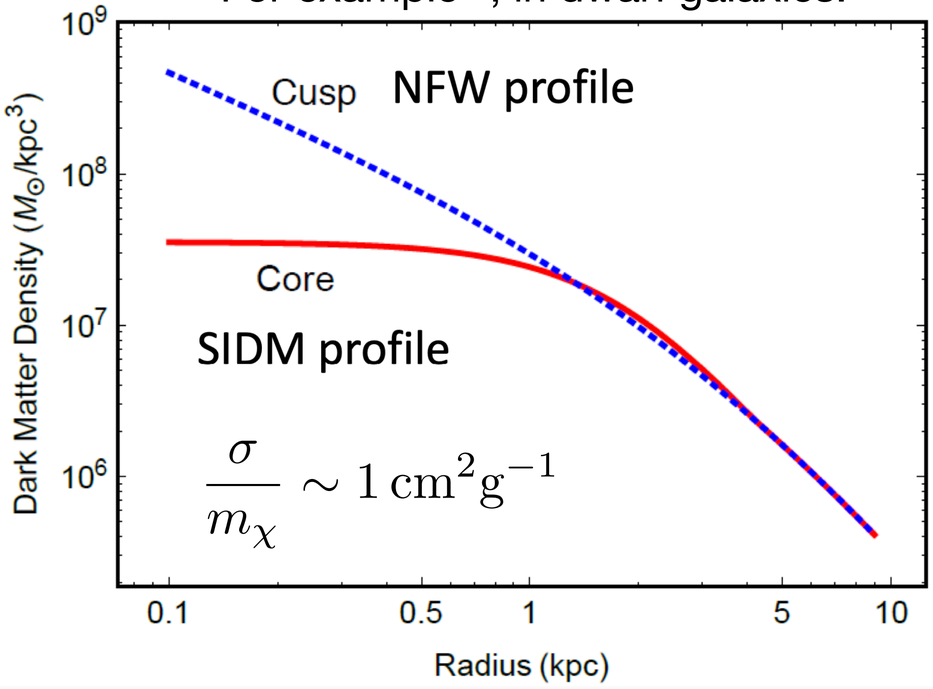
SIDM[2]



Heat transport



For example[3], in dwarf galaxies:

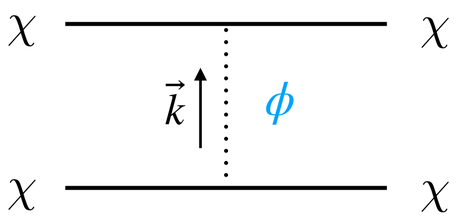


[1] Review: Rev. Mod. Phys. **97**, 045004 [2] Phys.Rev.Lett. 84 (2000) 3760-3763 [3] arXiv:1705.02358

Dissipative Self-Interacting Dark Matter (dSIDM)

SIDM

Heat transport

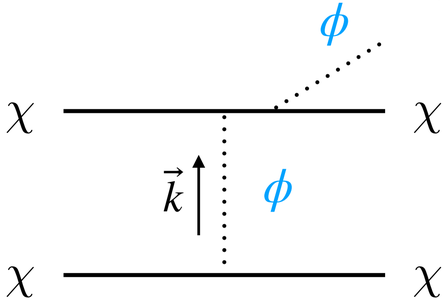


$$\frac{\sigma}{m_\chi} \gtrsim \mathcal{O}(1)\,\mathrm{cm}^2\mathrm{g}^{-1}$$

Light particles

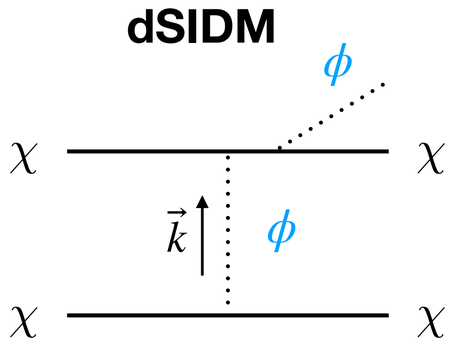
dSIDM

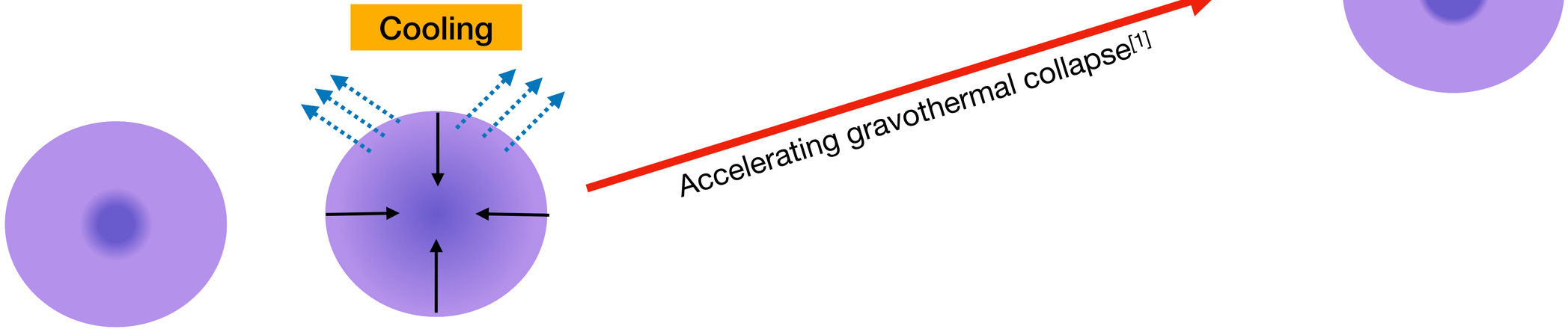
Cooling effect



1] Examples of simple dissipative dark sectors: arXiv: 0810.5126, arXiv:1807.04750, ArXiv: 1812.07000

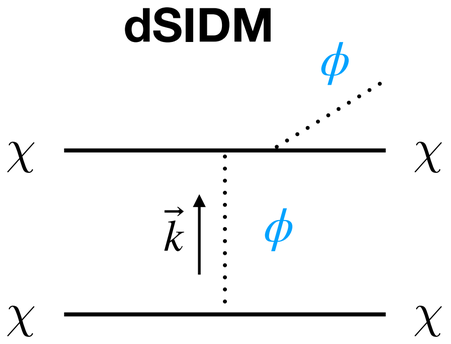
Dissipative Self-Interacting Dark Matter (dSIDM)

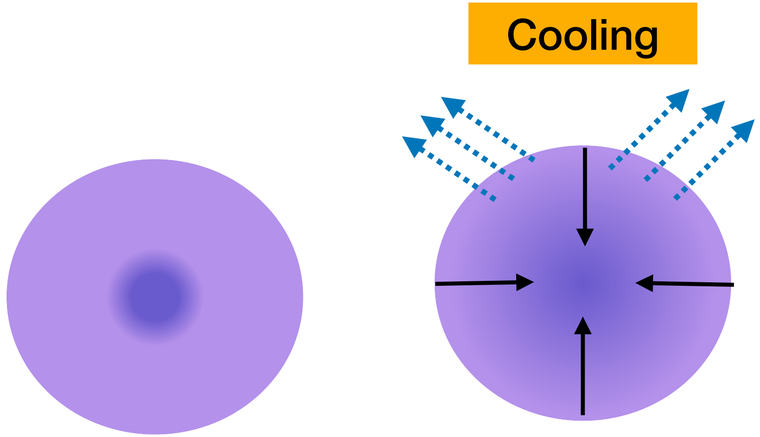




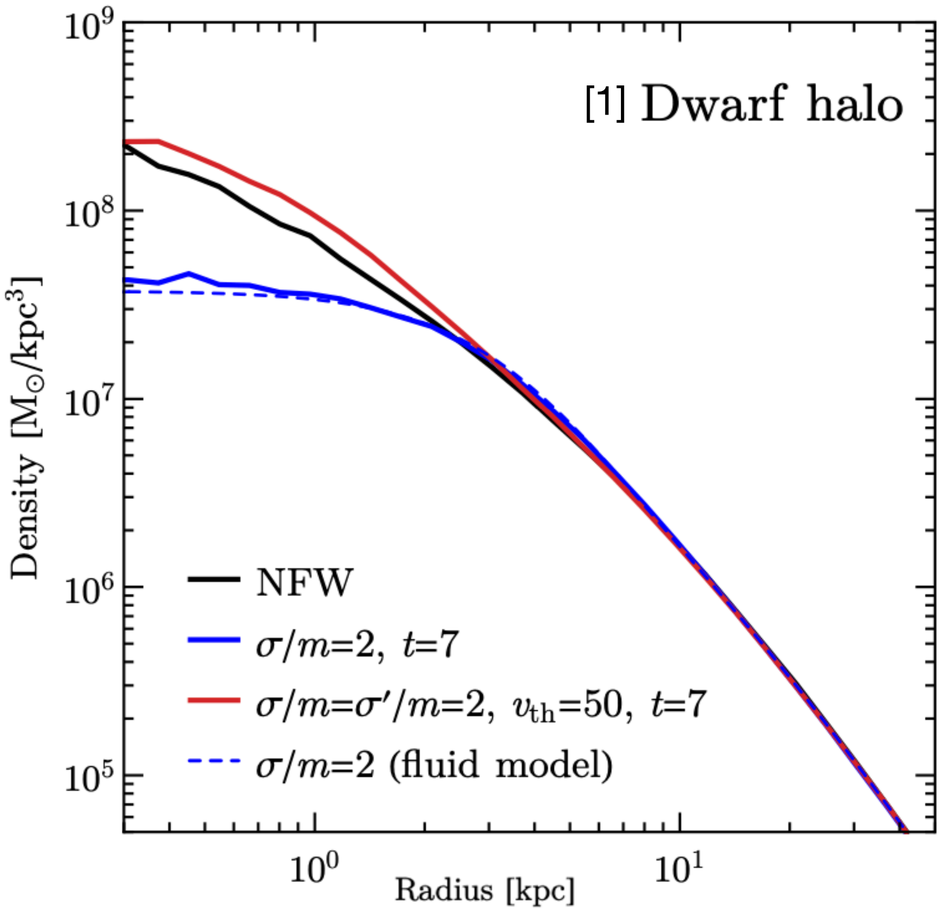
[1] ArXiv:1912.06757 [2] arXiv:2410.06252 [3] arXiv:1412.1839 [4] arXiv:1303.3271

Dissipative Self-Interacting Dark Matter (dSIDM)



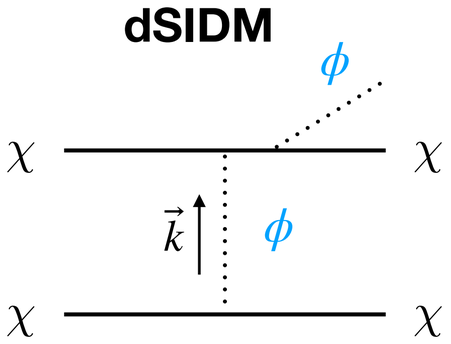


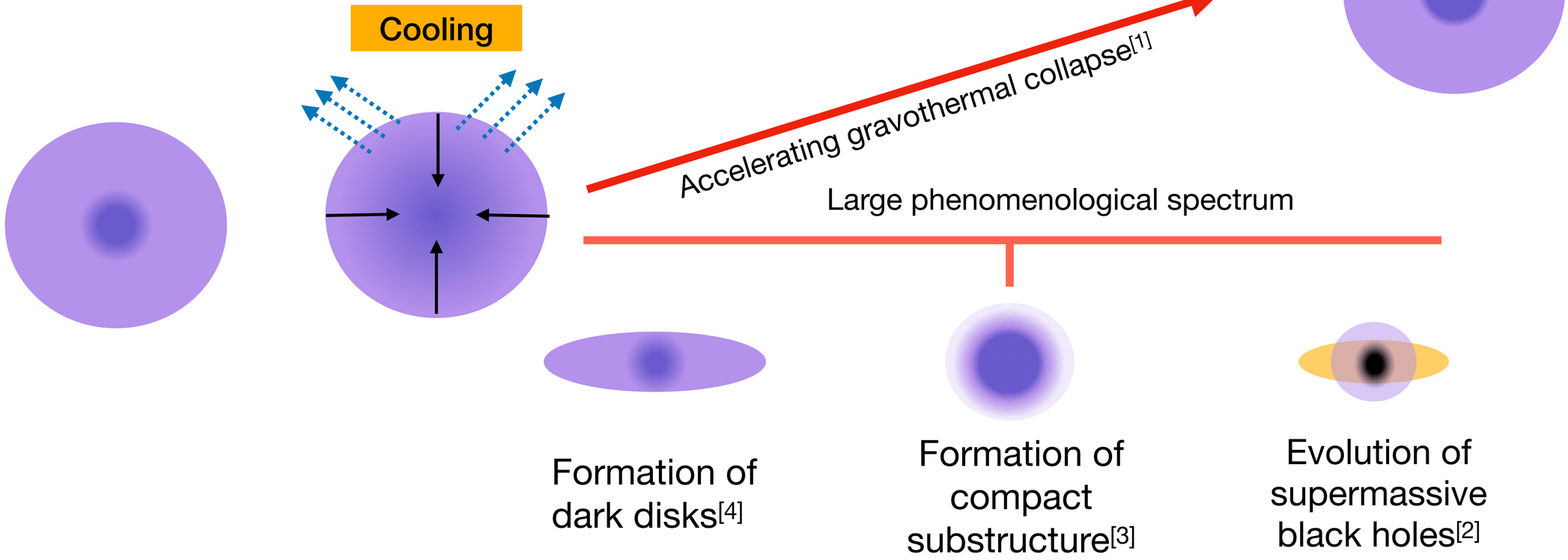
Gravothermal



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Dissipative Self-Interacting Dark Matter (dSIDM)

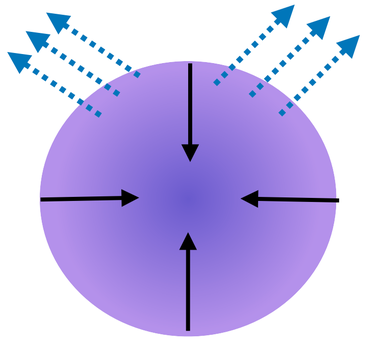




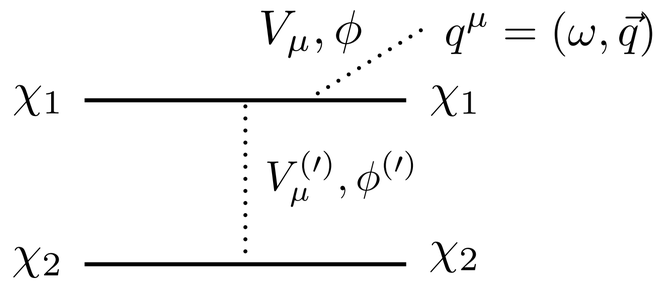
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dSIDM: microphysics

