Phenomenology of Axion-Meson Mixing and Scattering Processes

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1. Introduction
2. Axion Lagrangian
3. ALP-meson mixing & ALP mass
4. Amplitudes parameterizations
5. Axion (& axion-mediated) decays
6. Conclusions and outlook

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Introduction

- QCD $\equiv$ QFT of strong interactions
- Includes topological θ term:

$$\mathcal{L}_{\text{QCD}} \supset \theta \frac{\alpha_s}{8\pi} G^{a,\mu\nu} \tilde{G}^a_{\mu\nu}$$

- Strong $\mathcal{CP}$ problem:

$$\overline{\theta} = \theta - \arg(\det(Y_u Y_d)) < 10^{-10}$$

BSM Solution: QCD axion (Peccei-Quinn mechanism)

$$\mathcal{L} \supset \left(\bar{\theta} - \frac{a}{f_a} \right) \frac{\alpha_s}{8\pi} G^{a,\mu\nu} \tilde{G}^a_{\mu\nu}$$

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Introduction

- **Motivation**
  - Effects of ALPs in low energy QCD
  - Formalism of axion-meson interaction in χPT ($\equiv$ Low energy EFT of QCD)

- **Objectives**
  - $\eta^{(\prime)}$ decays involving a virtual axion: $\eta^{(\prime)} \to \pi\pi a \to \pi\pi(\ell^+\ell^-, \gamma\gamma)\ldots$
  - Study of axion-meson interactions in the early universe
    - $\pi\pi \leftrightarrow \pi a$
    - $K\pi \leftrightarrow Ka$

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ALP-QCD Lagrangian

Axion-Like-Particle (ALP) Lagrangian:

$$\mathcal{L} \supset -\frac{1}{4} G^{a,\mu\nu} G^{a}_{\mu\nu} + \overline{q}(i\not{D} - \mathcal{M})q$$
$$- \frac{a}{f_a} \frac{\alpha_s}{8\pi} G^{a,\mu\nu} \tilde{G}^{a}_{\mu\nu} + \frac{1}{2}(\partial_\mu a)^2 - \frac{1}{2} m_0^2 a^2 - \frac{\partial^\mu a}{2 f_a} c_q \overline{q} \gamma_\mu \gamma_5 q$$

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$$- \frac{\alpha_{\text{em}}}{8\pi f_a} c_\gamma \, a F_{\mu\nu} \tilde{F}^{\mu\nu} - \sum_{\ell=e,\mu} \frac{c_\ell}{2 f_a} (\partial_\mu a) \, \bar{\ell} \gamma^\mu \gamma^5 \ell$$

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$$q \mapsto \exp\left(i\tfrac{a}{2f_a}\kappa\gamma_5\right) q$$

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Equivalent ALP Lagrangian:

$$\mathcal{L} \supset -\frac{1}{4} G^{a,\mu\nu} G^{a}_{\mu\nu} + \bar{q}(i\not{D} - \tilde{\mathcal{M}}(a))q + \frac{1}{2}(\partial_\mu a)^2 - \frac{1}{2}m_0^2 a^2 - \frac{\partial^\mu a}{2f_a}\bar{c}_q\bar{q}\gamma_\mu\gamma_5 q$$
$$- \frac{\alpha_{\text{em}}}{8\pi f_a}\bar{c}_\gamma\, a F_{\mu\nu}\tilde{F}^{\mu\nu} - \sum_{\ell=e,\mu} \frac{c_\ell}{2f_a}(\partial_\mu a)\,\bar{\ell}\gamma^\mu\gamma^5\ell$$

Chiral Perturbation Theory

Leading order Large-N_c χPT Lagrangian:

$$\mathcal{L} \supset \frac{F^2}{4}\langle (D_\mu \mathcal{U})(D^\mu \mathcal{U})^\dagger \rangle + \frac{F^2}{4}\langle \chi \mathcal{U}^\dagger + \chi^\dagger \mathcal{U} \rangle - \frac{1}{2} M_0^2 \eta_0^2$$

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ALP enters as an external source:

$$D_\mu \mathcal{U} = \partial_\mu \mathcal{U} - i\frac{\partial_\mu a}{2f_a}\{\bar{c}, \mathcal{U}\}$$

$$\chi = 2B_0 \exp\left(-i\frac{a}{f_a}\kappa\right)\mathcal{M} = 2B_0\hat{\mathcal{M}}(a)$$

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10 parameters: 5 from the hadronic sector and 5 from the ALP sector

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ALP-meson mixing

For the neutral fields, $\varphi = (a_0, \pi_3, \eta_8, \eta_0)^T$:

$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu \varphi)^T K (\partial^\mu \varphi) - \frac{1}{2}\varphi^T M^2_{\text{mix}} \varphi \neq \sum_i \frac{1}{2}(\partial_\mu \varphi_i)(\partial^\mu \varphi_i) - \frac{1}{2} m_i^2 \varphi_i^2$$

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$$\eta_8 = \cos(\theta_{\eta\eta'}) \left(\eta + \theta_{\pi\eta}\pi^0\right) + \sin(\theta_{\eta\eta'}) \left(\eta' + \theta_{\pi\eta'}\pi^0\right) + \gamma_{a\eta} a$$

$$\eta_0 = \cos(\theta_{\eta\eta'}) \left(\eta' + \theta_{\pi\eta'}\pi^0\right) - \sin(\theta_{\eta\eta'}) \left(\eta + \theta_{\pi\eta}\pi^0\right) + \gamma_{a\eta'} a$$

$$\pi_3 = \pi^0 - \theta_{\pi\eta}\eta - \theta_{\pi\eta'}\eta' + \gamma_{a\pi} a$$

$$a_0 = a + h_\pi \pi^0 + h_{\eta'}\eta' + h_\eta \eta$$

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Teal terms: $\mathcal{O}(F/f_a)$