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Cooling



$$\frac{\dot{\epsilon}}{n_1 n_2} \propto \frac{\mu^{3/2}}{T^{3/2}} \int_{v_{\min}}^{\infty} dv_i \, f(v_i) \int_{m_{\phi,V}}^{\omega_{\max}} d\omega \, \omega \frac{d\sigma}{d\omega}$$

→ Depends on the microphysics

The energy lost per unit of time by the system

This talk: how the microscopic properties of the DM candidates shape the resulting cooling in minimal dissipative scenarios

dSIDM Models

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Hidden DM sector

DM candidates
- Fermion
- Scalar

$\chi_1\chi_2 \to \chi_1\chi_2 V \quad \tilde{\chi}_1\tilde{\chi}_2 \to \tilde{\chi}_1\tilde{\chi}_2\phi \quad \chi_1\chi_2 \to \chi_1\chi_2\phi$

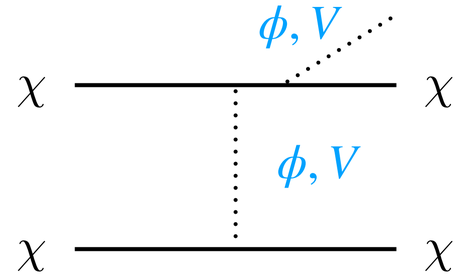
$S_1 S_2 \to S_1 S_2 V \quad \tilde{S}_1\tilde{S}_2 \to \tilde{S}_1\tilde{S}_2\phi \quad S_1 S_2 \to S_1 S_2\phi$

Emission/Mediation:
- Gauge vector V
- Scalar ϕ

Including massive emission

Two mediation regimes:

Long-range Short-range



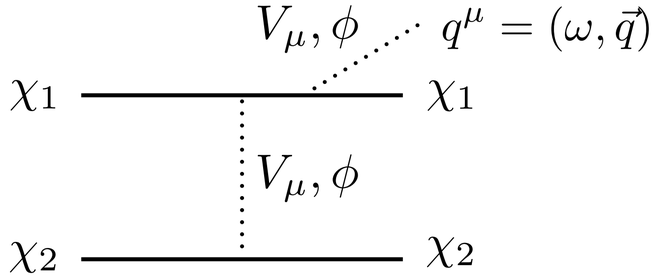
$$\mathcal{L}_{\rm int}^{V} = g\sum_{j=1,2}\left\{-Q_{\chi_j}\bar{\chi}_j \not{V}\chi_j + iQ_{S_j}V^{\mu}\left[(\partial_\mu S_j^{\dagger})S_j - S_j^{\dagger}\partial_\mu S_j\right]\right\} + g^2\sum_{j=1,2}Q_{S_j}^2 S_j^{\dagger}S_j V_\mu V^\mu$$

$$\mathcal{L}_{\rm int}^{\phi} = \sum_{j=1,2}\left\{-y_j\bar{\chi}_j\chi_j\phi - \frac{1}{2}\tilde{y}_j\tilde{\chi}_j^T C\tilde{\chi}_j\phi - A_j|S_j|^2\phi - \frac{1}{2}\tilde{A}_j\tilde{S}_j^2\phi - \frac{1}{2}\lambda_j|S_j|^2\phi^2 - \frac{1}{4}\tilde{\lambda}_j\tilde{S}_j^2\phi^2\right\}$$

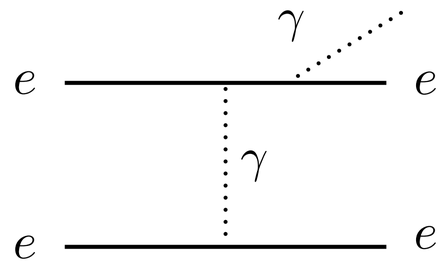
$$\mathcal{L}_{\rm int}^{\rm self} = -\frac{1}{3!}A_\phi\phi^3 - \frac{1}{4!}\lambda_\phi\phi^4 - \frac{1}{(1+3\delta_{S_1S_2})}\lambda_S|S_1|^2|S_2|^2 - \frac{1}{4(1+2\delta_{\tilde{S}_1\tilde{S}_2})!}\tilde{\lambda}_S\tilde{S}_1^2\tilde{S}_2^2$$

Bremsstrahlung

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In DM halos: $v_i \lesssim 100 - 1000\,\mathrm{km.s}^{-1}$

Non-relativistic scattering:

Long range: $m_{\phi,V} \ll |\vec{k}|$

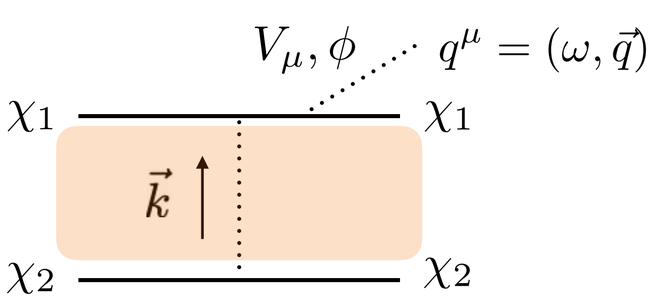
$$V(r) = \frac{a_1 a_2}{4\pi r}$$

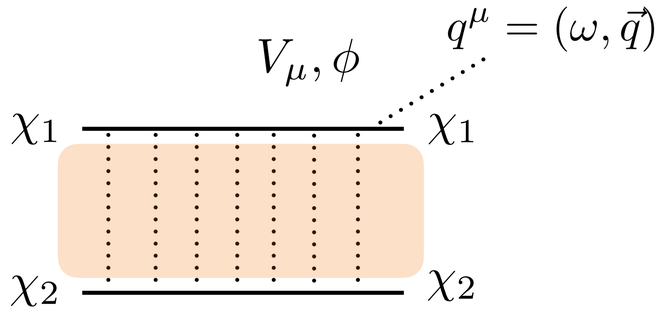
$$r = |x_1 - x_2|$$

$$\mathcal{T}_{f(V,\phi),i} = -2\pi i \delta(E_i - E_f - \omega) \langle \psi_f | a(\vec{q}, \lambda) V_{\mathrm{int}} | \psi_i \rangle$$

Textbook discussion: Lectures on Quantum Mechanics, Steven Weinberg

Bremsstrahlung: kinematic regimes





Long range: $m_{\phi,V} \ll |\vec{k}|$

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Initial and final states:

$$H_0 \left| \psi_{i(f)} \right\rangle = E_{i(f)} \left| \psi_{i(f)} \right\rangle$$

Analytical solution

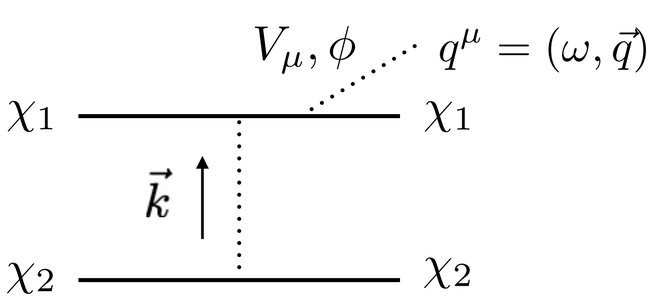
all orders in the Coulomb potential

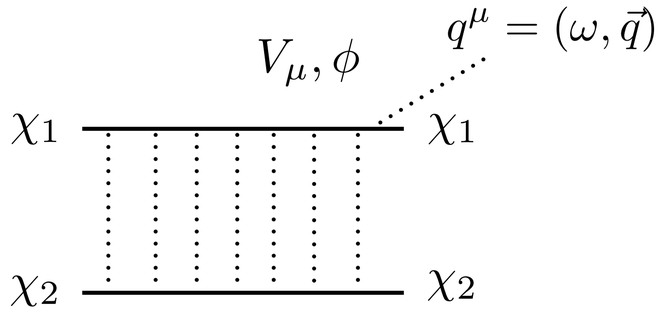
$$\chi^{(+)}_{\vec{p}_i}(\vec{r}) = \Gamma(1 - i\nu_i) e^{\frac{\pi}{2}\nu_i} e^{i\vec{p}_i \cdot \vec{r}} {}_1F_1\left(i\nu_i, 1; i\zeta_i^-\right)$$

$$\chi^{(-)}_{\vec{p}_f}(\vec{r}) = \Gamma(1 + i\nu_f) e^{\frac{\pi}{2}\nu_f} e^{i\vec{p}_f \cdot \vec{r}} {}_1F_1\left(-i\nu_f, 1; -i\zeta_f^+\right)$$

Textbook discussion: Quantum Mechanics: Non-Relativistic Theory, Landau

Bremsstrahlung: kinematic regimes





Long range: $m_{\phi,V} \ll |\vec{k}|$

Non-relativistic scattering:

$$V(r) = \frac{a_1 a_2}{4\pi r}$$

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Analytical solution to

all orders in the Coulomb potential

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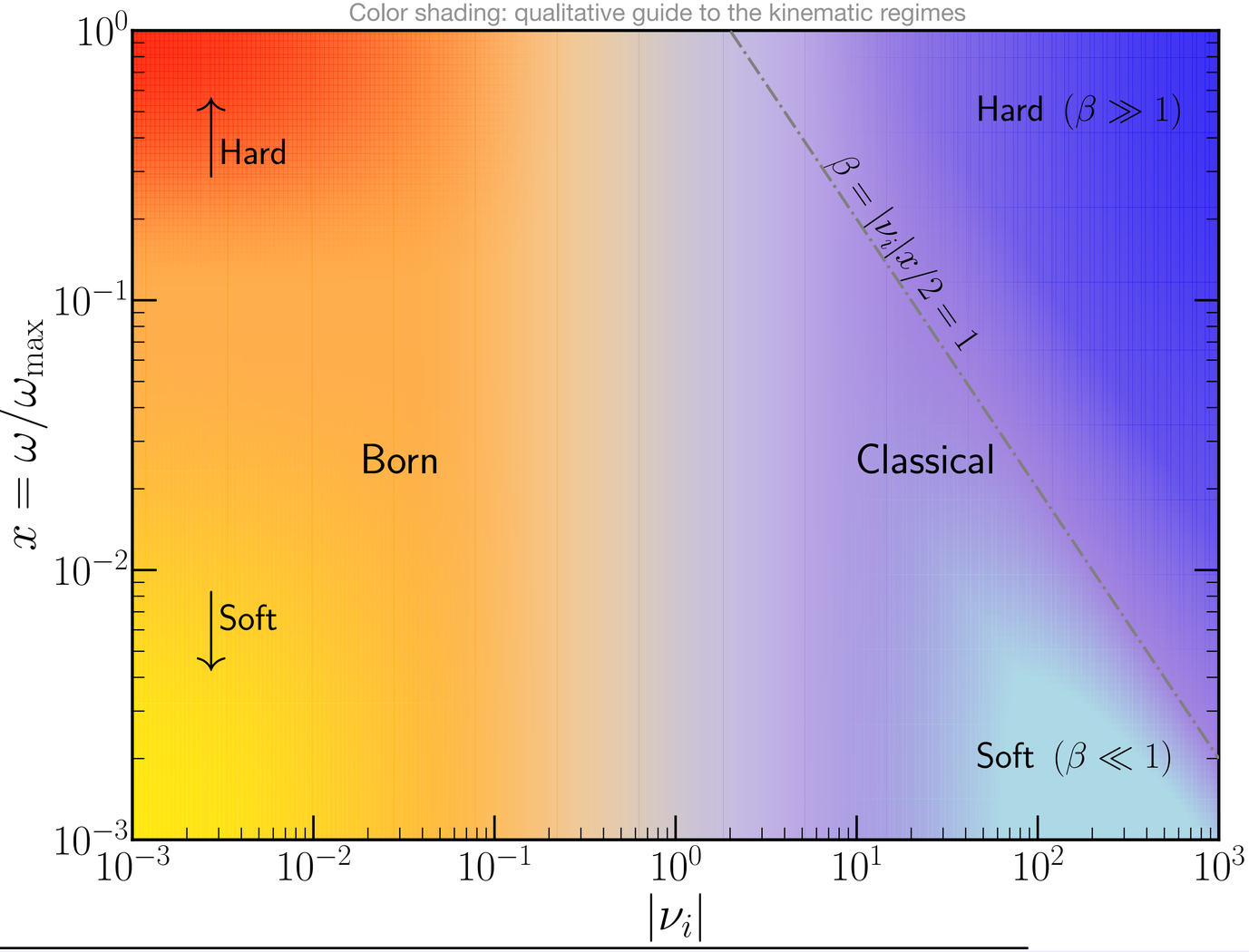
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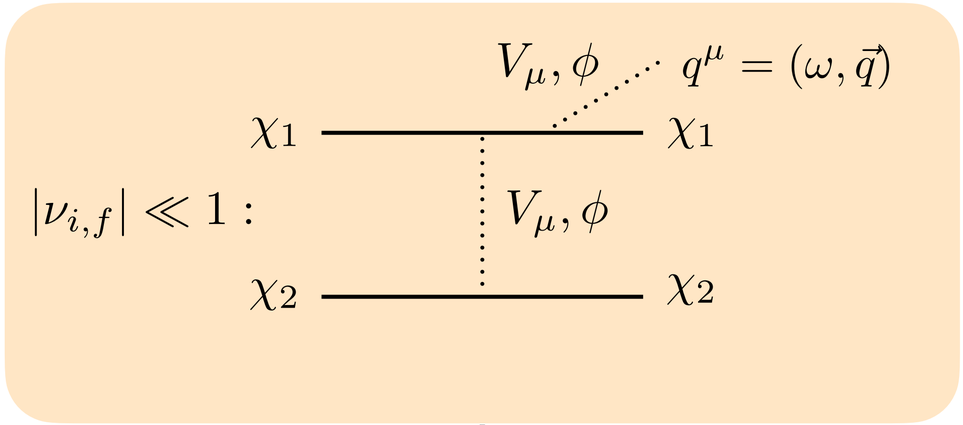
The different scattering regimes are characterized by the Sommerfeld parameters: $|\nu_{if}| = \frac{a_1 a_2}{4\pi v_{i,f}}$

Textbook discussion: Quantum Mechanics: Non-Relativistic Theory, Landau

Bremsstrahlung: kinematic regimes

$$\mathcal{T}_{f(V,\phi),i} = -2\pi i \delta(E_i - E_f - \omega)\langle \psi_f | a(\vec{q},\lambda) V_{\rm int} | \psi_i \rangle$$





$\psi_{i,f} = e^{i\vec{p}_{i,f}\cdot\vec{r}}$ ↓ Born

Quantum Field Theory,
Tree level

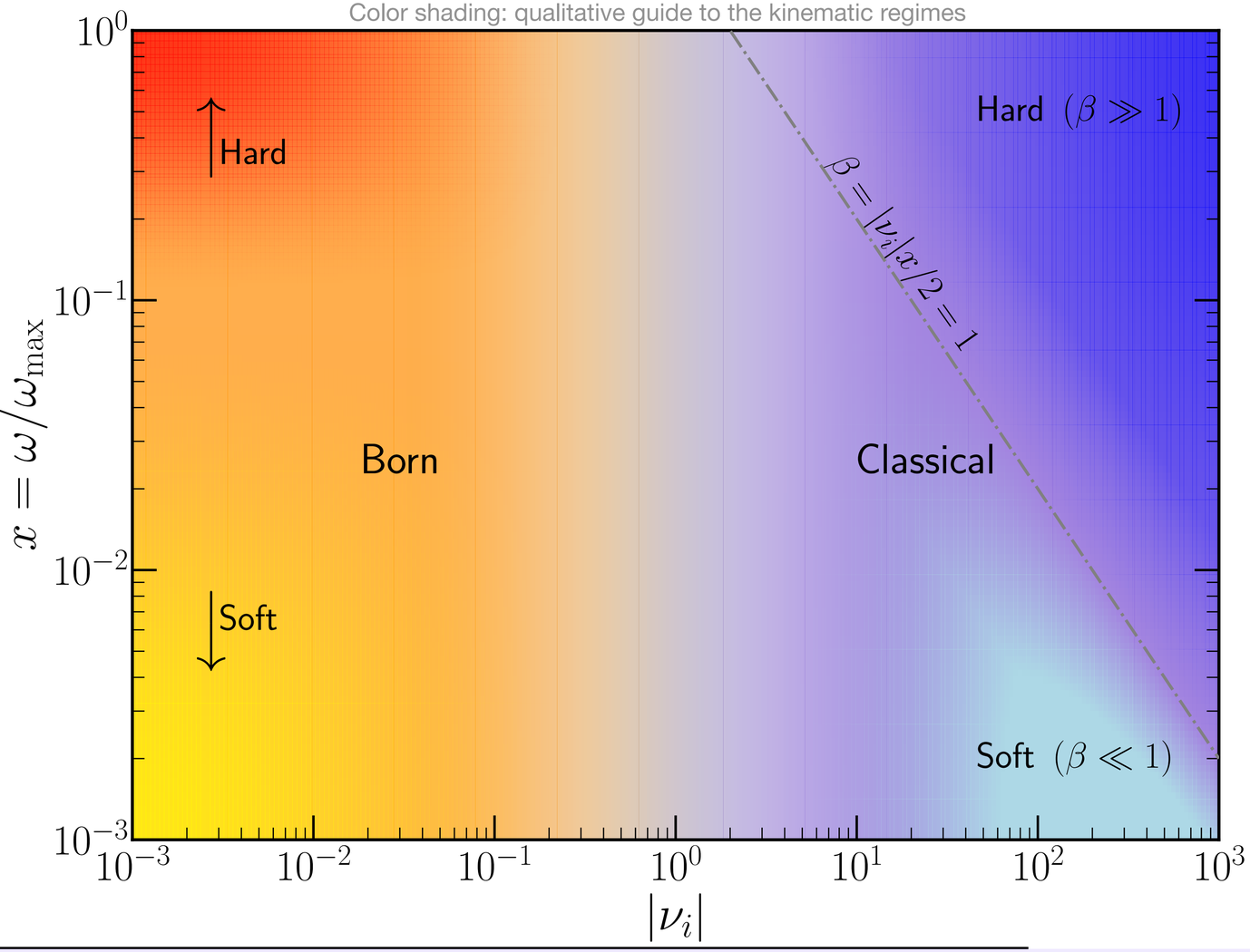
$$\mathcal{M}_{2\to 3}$$

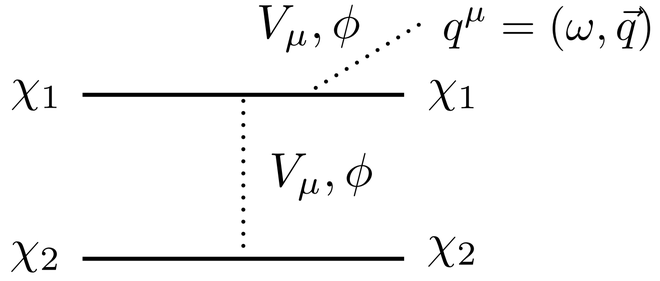
Multipole expansion: $v_i \ll 1$

V-emission and Coulomb potential: arXiv:2007.06592

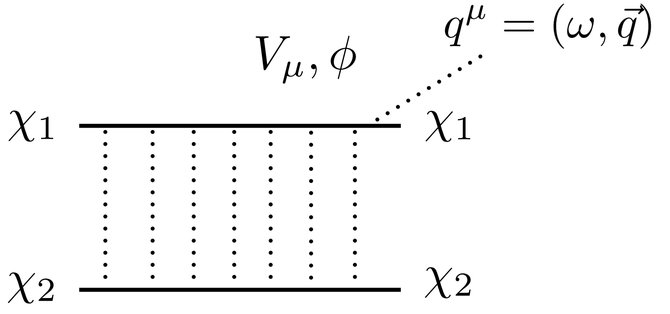
Bremsstrahlung: kinematic regimes

$$\mathcal{T}_{f(V,\phi),i} = -2\pi i \delta(E_i - E_f - \omega)\langle\psi_f|a(\vec{q},\lambda)V_{\rm int}|\psi_i\rangle$$





$|\nu_{i,f}| \sim 1$: Intermediate regime, need the full Coulomb state

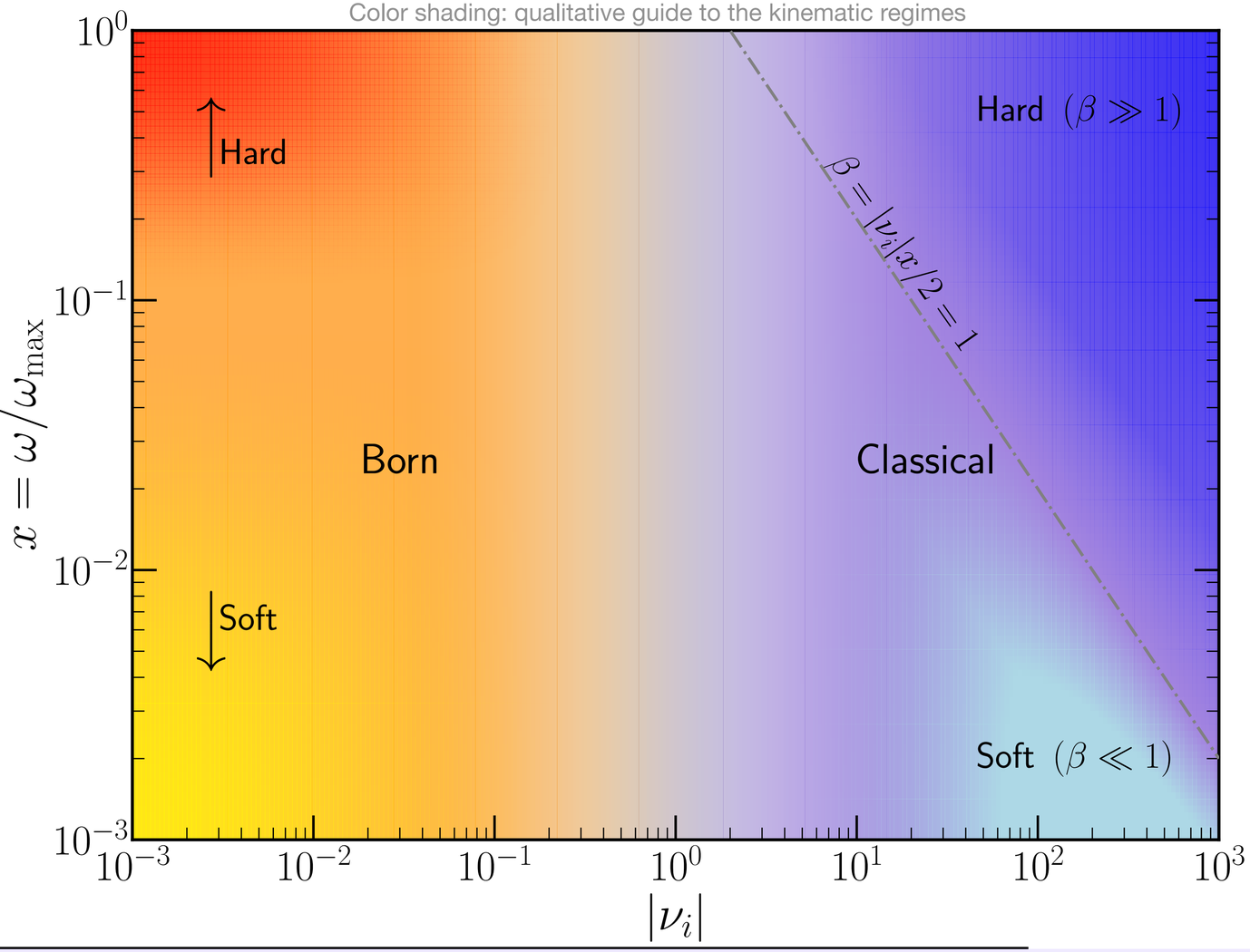


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Bremsstrahlung: kinematic regimes

arXiv:26XX.XXXX

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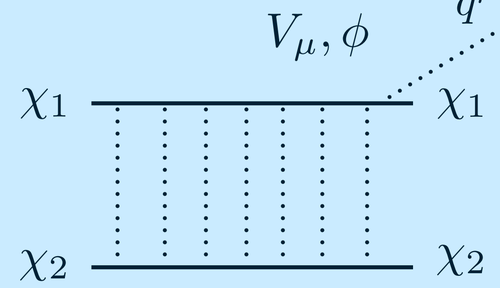


Particles moving along a classical path

I → Retarded potentials: $\vec{A}, \phi$

Quasi-Classical

$|\nu_{i,f}| \gg 1$:



V-emission and Coulomb potential: arXiv:2007.06592

Bremsstrahlung: multipole expansion

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Long range

Non-relativistic scattering:

$$V(r) = \frac{a_1 a_2}{4\pi r}$$

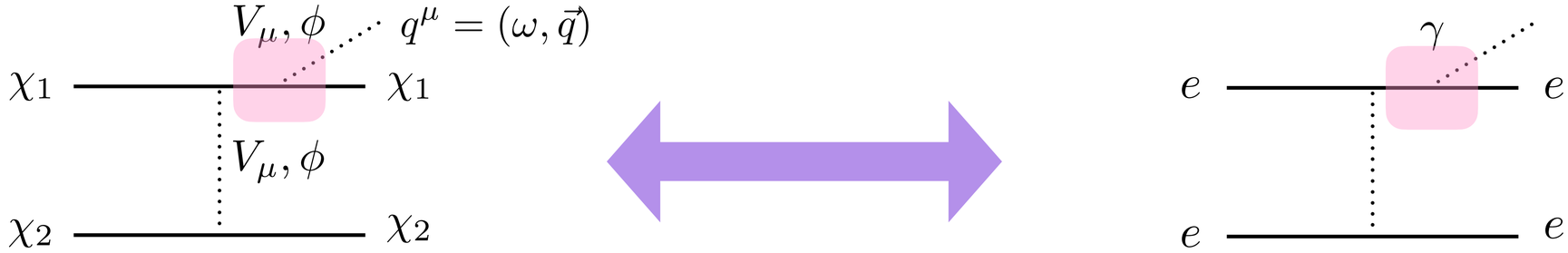
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Model-dependent:
- nature of the emitted particle
- DM candidates

Textbook discussion: Lectures on Quantum Mechanics, Steven Weinberg
V-emission: arXiv:2007.06592v3, ϕ-emission: arXiv:2303.00778v2

Bremsstrahlung: multipole expansion

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$$\phi(\vec{x}_n),\ \vec{A}(\vec{x}_n)$$

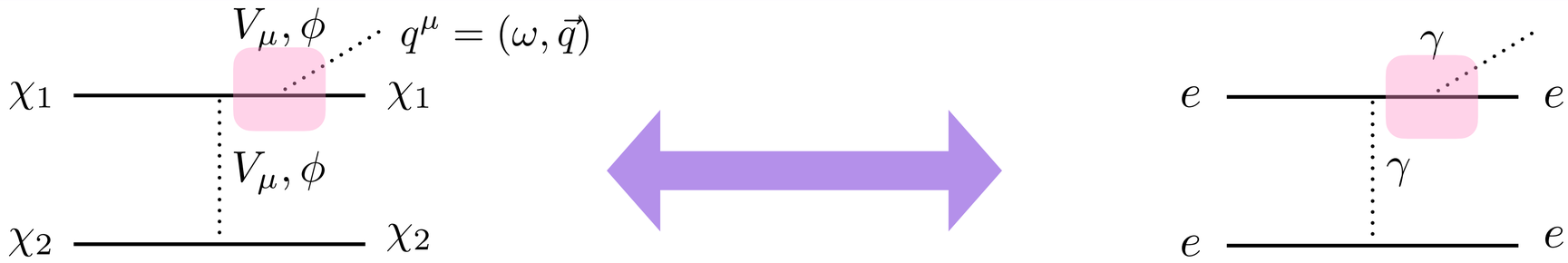
$$\mathcal{T}_{fi} \propto \frac{a_1}{\sqrt{2\omega}} \left\langle \varphi_{\bar{f}} \left| e^{-i\mu\vec{r}\cdot\vec{q}/m_1} \mathcal{O}_1 \right| \varphi_i \right\rangle + \frac{a_2}{\sqrt{2\omega}} \left\langle \varphi_{\bar{f}} \left| e^{i\mu\vec{r}\cdot\vec{q}/m_2} \mathcal{O}_2 \right| \varphi_i \right\rangle$$

$$\vec{r}\cdot\vec{q} \sim v_i \ll 1 \qquad e^{i\vec{q}\cdot\vec{r}} = 1 + i\vec{q}\cdot\vec{r} - \frac{1}{2}(\vec{q}\cdot\vec{r})^2 + \cdots$$

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Bremsstrahlung: multipole expansion

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$$\phi(\vec{x}_n),\ \vec{A}(\vec{x}_n)$$

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$$\vec{r}\cdot\vec{q} \sim v_i \ll 1 \qquad e^{i\vec{q}\cdot\vec{r}} = 1 + i\vec{q}\cdot\vec{r} - \frac{1}{2}(\vec{q}\cdot\vec{r})^2 + \cdots$$

$$\mathcal{T}_{fi} = -i(2\pi)^4 \delta(E_i - E_f - \omega)\delta^{(3)}(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4 - \vec{q})\mathcal{M}$$

Dipole Quadrupole

$$\mathcal{M} \simeq \left(\frac{a_1}{m_1} - \frac{a_2}{m_2}\right)\langle f|\mathcal{O}_{\mathcal{D}}|i\rangle + \left(\frac{a_1}{m_1^2} + \frac{a_2}{m_2^2}\right)\langle f|\mathcal{O}_{\mathcal{Q}}|i\rangle$$

Quadrupole emission amplitude is **suppressed** by $\vec{r}\cdot\vec{q} \sim v_i$ compared to dipole