ALP mass

Full $\mathcal{O}(F^2/f_a^2)$ calculation of the pseudoscalar masses yields a complete expression for the ALP mass:

$$m_a^2 = m_0^2 + m_{a,\text{QCD}}^2 \left(1 + R_0 + \sum_q R_q c_q + \sum_{qq'} R_{qq'} c_q c_{q'}\right)$$

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QCD axion mass:

$$m_{a,\text{QCD}}^2 = \left(\frac{F}{f_a}\right)^2 \frac{M_\pi^2}{4} \frac{M_K^2 - M_\pi^2/2}{M_K^2 - M_\pi^2/4} \left(1 + \frac{3M_\pi^2}{2M_0^2} \frac{M_K^2 - M_\pi^2/2}{M_K^2 - M_\pi^2/4}\right)^{-1}$$

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- $R_0,\ R_{u,d,s},\ R_{uu,dd,ss} \sim (10^{-4} - 10^{-5}) \frac{m_0^2}{1\ \text{MeV}^2}$
- $R_{ud} \sim 10^{-9} \frac{m_0^2}{1\ \text{MeV}^2},\ R_{us,ds} \sim 10^{-10} \frac{m_0^4}{1\ \text{MeV}^4}$

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Contributions to one ALP amplitudes

Full 4-body expansion of the LO Lagrangian:

$$\mathcal{L}^{(4)} = \frac{1}{48F^2}\langle[\Phi,\partial_\mu\Phi][\Phi,\partial^\mu\Phi]\rangle + \frac{1}{12Ff_a}(\partial_\mu a)\langle\bar{c}[\Phi,[\Phi,\partial^\mu\Phi]]\rangle +$$
$$+ \frac{1}{16f_a^2}(\partial_\mu a)(\partial^\mu a)\langle[\bar{c},\Phi][\bar{c},\Phi]\rangle +$$
$$+ \frac{B_0}{24F^2}\langle\mathcal{M}\Phi^4\rangle + \frac{B_0}{6Ff_a}\frac{a}{\langle\mathcal{M}^{-1}\rangle}\langle\Phi^3\rangle + \frac{B_0}{4f_a^2}\frac{a^2}{\langle\mathcal{M}^{-1}\rangle^2}\langle\mathcal{M}^{-1}\Phi^2\rangle +$$
$$+ \frac{B_0F}{6f_a^3}\frac{a^3}{\langle\mathcal{M}^{-1}\rangle^3}\langle(\mathcal{M}^{-1})^2\Phi\rangle + \frac{B_0F^2}{24f_a^4}\frac{a^4}{\langle\mathcal{M}^{-1}\rangle^4}\langle(\mathcal{M}^{-1})^3\rangle$$

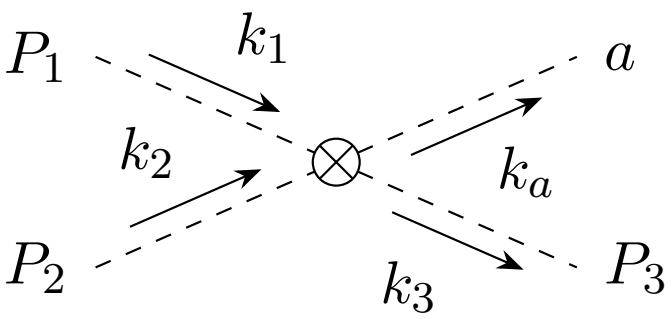
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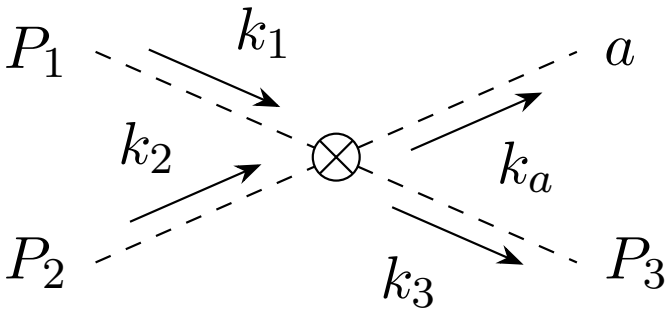
- Red terms: ALP enters via mixing
- Blue terms: ALP enters via external current
- Teal terms: Contribution beyond $\mathcal{O}(F/f_a)$ to 1 ALP processes

Parameterization of one ALP scattering amplitude



$$\mathcal{L}^{(4)} \supset \frac{1}{48F^2}\langle[\Phi,\partial_\mu\Phi][\Phi,\partial^\mu\Phi]\rangle + \\ + \frac{1}{12Ff_a}(\partial_\mu a)\langle\overline{c}[\Phi,[\Phi,\partial^\mu\Phi]]\rangle + \\ + \frac{B_0}{24F^2}\langle\mathcal{M}\Phi^4\rangle + \frac{B_0}{6Ff_a}\frac{a}{\langle\mathcal{M}^{-1}\rangle}\langle\Phi^3\rangle$$

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$$\mathcal{M} = c_{s_4}^{P}\gamma_{aP} + c_{s_3} + \sum_{i=1,2}\left(c_{\mathsf{ax}}^{aP_i} + c_{\mathsf{D}}^{aP_i;P}\gamma_{aP}\right)(k_a\cdot k_i) - \left(c_{\mathsf{ax}}^{aP_3} + c_{\mathsf{D}}^{aP_3;P}\gamma_{aP}\right)(k_a\cdot k_3) - c_{\mathsf{D}}^{P_1P_2;P}\gamma_{aP}(k_1\cdot k_2) + \sum_{i=1,2} c_{\mathsf{D}}^{P_iP_3;P}\gamma_{aP}(k_i\cdot k_3)$$

Parameterization of ALP-meson amplitudes

- Complete parameterization of any 4-body amplitude with 1 to 4 ALPs

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$$\sum |\mathcal{M}_{aK\to\pi K}|^2 = \frac{3t^2}{16F^2 f_a^2}(1-\epsilon^2) + \frac{(s-M_K^2)(s+t-M_K^2)}{F^2 f_a^2}\epsilon^2$$

From our result: $M_{\eta'} \to \infty$, $M_\pi \to 0$, $m_a \to 0$, $c_u = c_d = c_s = 0$

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  - Axion-Pion scattering (L. Di Luzio et. al., PRL 126, 241801 (2021))

$$\sum |\mathcal{M}_{a\pi\to\pi\pi}|^2 = \left(\frac{3C_{a\pi}}{2F f_a}\right)^2 (s^2+t^2+u^2-3M_\pi^2)$$

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Axion decays

Lagrangian involved in ALP decays

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Allowed decays:

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Complete description of LO decays for $m_a < M_\eta + 2M_\pi = 822$ MeV

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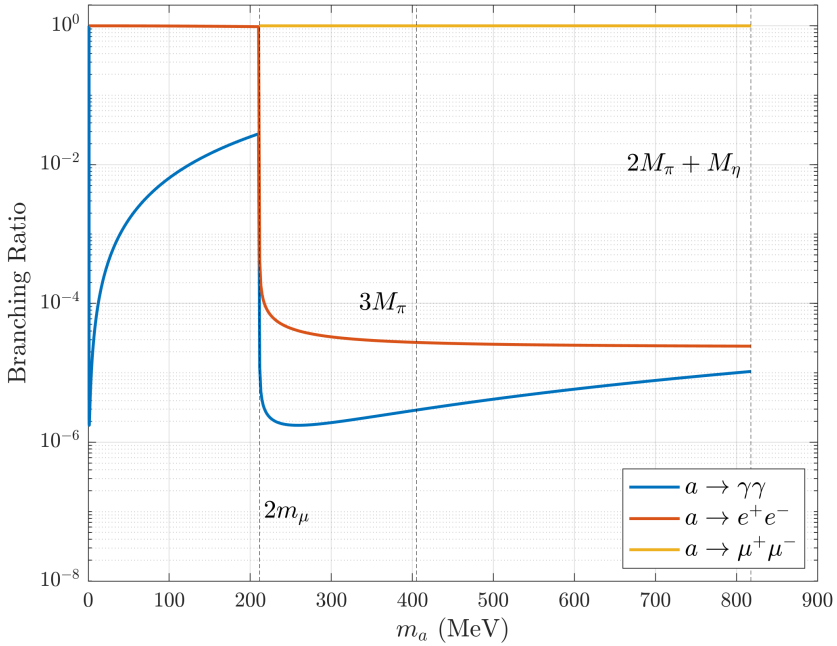
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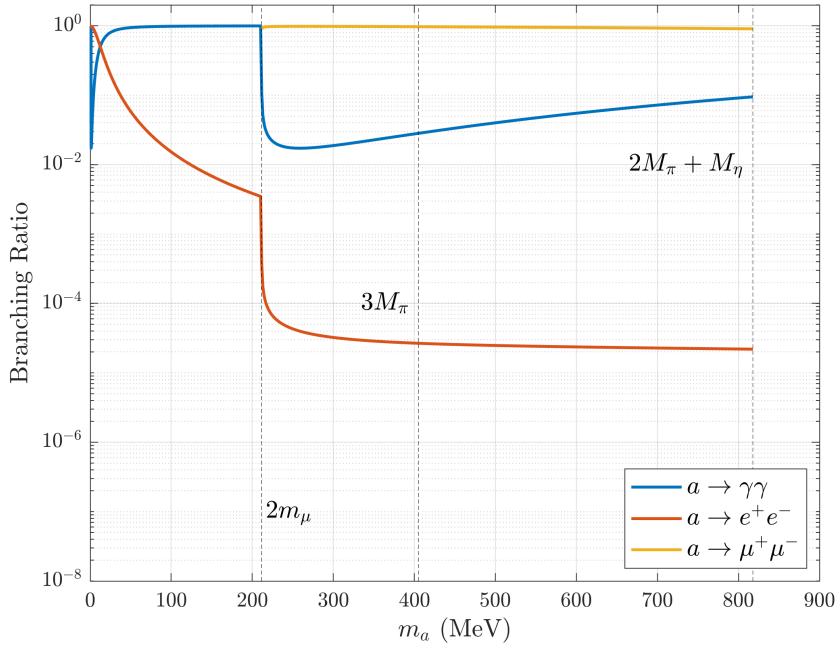
Complete description of LO decays for $m_a < M_\eta + 2M_\pi = 822$ MeV

$\Gamma_a/m_a \lesssim 10^{-9}$ for $\mathcal{O}(1)$ couplings and $f_a = 1$ TeV: NWA