Textbook discussion: Lectures on Quantum Mechanics, Steven Weinberg
V-emission: arXiv:2007.06592v3, ϕ-emission: arXiv:2303.00778v2

Born

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Hidden DM sector

DM candidates
- Fermion
- Scalar

$\chi_1\chi_2 \to \chi_1\chi_2 V \quad \tilde{\chi}_1\tilde{\chi}_2 \to \tilde{\chi}_1\tilde{\chi}_2\phi \quad \chi_1\chi_2 \to \chi_1\chi_2\phi$

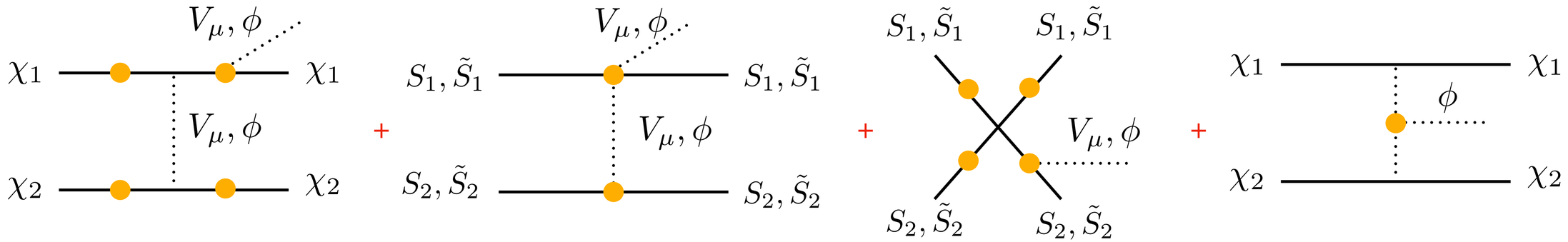
$S_1 S_2 \to S_1 S_2 V \quad \tilde{S}_1\tilde{S}_2 \to \tilde{S}_1\tilde{S}_2\phi \quad S_1 S_2 \to S_1 S_2\phi$

Emission/Mediation:
- Gauge vector
- Scalar

Including massive emission

Long-range

$\mathcal{M}_{2\to 3} =$



Finite but large set of dissipative channels to evaluate

Born: Dipole vs Quadrupole emission

JCAP03(2026)034

In DM halos: $v_i \lesssim 100 - 1000\,\mathrm{km.s}^{-1}$

$p_i/\mu \sim v_i, \omega/\mu \sim v_i^2 \ll 1$

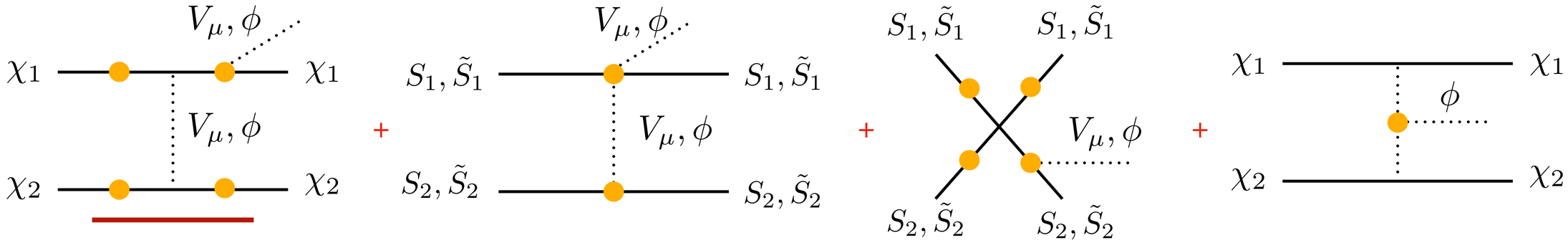
Separation of scales

$$\mathcal{M}_{2\to3} \simeq \mathcal{M}_{\mathcal{O}(v_i^{-n})} + \mathcal{M}_{\mathcal{O}(v_i^{-n+1})} + \dots$$

Born: Dipole vs Quadrupole emission

JCAP03(2026)034

$\mathcal{M}_{2\to3} =$



$$\mathcal{M}_{2\to3} \simeq \mathcal{M}_{\mathcal{O}(v_i^{-n})} + \mathcal{M}_{\mathcal{O}(v_i^{-n+1})} + \dots$$

$$\langle |\mathcal{M}_{\mathcal{D}}|^2 \rangle_{\hat{\mathbf{q}}} \propto a_1^2 a_2^2 \left(\frac{a_1}{m_1} - \frac{a_2}{m_2} \right)^2 \mathcal{H}(v_{i,f}, \omega)$$

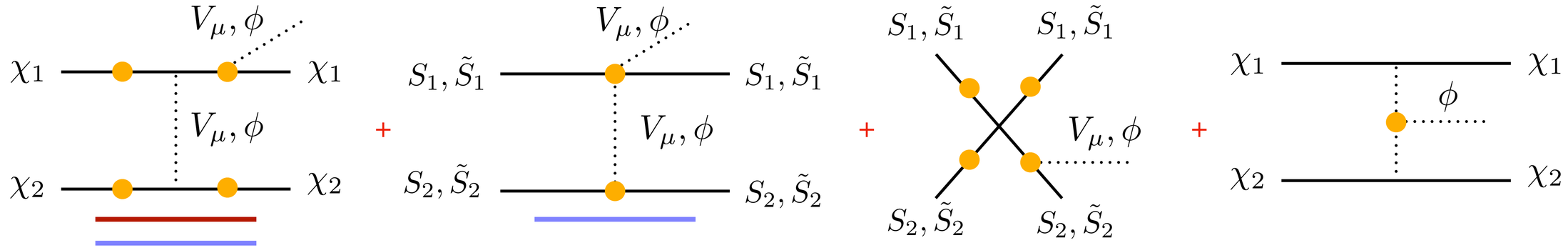
Dipole

Model-independent
Dependencies on the emission type and DM candidates are v_i-suppressed

Born: Dipole vs Quadrupole emission

JCAP03(2026)034

$\mathcal{M}_{2\to3} =$



$$\mathcal{M}_{2\to3} \simeq \mathcal{M}_{\mathcal{O}(v_i^{-n})} + \mathcal{M}_{\mathcal{O}(v_i^{-n+1})} + \dots$$

$$\langle|\mathcal{M}_{\mathcal{D}}|^2\rangle_{\hat{\mathbf{q}}} \propto a_1^2 a_2^2 \left(\frac{a_1}{m_1} - \frac{a_2}{m_2}\right)^2 \mathcal{H}(v_{i,f}, \omega)$$

$$\langle|\mathcal{M}_{\mathcal{Q}}|^2\rangle_{\hat{\mathbf{q}}} \propto a_1^2 a_2^2 \left(\frac{a_1}{m_1^2} + \frac{a_2}{m_2^2}\right)^2 \mathcal{G}(\alpha, \beta, v_{i,f}, \omega)$$

Dipole

Model-independent
Dependencies on the emission type and DM candidates are v_i-suppressed

$\chi_1\chi_2 \to \chi_1\chi_2 V$

$S_1 S_2 \to S_1 S_2 V$

$\tilde{S}_1\tilde{S}_2 \to \tilde{S}_1\tilde{S}_2\phi$

$S_1 S_2 \to S_1 S_2 \phi$

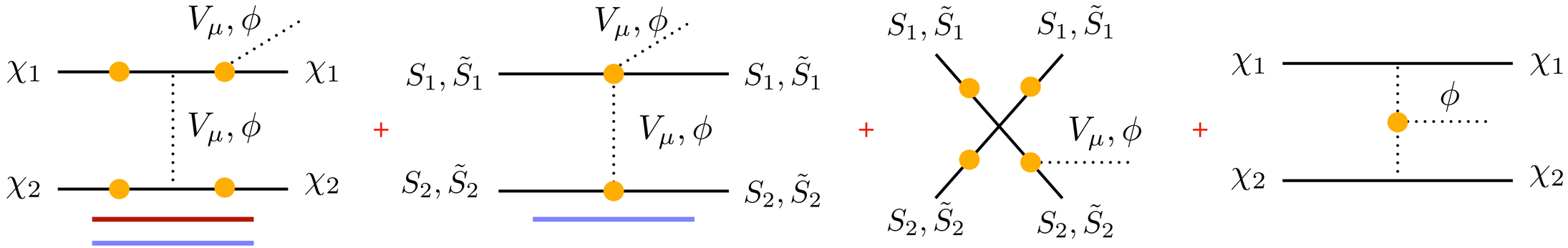
$\chi_1\chi_2 \to \chi_1\chi_2\phi$

$\tilde{\chi}_1\tilde{\chi}_2 \to \tilde{\chi}_1\tilde{\chi}_2\phi$

Born: Dipole vs Quadrupole emission

JCAP03(2026)034

$$\mathcal{M}_{2\to3} =$$



$$\mathcal{M}_{2\to3} \simeq \mathcal{M}_{\mathcal{O}(v_i^{-n})} + \mathcal{M}_{\mathcal{O}(v_i^{-n+1})} + \dots$$

$$\langle|\mathcal{M}_{\mathcal{D}}|^2\rangle_{\hat{\mathbf{q}}} \propto a_1^2 a_2^2 \left(\frac{a_1}{m_1} - \frac{a_2}{m_2}\right)^2 \mathcal{H}(v_{i,f}, \omega)$$

$$\langle|\mathcal{M}_{\mathcal{Q}}|^2\rangle_{\hat{\mathbf{q}}} \propto a_1^2 a_2^2 \left(\frac{a_1}{m_1^2} + \frac{a_2}{m_2^2}\right)^2 \mathcal{G}(\alpha, \beta, v_{i,f}, \omega)$$

Dipole

Model-independent
Dependencies on the emission type and DM candidates are v_i-suppressed

Multipole expansion
Compact analytical expression

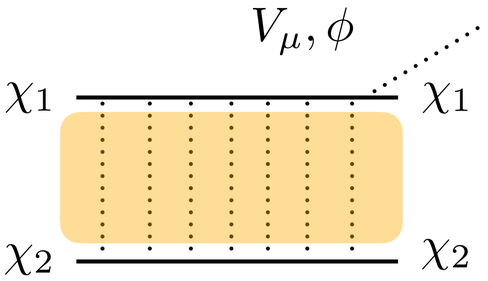
Quadrupole

Model-dependent
Spin effects and annihilation channels are v_i-suppressed, yielding only three distinct expressions

Beyond Born

To appear soon: arXiv:26XX.XXXX

Initial and final states are known:

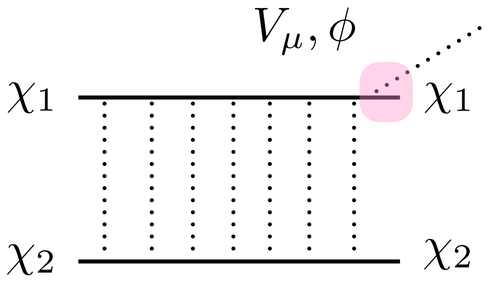


$$\chi^{(+)}_{\vec{p}_i}(\vec{r}) = \Gamma(1-i\nu_i)e^{\frac{\pi}{2}\nu_i}e^{i\vec{p}_i\cdot\vec{r}}{}_1F_1\left(i\nu_i, 1; i\zeta_i^-\right),\ \chi^{(-)}_{\vec{p}_f}(\vec{r}) = \Gamma(1+i\nu_f)e^{\frac{\pi}{2}\nu_f}e^{i\vec{p}_f\cdot\vec{r}}{}_1F_1\left(-i\nu_f, 1; -i\zeta_f^+\right)$$

Beyond Born

To appear soon: arXiv:26XX.XXXX

Initial and final states are known:



$$\chi^{(+)}_{\vec{p}_i}(\vec{r}) = \Gamma(1-i\nu_i)e^{\frac{\pi}{2}\nu_i}e^{i\vec{p}_i\cdot\vec{r}}{}_1F_1\left(i\nu_i, 1; i\zeta_i^-\right), \; \chi^{(-)}_{\vec{p}_f}(\vec{r}) = \Gamma(1+i\nu_f)e^{\frac{\pi}{2}\nu_f}e^{i\vec{p}_f\cdot\vec{r}}{}_1F_1\left(-i\nu_f, 1; -i\zeta_f^+\right)$$

Emission operators are obtained from the full relativistic theory and are model dependent:

Dipole Quadrupole

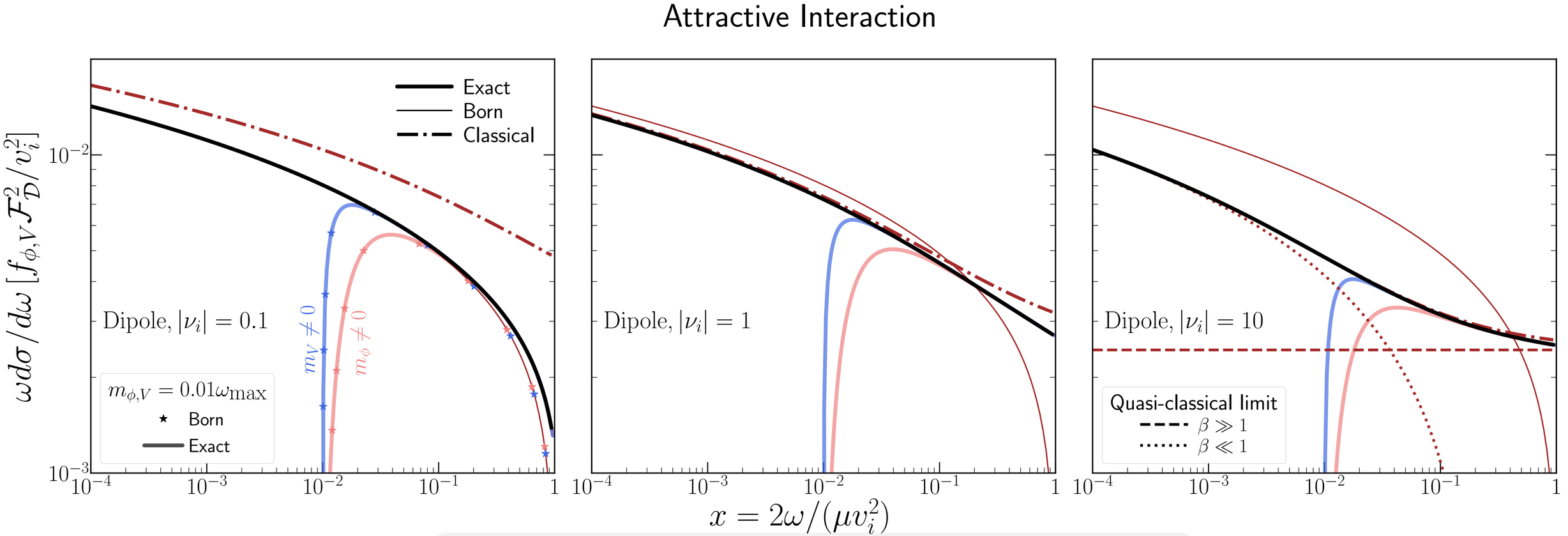
$$\mathcal{M} \simeq \left(\frac{a_1}{m_1} - \frac{a_2}{m_2}\right)\langle f|\mathcal{O}_{\mathcal{D}}|i\rangle + \left(\frac{a_1}{m_1^2} + \frac{a_2}{m_2^2}\right)\langle f|\mathcal{O}_{\mathcal{Q}}|i\rangle$$