Axion decays

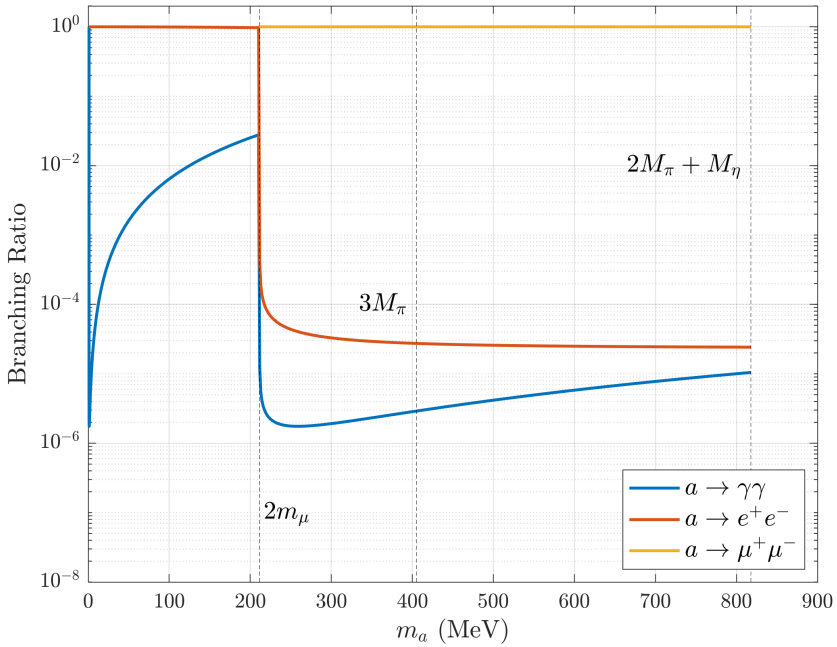


(a) $c_q = \bar{c}_\gamma = c_e = c_\mu = 1$.

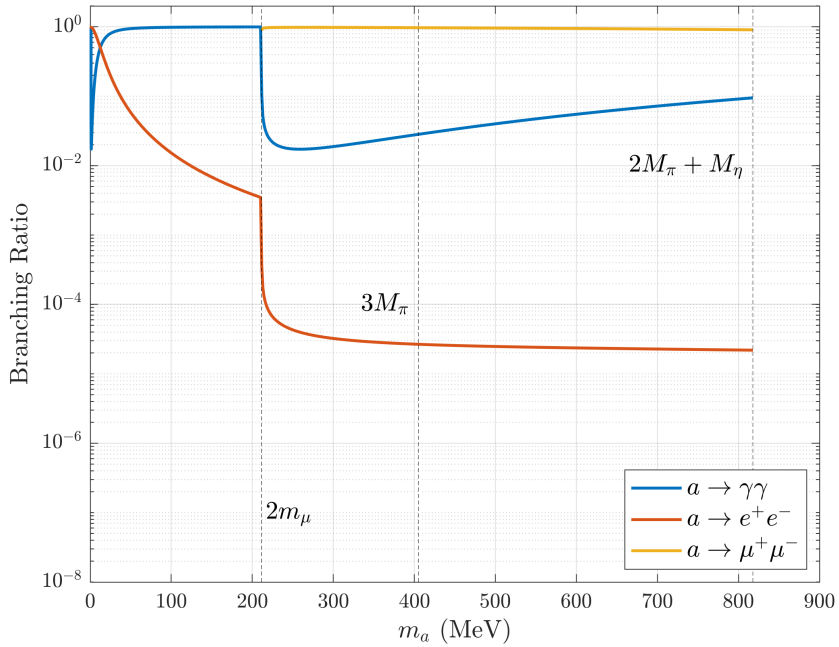


(b) $c_q = \bar{c}_\gamma = 1$, $c_e = c_\mu = 10^{-2}$.

Axion decays



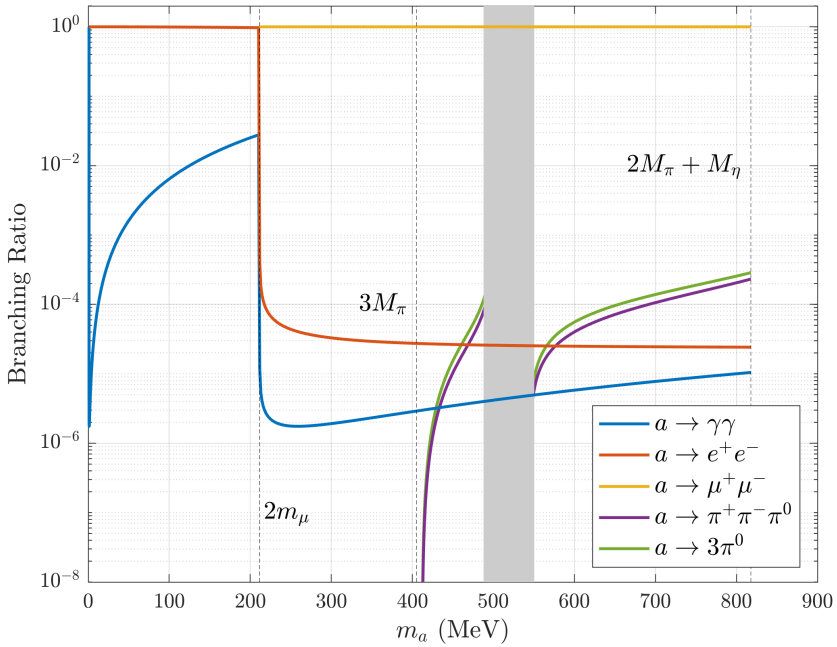
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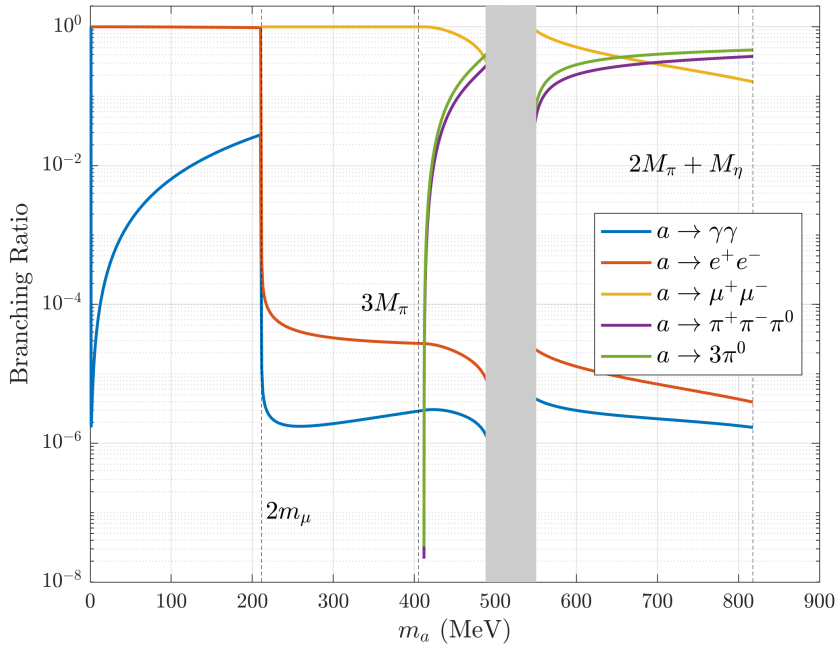
(b) $c_q = \bar{c}_\gamma = 1$, $c_e = c_\mu = 10^{-2}$.