Model-independent **Model-dependent**

$$\omega\frac{d\sigma_I}{d\omega} = \frac{\mu^2\omega^3}{4\pi^3}\sqrt{1-\frac{m_{\phi,V}^2}{\omega^2}}\sqrt{1-\frac{2\omega}{\mu v_i^2}}\int d\cos\theta\langle|\mathcal{M}_I|^2\rangle_{\vec{\tilde{q}}}$$

Beyond Born: energy-differential cross section

To appear soon: arXiv:26XX.XXXX



$$\omega\frac{d\sigma}{d\omega}\bigg|_{\text{Exact}} = \frac{1}{24\pi^3 v_i^2}\mathcal{F}_D^2 S_i S_f \frac{z_{\min}}{4\nu_i\nu_f}\frac{d}{dz_{\min}}\,|{}_2F_1(i\nu_f, i\nu_i; 1; z_{\min})|^2$$

$|\nu_i| \ll 1$

$$\omega\frac{d\sigma}{d\omega}\bigg|_{\text{Born}} = \frac{\mathcal{F}_{\mathcal{D}}^2}{24\pi^3 v_i^2}\ln\left(\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}}\right)$$

Independent calculations

$|\nu_i| \gg 1$

$$\omega\frac{d\sigma}{d\omega}\bigg|_{\text{classical}} = \frac{\mathcal{F}_{\mathcal{D}}^2}{96\pi v_i^2}\beta|H_{i\beta}^{(1)}(i\beta)|H_{i\beta}^{(1)\prime}(i\beta)$$

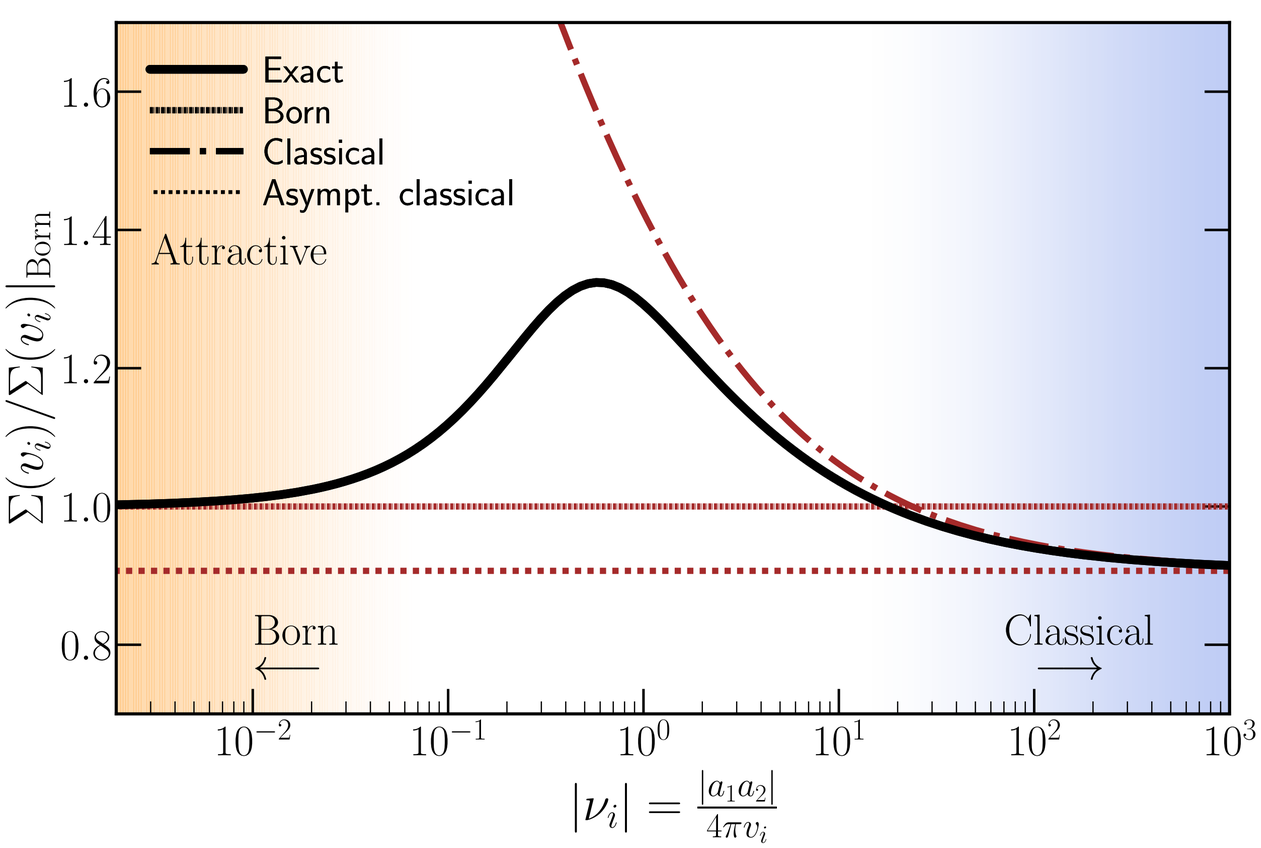
V-emission: ArXiv2007.06592

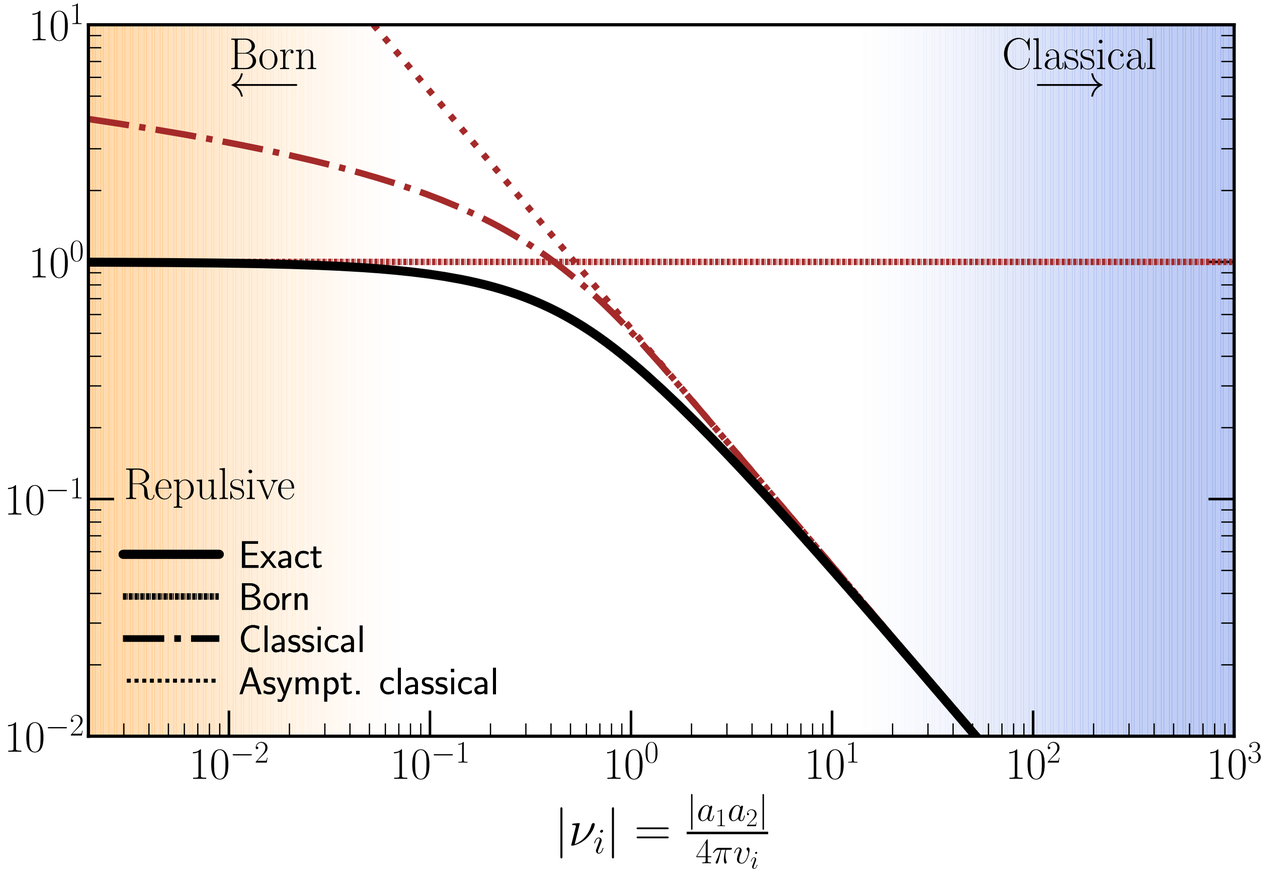
Beyond Born: effective energy loss rate

To appear soon: arXiv:26XX.XXXX

$$\frac{\dot{\epsilon}}{n_1 n_2} \propto \frac{\mu^{3/2}}{T^{3/2}} \int_{v_{\rm min}}^{\infty} dv_i \, f(v_i) \int_{m_{\phi,V}}^{\omega_{\rm max}} d\omega \, \omega \frac{d\sigma}{d\omega} \rightarrow \Sigma(v_i)$$

Dipole emission



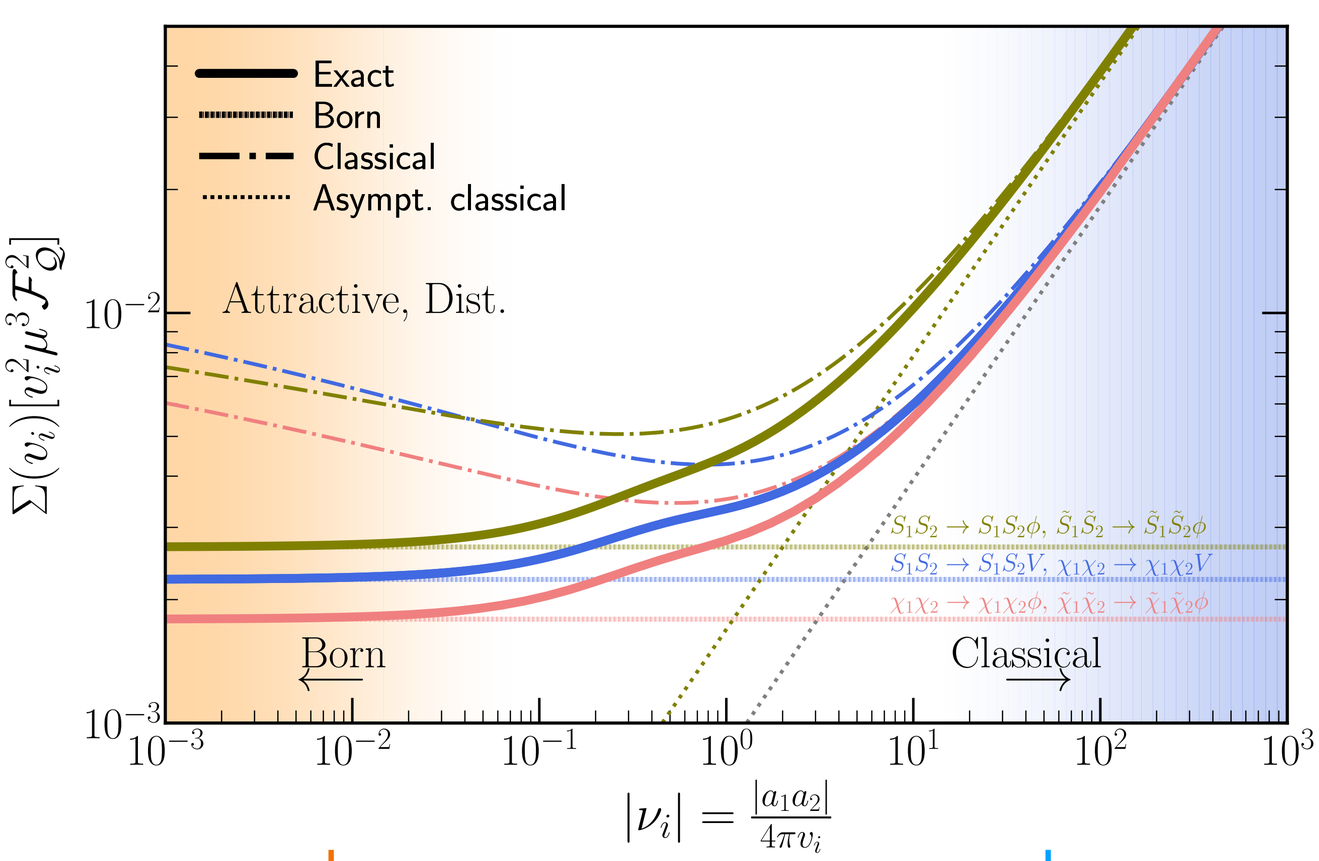


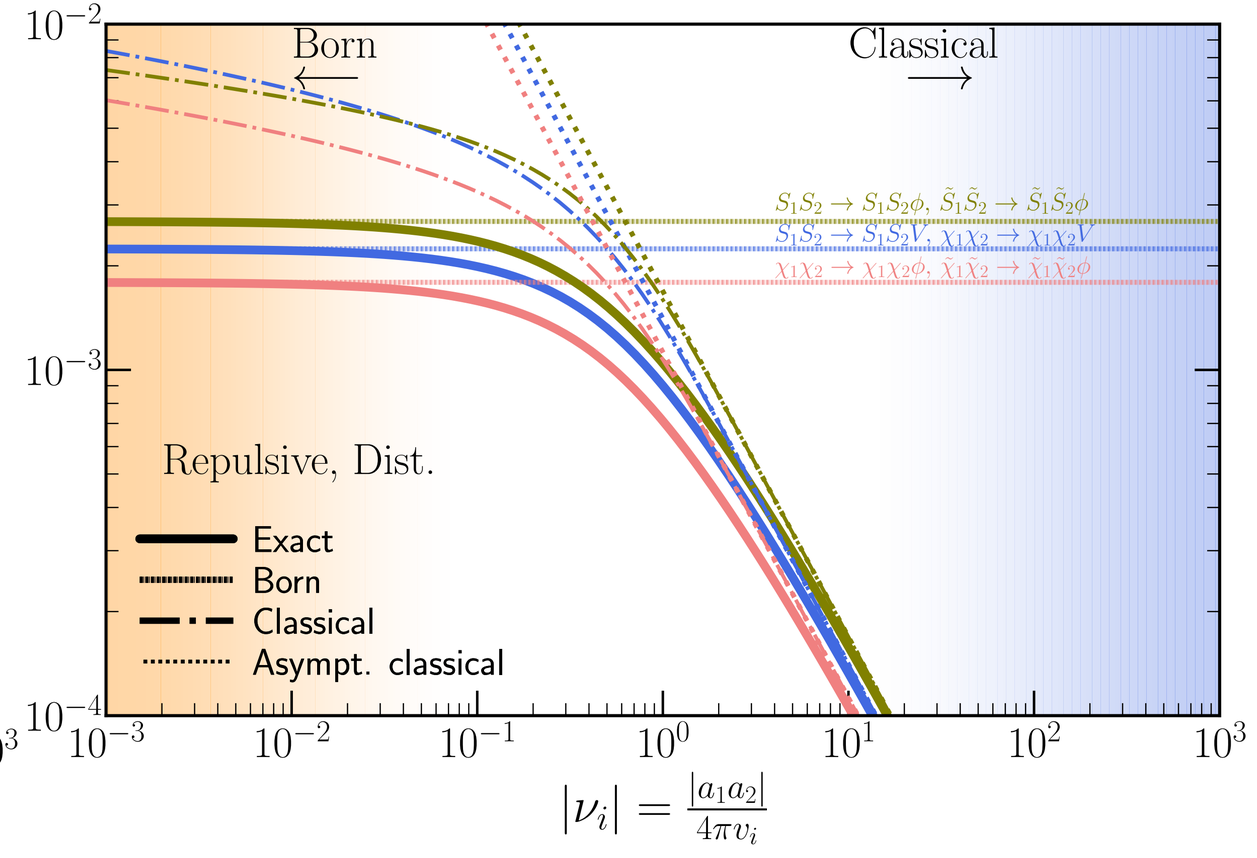
Beyond Born: effective energy loss rate

To appear soon: arXiv:26XX.XXXX

$$\Sigma(v_i) = \int_{m_{\phi,V}}^{\mu v_i^2/2} d\omega\, \omega \frac{d\sigma}{d\omega}$$