$|\mathcal{M}(a \to 3\pi)|^2 \propto (c_u - c_d)$: no hadronic decay for isospin conserving couplings

Axion decays



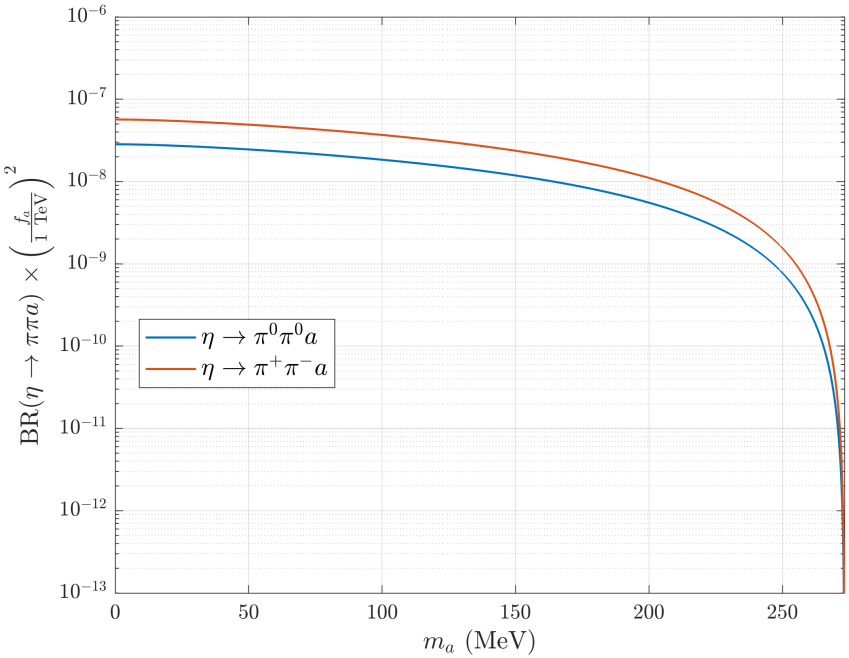
(c) $c_{u,s} = \bar{c}_\gamma = c_e = c_\mu = 1,\ c_d = 0.$



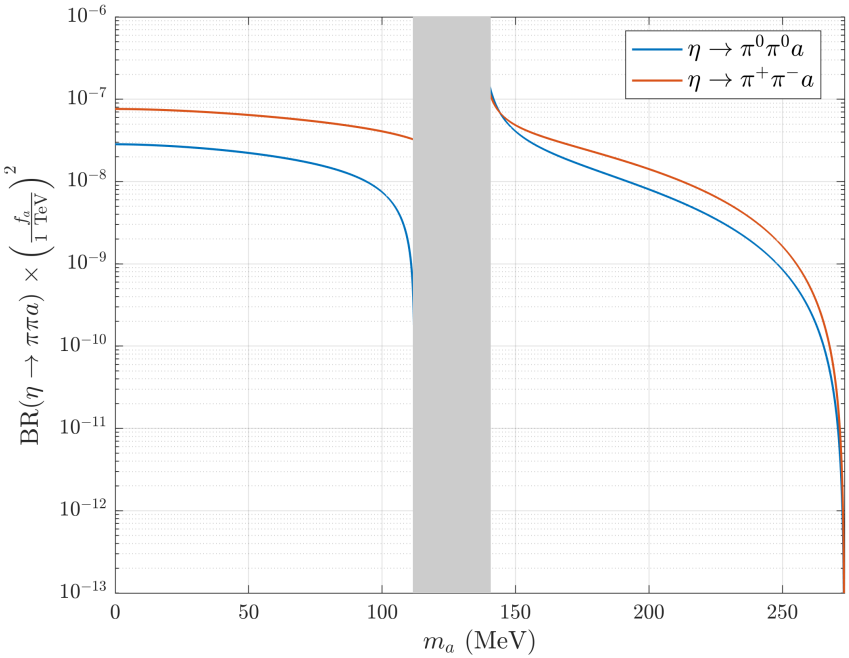
(d) $c_{u,s} = 1,\ \bar{c}_\gamma = c_e = c_\mu = 10^{-2},\ c_d = 0.$

$|\mathcal{M}(a \to 3\pi)|^2 \propto (c_u - c_d)$: no hadronic decay for isospin conserving couplings

$\eta^{(\prime)} \to \pi\pi a$ decays

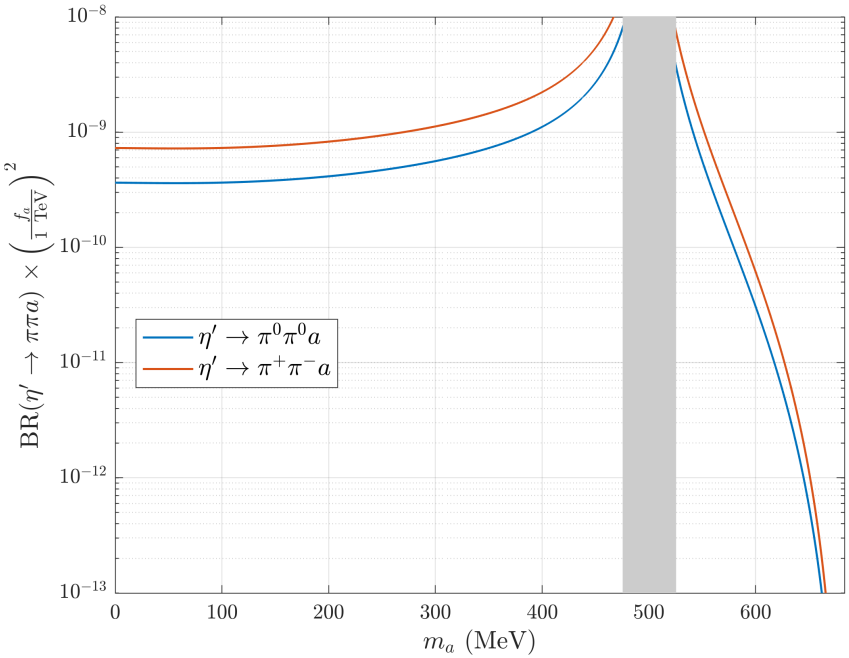


(a) η decays with $c_q = 1$.

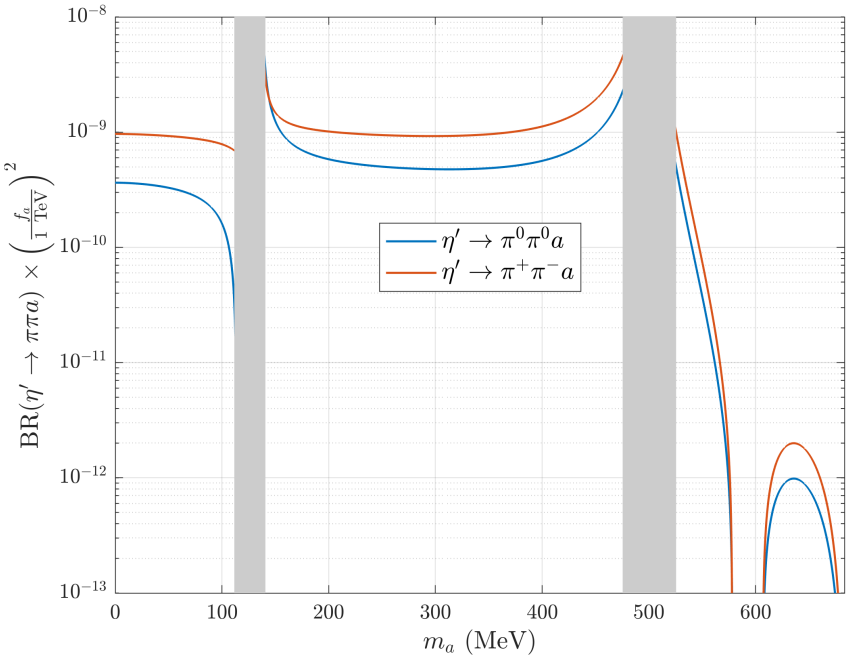


(b) η decays with $c_u = c_s = 1$, $c_d = 0$.

$\eta^{(\prime)} \to \pi\pi a$ decays



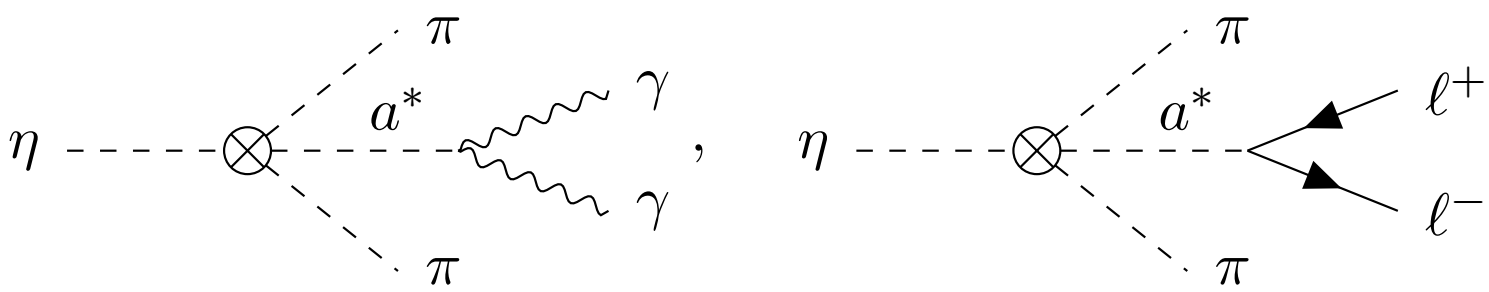
(c) η' decays with $c_q = 1$.



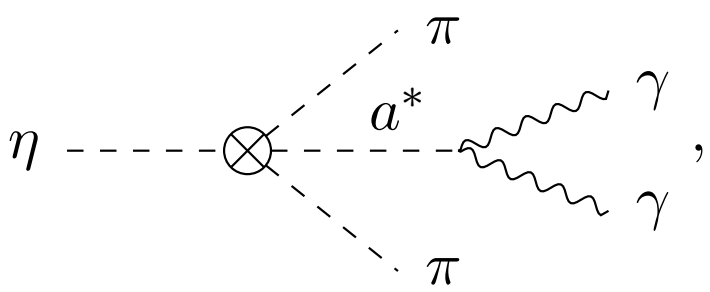
(d) η' decays with $c_u = c_s = 1$, $c_d = 0$.