Quadrupole emission: distinguishable particle scattering





$$\frac{\Sigma(v_i)_\mathcal{Q}}{\Sigma(v_i)_\mathcal{D}} \propto v_i^2$$

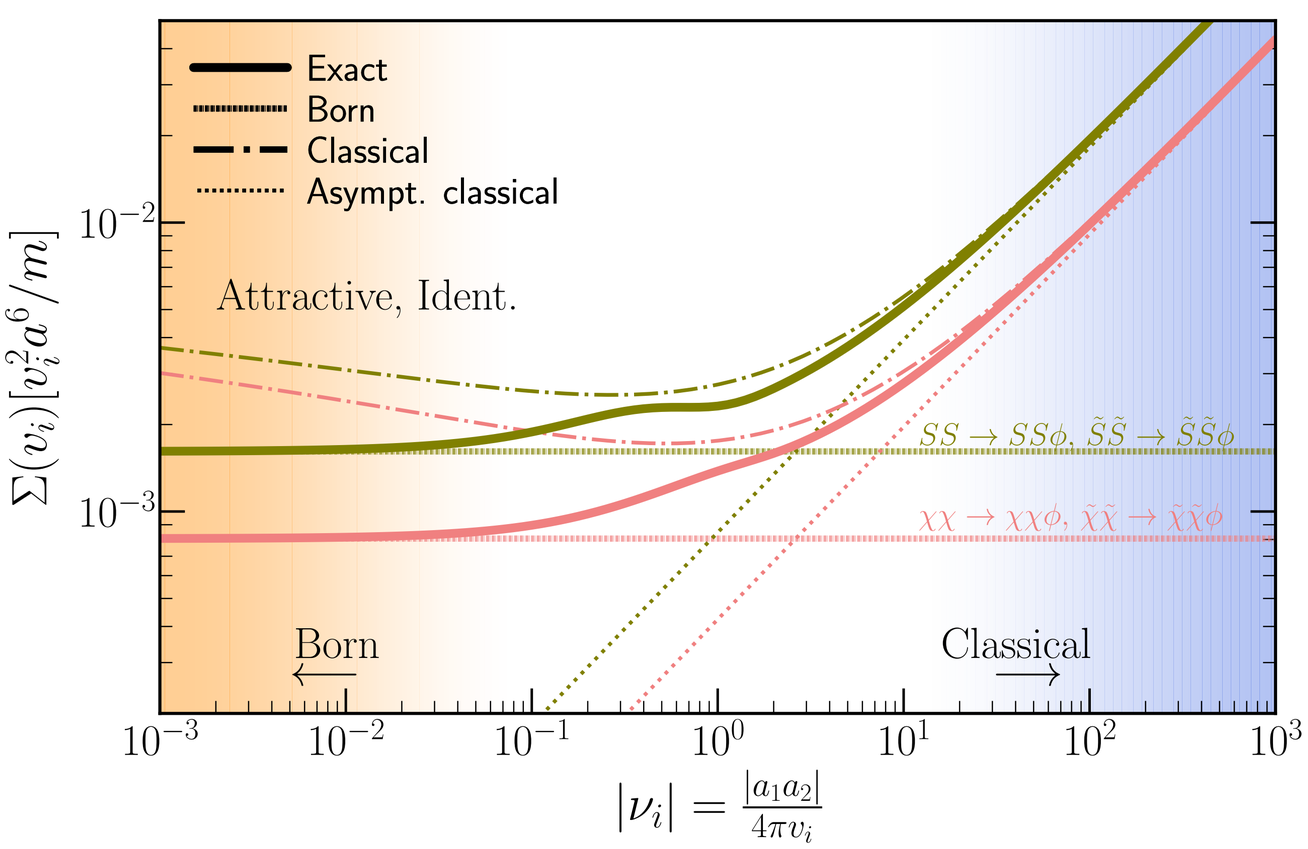
$$\frac{\Sigma(v_i)_\mathcal{Q}}{\Sigma(v_i)_\mathcal{D}} \propto v_i^2 |\nu_i|^{2/3}$$

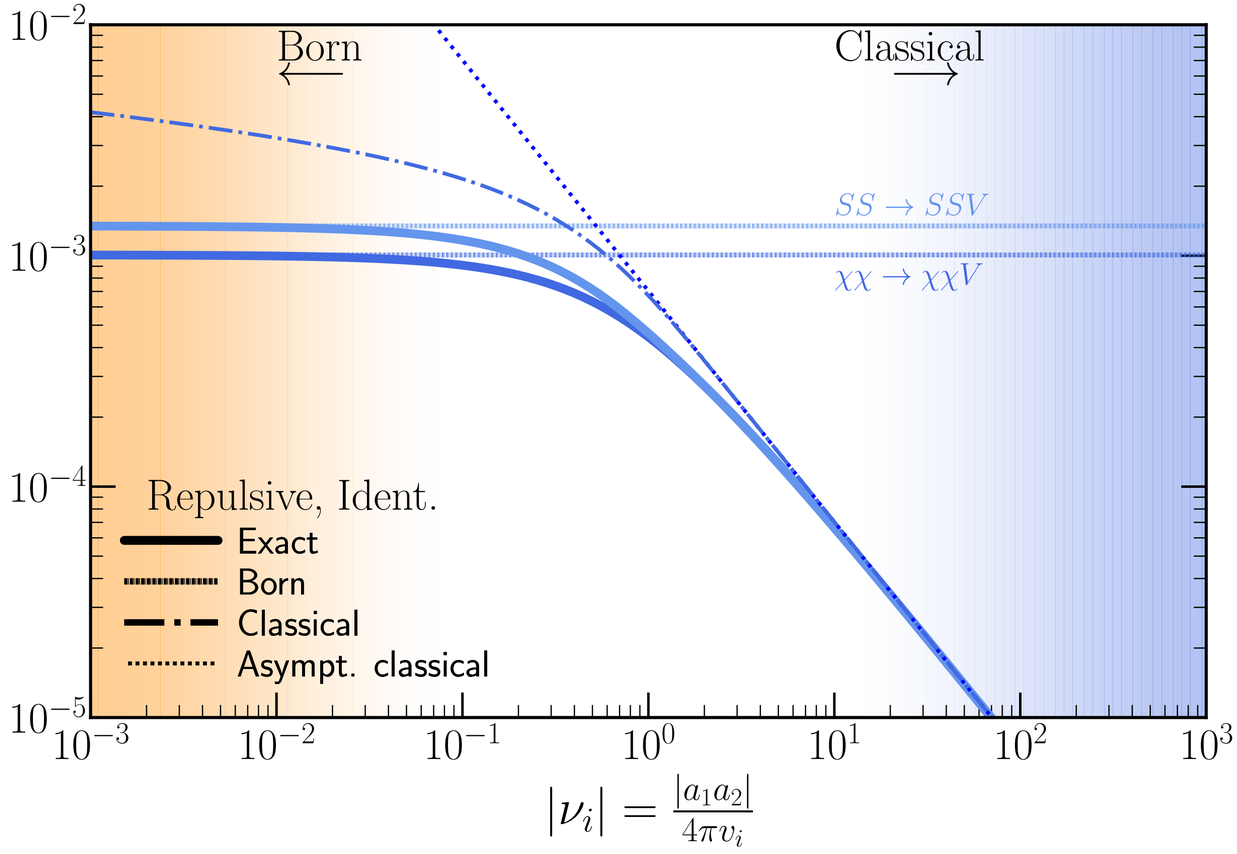
$$\frac{\Sigma(v_i)_\mathcal{Q}}{\Sigma(v_i)_\mathcal{D}} \propto v_i^2$$

Beyond Born: effective energy loss rate

To appear soon: arXiv:26XX.XXXX

Quadrupole emission: Identical particle scattering





Summary

arXiv:2509.12317

We provide a ready-to-use catalogue of energy-differential cross sections and energy-loss rates for representative dSIDM models

$\dot{\epsilon}$	**Gauge emission V $(f_{\phi,V}=2)$**		**Scalar emission ϕ $(f_{\phi,V}=1)$**	
dSIDM	$\chi_1\chi_2\to\chi_1\chi_2V$	$S_1S_2\to S_1S_2V$	$\chi_1\chi_2\to\chi_1\chi_2\phi$, $\tilde{\chi}_1\tilde{\chi}_2\to\tilde{\chi}_1\tilde{\chi}_2\phi$	$S_1S_2\to S_1S_2\phi$, $\tilde{S}_1\tilde{S}_2\to\tilde{S}_1\tilde{S}_2\phi$
Dipole radiation				
Mediation				
Long-range	LO (5.6)			
Short-range	LO (5.8)			
Quadrupole radiation				
Distinguishable particle scattering $\{\alpha,\beta\}$				
Mediation				
Long-range: NLO (5.6)	{3/4, 13/4}		{1, 6}	{1, 11}
Short-range: NLO (5.8)	{3/4, 1/4}		{1, 7}	{1, 2}
Identical particle scattering				
Mediation				
Long-range	(5.14a)	(5.15a)	(5.14b)	(5.15b)
Short-range	(5.16a)	(5.17a)	(5.16b)	(5.17b)
Additional radiation processes				
Mediation				
Long-range	–	Contact: App. B.1	VIB: (B.8)	Vertex: (5.7), (5.15c) Contact: App. B.1 VIB: (B.8)
Short-range	–	–	VIB: (B.15)	Vertex: (5.9), (5.18) VIB: (B.15)

$\omega\,(d\sigma/d\omega)$	**Gauge emission V $(f_{\phi,V}=2)$**		**Scalar emission ϕ $(f_{\phi,V}=1)$**	
dSIDM	$\chi_1\chi_2\to\chi_1\chi_2V$	$S_1S_2\to S_1S_2V$	$\chi_1\chi_2\to\chi_1\chi_2\phi$, $\tilde{\chi}_1\tilde{\chi}_2\to\tilde{\chi}_1\tilde{\chi}_2\phi$	$S_1S_2\to S_1S_2\phi$, $\tilde{S}_1\tilde{S}_2\to\tilde{S}_1\tilde{S}_2\phi$
Dipole radiation				
Full spectrum (massive emission)				
Mediation				
Long-range	(3.28)			
Short-range	(3.32)			
Soft spectrum				
Mediation				
Long-range	(4.12b)			
Short-range	(4.12a)			
Quadrupole radiation				
Distinguishable particle scattering $\{\alpha,\beta\}$				
Full spectrum (massive emission)				
Mediation				
Long-range	(F.9)		(F.14)	(F.16)
Short-range	(F.19)		(F.22)	(F.24)
Soft spectrum				
Mediation				
Long-range: (4.26)	{3/4, 13/4}		{1, 6}	{1, 11}
Short-range: (4.25)	{3/4, 1/4}		{1, 7}	{1, 2}
Identical particle scattering				
Full spectrum (massive emission)				
Mediation				
Long-range	(3.43)	(3.46)	(3.47)	(3.48)
Short-range	Sec. 3.5.2	(3.50)	Sec. 3.5.2	(3.52)
Soft spectrum				
Long-range	(4.27a)	(4.27b)	(4.29)	(4.28)
Short-range			Sec. 4.4	
Additional radiation processes				
Mediation				
Long-range	–	Contact: App. B.1	VIB: (B.6), (B.10b)	Vertex: (3.37), (3.49) Contact: App. B.1 VIB: (B.6), (B.10a)
Short-range	–	–	VIB: (B.14)	Vertex: (3.42), (3.53) VIB: (B.14)

Summary

arXiv:2509.12317

We provide a ready-to-use catalogue of energy-differential cross sections and energy-loss rates for representative dSIDM models

$\dot{\epsilon}$	**Gauge emission V $(f_{\phi,V}=2)$**		**Scalar emission ϕ $(f_{\phi,V}=1)$**	
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Dipole radiation				
Mediation				
Long-range	LO (5.6)			
Short-range	LO (5.8)			
Quadrupole radiation				
Distinguishable particle scattering $\{\alpha,\beta\}$				
Mediation				
Long-range: NLO (5.6)	{3/4, 13/4}		{1, 6}	
Short-range: NLO (5.8)	{3/4, 1/4}		{1, 7}	
Identical particle scattering				
Mediation				
Long-range	(5.14a)	(5.15a)	(5.14b)	
Short-range	(5.16a)	(5.17a)	(5.16b)	
Additional radiation processes				
Mediation				
Long-range	–	Contact: App. B.1	VIB: (B.8)	Vertex: (5.7), (5.15c) Contact: App. B.1 VIB: (B.8)
Short-range	–	–	VIB: (B.15)	Vertex: (5.9), (5.18) VIB: (B.15)

Can be adapted to a wide range of coupling, mass, and mediator properties

A comprehensive catalogue of beyond Born results coming soon!

$\omega\,(d\sigma/d\omega)$	**Gauge emission V $(f_{\phi,V}=2)$**		**Scalar emission ϕ $(f_{\phi,V}=1)$**	
dSIDM	$\chi_1\chi_2\to\chi_1\chi_2V$	$S_1S_2\to S_1S_2V$	$\chi_1\chi_2\to\chi_1\chi_2\phi$, $\tilde{\chi}_1\tilde{\chi}_2\to\tilde{\chi}_1\tilde{\chi}_2\phi$	$S_1S_2\to S_1S_2\phi$, $\tilde{S}_1\tilde{S}_2\to\tilde{S}_1\tilde{S}_2\phi$
Dipole radiation				
Full spectrum (massive emission)				
Mediation				
Long-range	(3.28)			
Short-range	(3.32)			
Soft spectrum				
Mediation				
Long-range	(4.12b)			
Short-range	(4.12a)			
n				
scattering $\{\alpha,\beta\}$				
emission)				
	(F.9) (F.19)		(F.14) (F.22)	(F.16) (F.24)
Mediation				
Long-range: (4.26)	{3/4, 13/4} {3/4, 1/4}		{1, 6} {1, 7}	{1, 11} {1, 2}
ring				
emission)				
	(3.43) Sec. 3.5.2	(3.46) (3.50)	(3.47) Sec. 3.5.2	(3.48) (3.52)
Long-range	(4.27a)	(4.27b)	(4.29)	(4.28)
Short-range	Sec. 4.4			
Additional radiation processes				
Mediation				
Long-range	–	Contact: App. B.1	VIB: (B.6), (B.10b)	Vertex: (3.37), (3.49) Contact: App. B.1 VIB: (B.6), (B.10a)
Short-range	–	–	VIB: (B.14)	Vertex: (3.42), (3.53) VIB: (B.14)

Conclusion

DM could be self-interacting on small cosmic scales → Beyond elastic SIDM: **dissipation**

In this talk, a set of dSIDM models was investigated:

- Using a perturbative QFT and an exact QM treatment relevant in **all** the kinematic regimes, including classical emission

The different kinematic regimes highlight different aspects of the scattering dynamics:

- Born: identical vs distinguishable particle scattering
- Classical: attractive vs repulsive interaction
- Intermediate regime: non-trivial dependence on the Sommerfeld parameters

Reconstructed the dipole and quadrupole pieces of the emission amplitude:

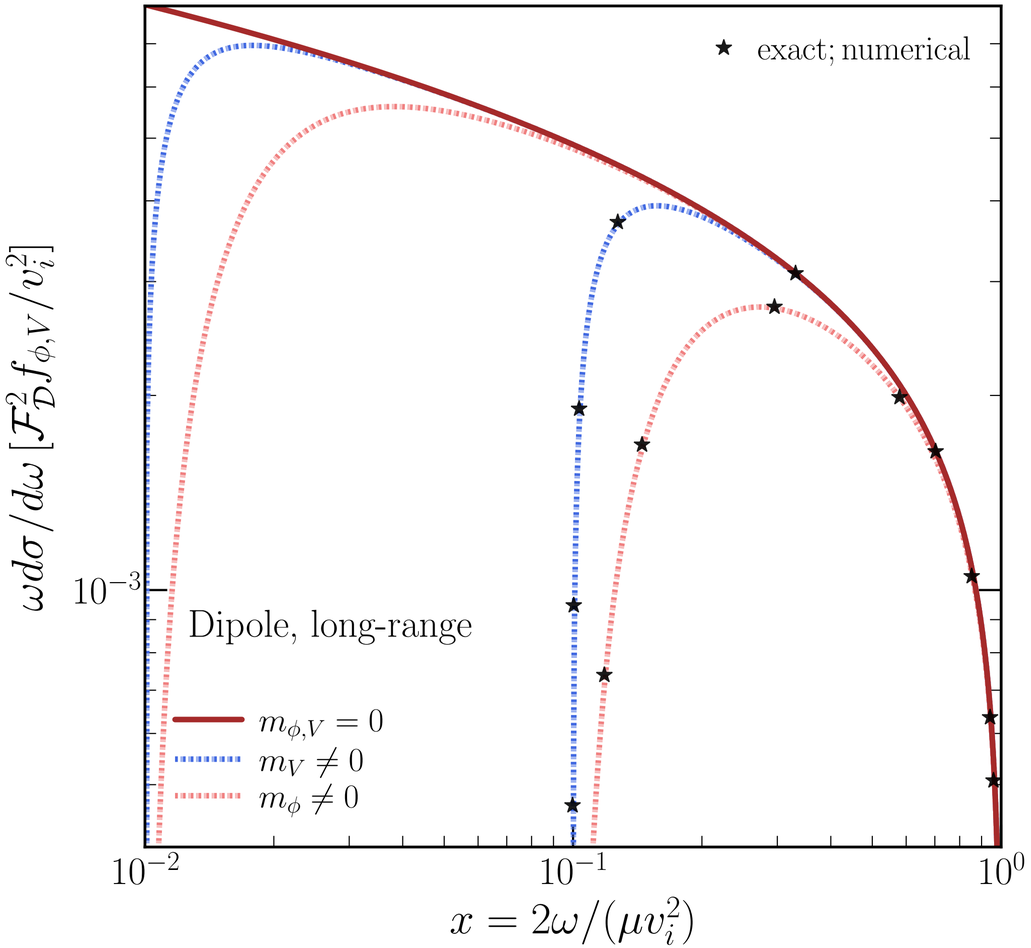
- Dipole radiation is universal, while quadrupole emission probes model-dependent relativistic corrections

Application: Constrain the parameter space of the SIDM model and the formation of small-scale structures

Born: energy-differential cross section

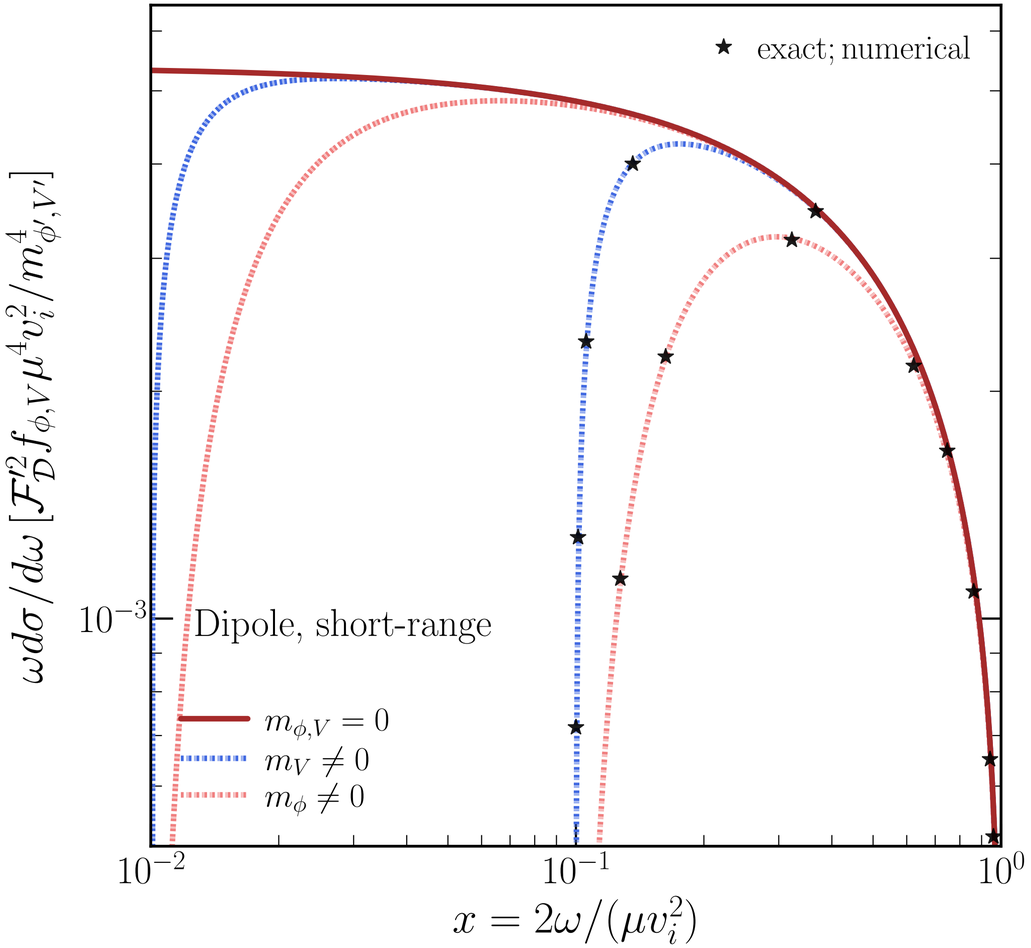
arXiv:2509.12317

Light mediator



$$\omega\frac{d\sigma}{d\omega} = f_{\phi,V}\mathcal{F}_{\mathcal{D}}^2\frac{1}{24\pi^3 v_i^2}\ln\left(\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}}\right)$$

Heavy mediator



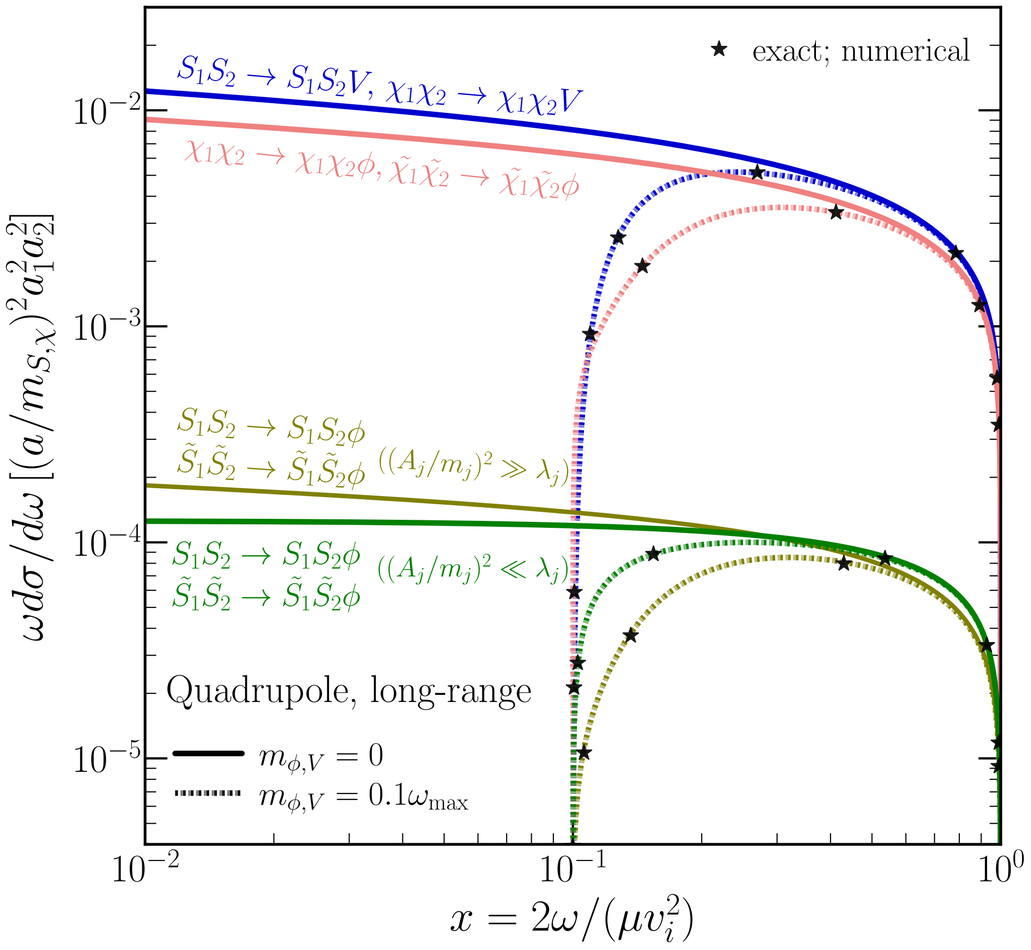
$$\omega\frac{d\sigma}{d\omega} = f_{\phi,V}\mathcal{F}_{\mathcal{D}}'^2\frac{\mu^4 v_i^2\sqrt{1-x}\,(2-x)}{12\pi^3 m_{\phi',V'}^4}$$

Born: energy differential cross-section

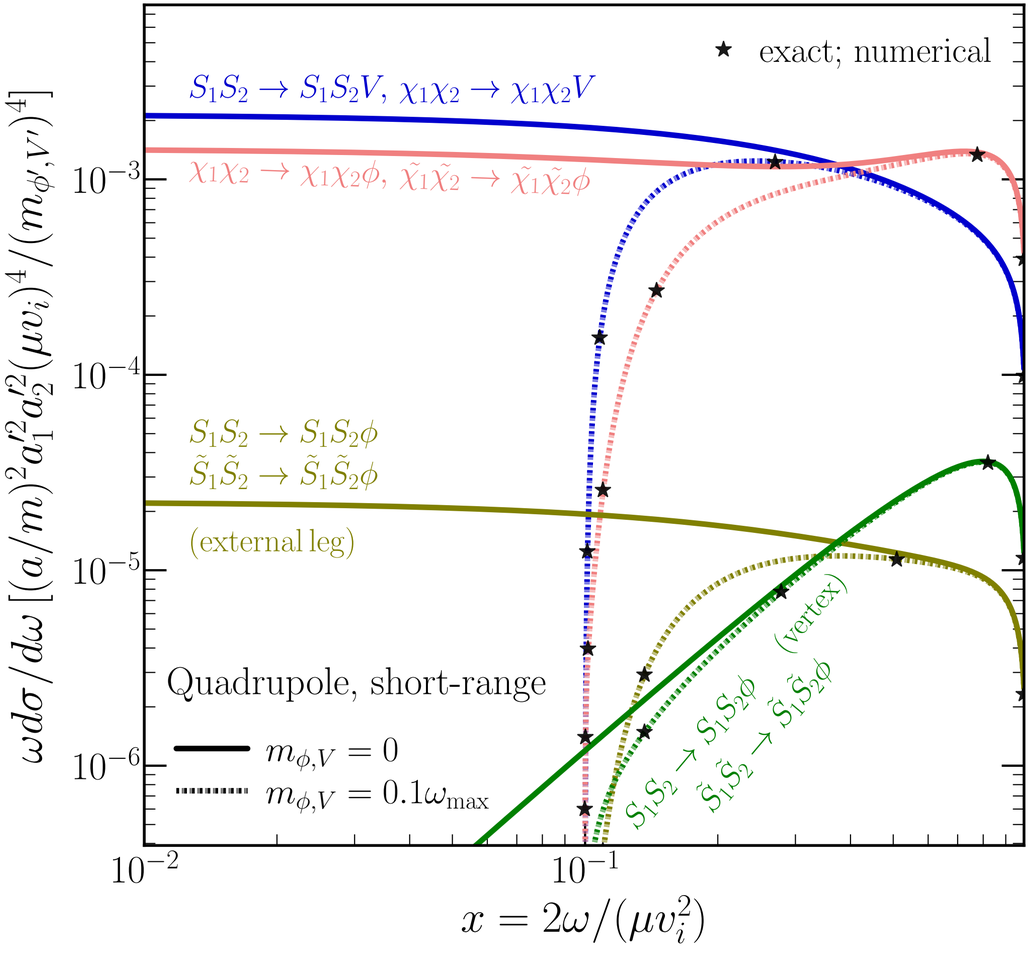
arXiv:2509.12317

Distinguishable particle scattering

Light mediator



Heavy mediator



$$\omega\frac{d\sigma}{d\omega} = f_{\phi,V}\mathcal{F}_{\mathcal{Q}}^2\frac{\mu^2}{60\pi^3}\left[(\beta-\alpha)\sqrt{1-x}+\alpha(2-x)\ln\left(\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}}\right)\right]$$

$$\omega\frac{d\sigma}{d\omega} = f_{\phi,V}\mathcal{F}_{\mathcal{Q}}'^2\frac{\mu^6 v_i^4\sqrt{1-x}}{180\pi^3 m_{\phi',V'}^4}\left[8\alpha'(1-x)+3x^2(\alpha'+\beta')\right]$$