$|\mathcal{M}(\eta^{(\prime)} \to \pi\pi a)|^2 \sim \dfrac{B_0(m_u - m_d)}{M_\pi^2 - m_a^2}(c_u - c_d)$: no *pion-pole* for isospin conserving couplings

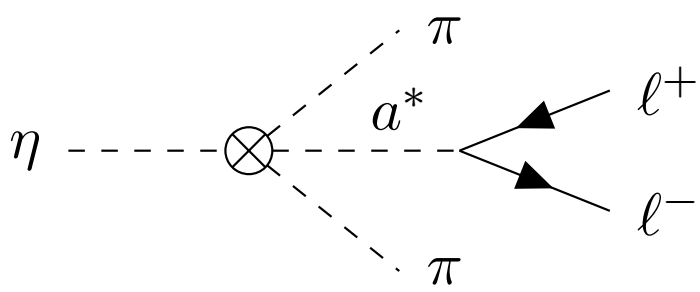
Axion-mediated η decays

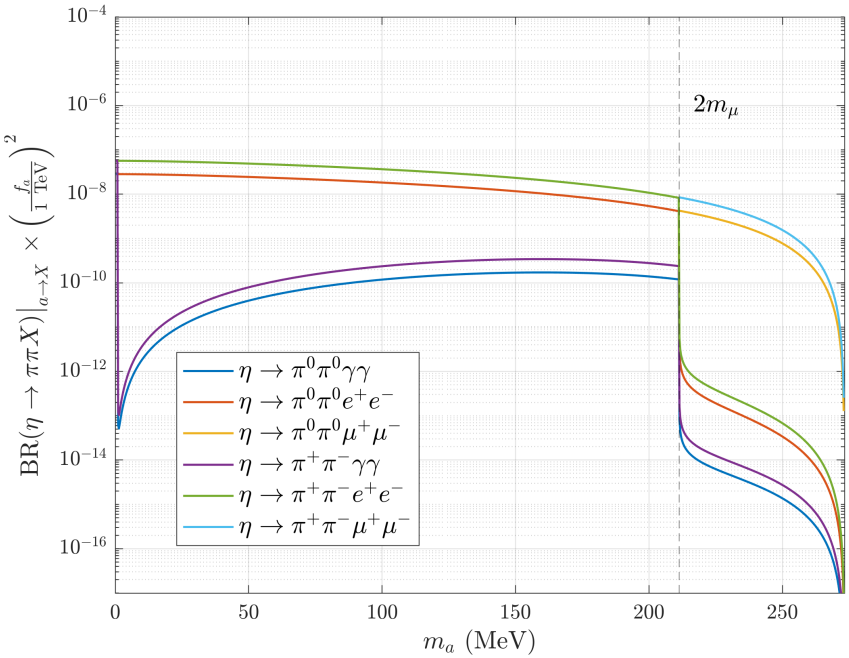


Axion-mediated η decays

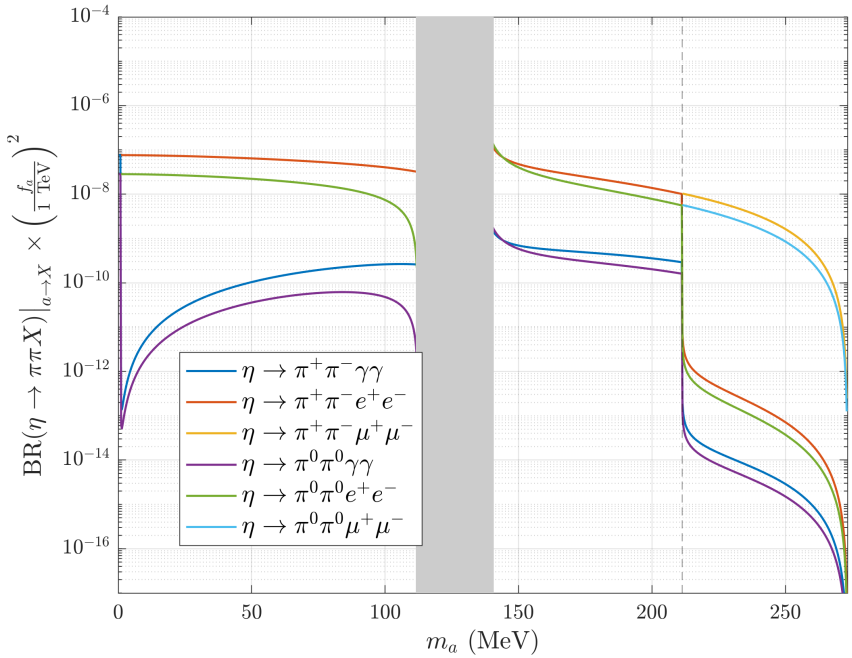


,



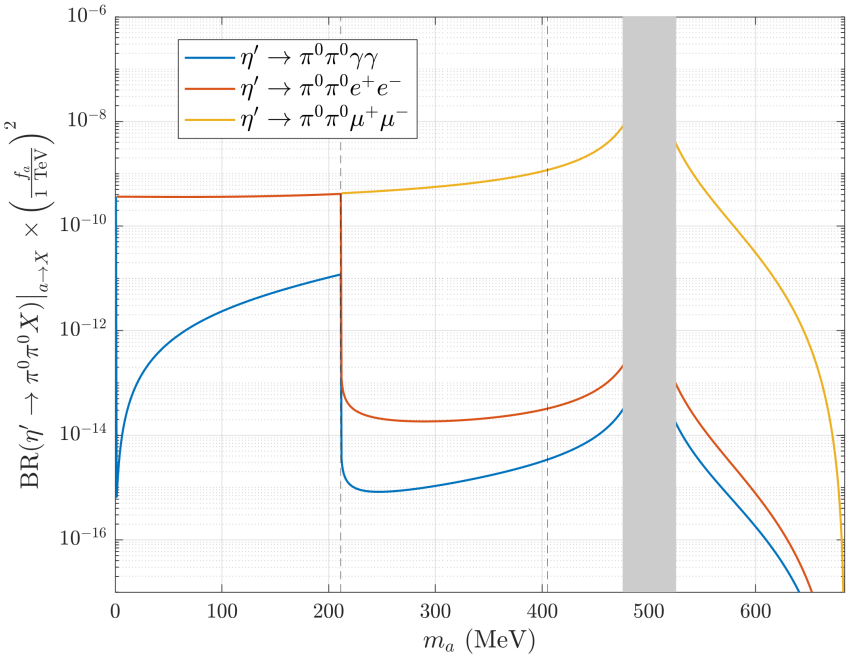


(a) $c_q = \bar{c}_\gamma = c_\ell = 1$.