Classical calculation

arXiv:26XX.XXXX

Particles moving along a classical path:

$$\vec{A} = \frac{1}{4\pi R_0} \int \vec{j}(t' + \vec{x} \cdot \hat{q})\, d^3x$$

Far from the source:
the radiated field acts as an
outgoing plane wave

$$\phi = \frac{1}{4\pi R_0} \int n_s(t' + \vec{x} \cdot \hat{q})\, d^3x$$

$v_i \ll c$ — Long wavelength approximation

Multipole expansion

$$\vec{A} \simeq \frac{1}{4\pi R_0} \mu \left(\frac{g_1}{m_1} - \frac{g_2}{m_2} \right) \partial_{t'} \vec{r} + \frac{1}{8\pi R_0} \mu^2 \left(\frac{g_1}{m_1^2} + \frac{g_2}{m_2^2} \right) \partial_{t'}^2 \left[\vec{r} (\vec{r} \cdot \hat{q}) \right] + \cdots$$

$$dI = \begin{matrix} |\partial_{t'} \vec{A} \times \hat{q}|^2 \\ |\partial_{t'} \phi|^2 \end{matrix} \Bigg\} \times R_0^2 d\Omega_{\hat{q}}$$

Observables are obtained from

$$\Sigma(v_i)|^{\text{tot}}_{\text{class.}} = \int_0^\infty d\rho\, 2\pi\rho \int_{-\infty}^{+\infty} dt'\, I$$

Landau & Lifshitz: the classical theory of fields

Energy loss rates

arXiv:2509.12317

Light mediator

$$\dot{\epsilon} = \frac{n_1 n_2 f_{\phi,V}}{6\pi^{7/2}\sqrt{2}} \left[\mathcal{F}_{\mathcal{D}}^2 (\mu T)^{1/2} + \frac{24}{45} \mathcal{F}_{\mathcal{Q}}^2 (\mu T)^{3/2} (4\alpha + \beta) \right]$$

$\chi_1 = \chi_2$

$$\dot{\epsilon} \propto g^6 \frac{n_\chi^2 T^{3/2}}{m_\chi^{5/2}}$$

Heavy mediator

$$\dot{\epsilon} = \frac{32\sqrt{2} n_1 n_2 f_{\phi,V}}{15 m_{\phi',V}^4 \pi^{7/2}} \left[\mathcal{F}_{\mathcal{D}}'^2 (\mu T)^{5/2} + \frac{8}{35} \mathcal{F}_{\mathcal{Q}}'^2 (\mu T)^{7/2} (8\alpha + \beta) \right]$$

$\chi_1 = \chi_2$

$$\dot{\epsilon} \propto g^2 g'^4 \frac{n_\chi^2 T^{7/2}}{m_\chi^{1/2} m_{\phi',V'}^4}$$

$$\frac{\dot{\epsilon}_{\text{Short}}}{\dot{\epsilon}_{\text{Long}}} \sim \frac{(\mu T)^2}{m_{\phi',V'}^4}$$

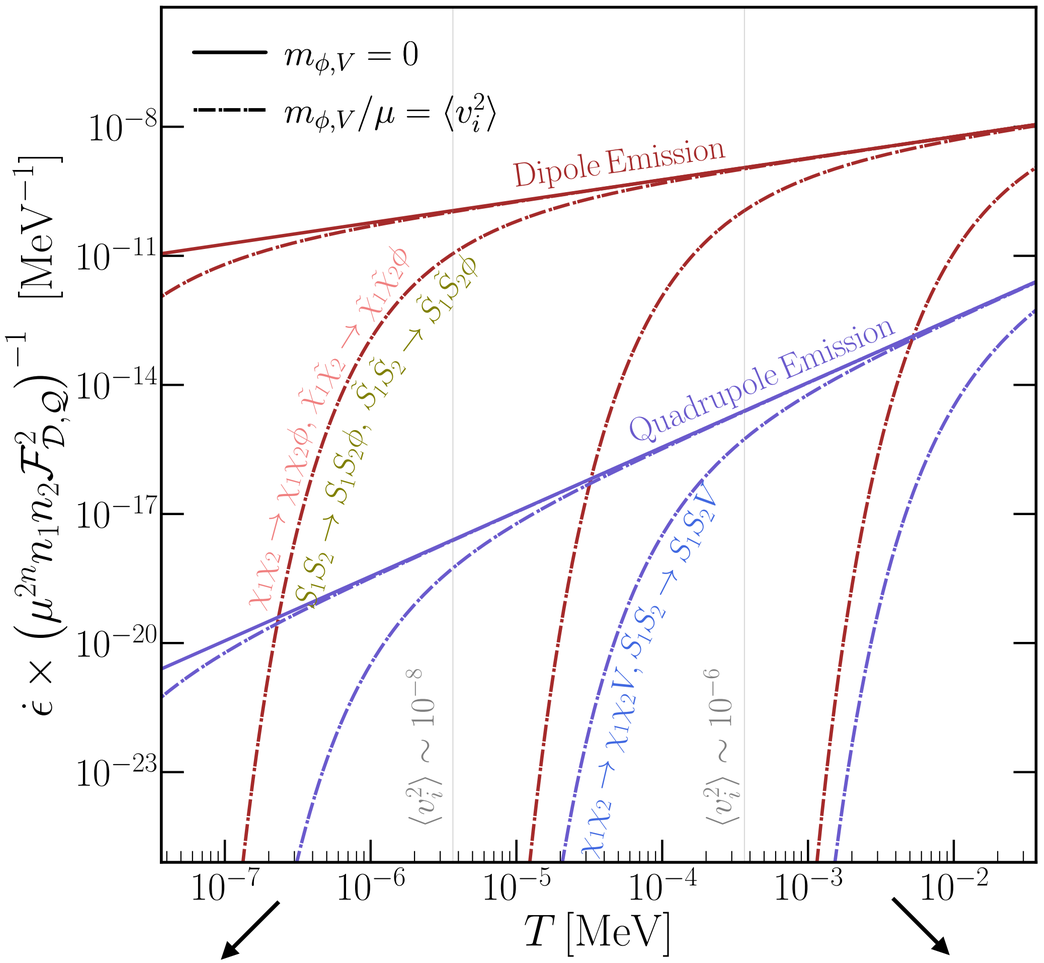
The long-range processes **dominate** the dissipative dynamics

At each multipole order and for each mediation regime, the differences between dSIDM scenarios remain **mild**: typically within **one to two orders of magnitude** for comparable couplings and masses

Energy loss rates

arXiv:2509.12317

Light mediator

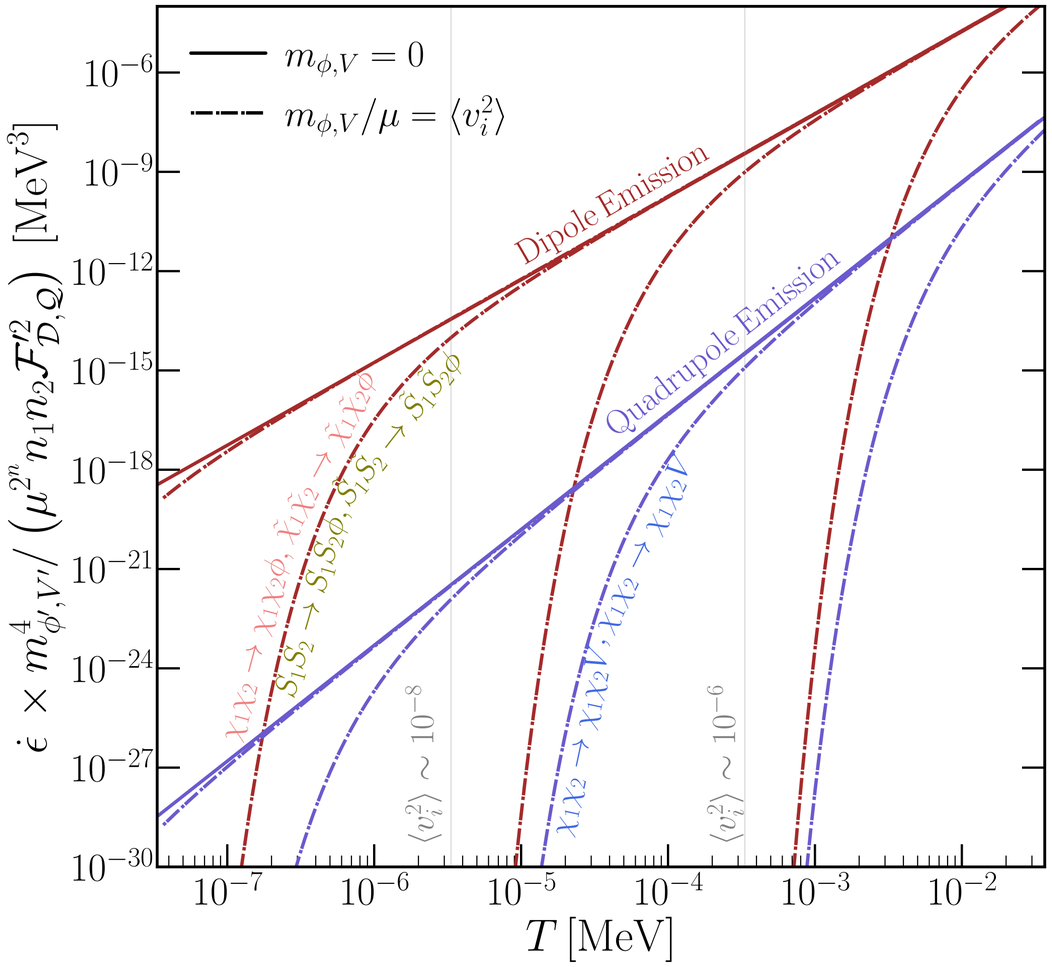


Dwarf galaxy

Galaxy Cluster

$$\frac{\dot{\epsilon}_{\rm Short}}{\dot{\epsilon}_{\rm Long}} \sim \frac{(\mu T)^2}{m^4_{\phi',V'}}$$

Heavy mediator

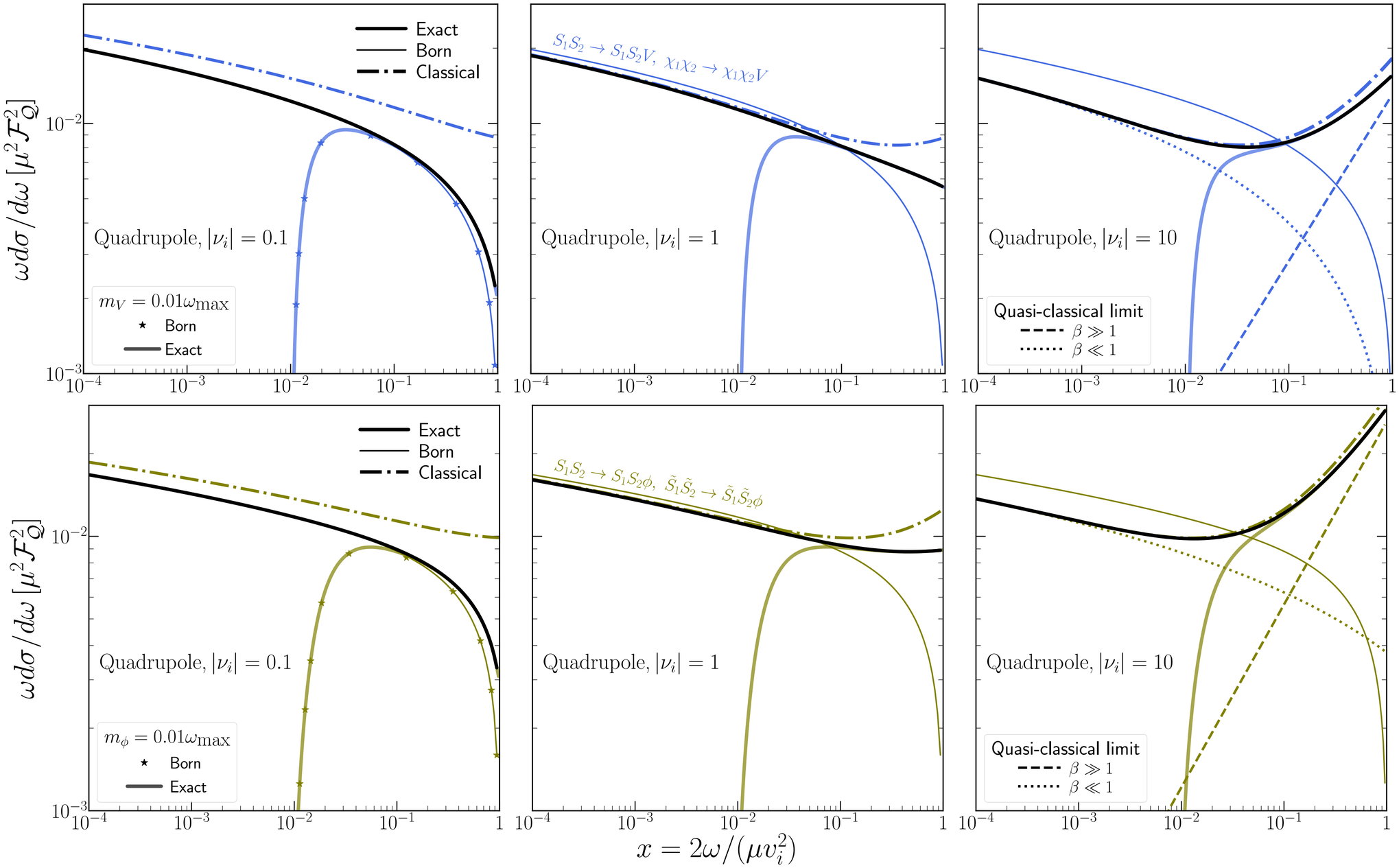


Massive emission naturally modulates the dissipative effects

Beyond Born: energy-differential cross section

arXiv:26XX.XXXX

Attractive Interaction



Beyond born: energy-differential cross section

arXiv:26XX.XXXX