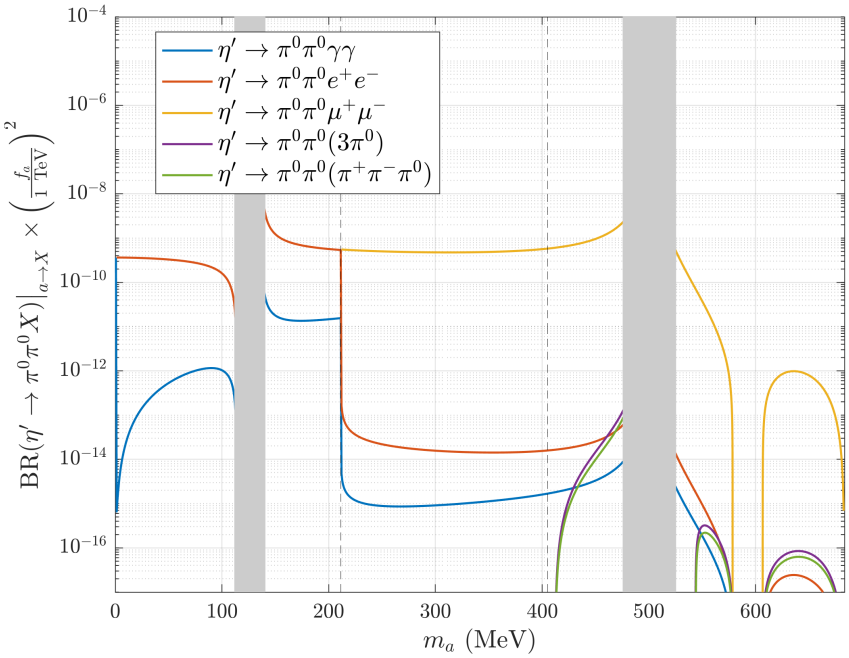


(b) $c_u = c_s = \bar{c}_\gamma = c_\ell = 1$, $c_d = 0$.

Axion-mediated η' decays



(a) $c_q = \bar{c}_\gamma = c_\ell = 1$.



(b) $c_u = c_s = \bar{c}_\gamma = c_\ell = 1$, $c_d = 0$.

Contents

1. Introduction
2. Axion Lagrangian
3. ALP-meson mixing & ALP mass
4. Amplitudes parameterizations
5. Axion (& axion-mediated) decays
6. Conclusions and outlook

Conclusions and outlook

- **Conclusions**
  - Theoretical study of insertion of ALPs in χPT at LO
  - Formal calculation of axion-meson mixing, including isospin breaking corrections
  - Complete determination of ALP mass at $\mathcal{O}(F^2/f_a^2)$
  - Complete parameterizations of 4-body amplitudes with 1 to 4 axions
  - Parameterization of axion-mediated $\eta^{(\prime)}$ decays for fully general couplings and $m_a \lesssim 822$ MeV

Conclusions and outlook

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- **Outlook**
  - Beyond LO (χPT@NLO, resonances, FSI...)
  - Application of these calculations to cosmological scenarios
  - Combination of axion-mediated decays with SM contribution

Back-up slides

BACK UP SLIDES

Kinetic and Mass Mixing

$$
\begin{aligned}
K_{PP'} &= \delta_{PP'}, \quad PP' = \pi_3, \eta_8, \eta_1 \\
K_{aP} &= \frac{F}{f_a}\frac{\langle \bar{c}\lambda_P \rangle}{2} \\
K_{aa} &= \frac{F^2}{f_a^2}\frac{\langle \bar{c}^2 \rangle}{2} \\
(M^2_{\mathsf{mix}})_{PP'} &= B_0 \langle \mathcal{M}\lambda_P\lambda_{P'} \rangle + \delta_{\eta_1 P}\delta_{\eta_1 P'} M_0^2 \\
(M^2_{\mathsf{mix}})_{aP} &= \frac{F}{f_a}\frac{B_0}{2}\langle \{\kappa, \mathcal{M}\}\lambda_P \rangle = \sqrt{6}\frac{F}{f_a}\frac{B_0}{\langle \mathcal{M}^{-1} \rangle}\delta_{0P} \\
(M^2_{\mathsf{mix}})_{aa} &= \frac{F}{f_a}\frac{B_0}{4}\langle \{\kappa, \{\kappa, \mathcal{M}\}\} \rangle = \frac{F}{f_a}\frac{B_0}{\langle \mathcal{M}^{-1} \rangle} + m_0^2
\end{aligned}
$$

ALP-Meson Mixing Angles

$$\theta_{a\eta'} = -\frac{F}{f_a}\frac{\sqrt{6}kM_\pi^2}{M_{\eta'}^2 - m_a^2}\cos\theta_{\eta\eta'} + \frac{M_{\eta'}^2 + m_a^2}{M_{\eta'}^2 - m_a^2}\left(\frac{K_{a0}}{2}\cos\theta_{\eta\eta'} + \frac{K_{a8}}{2}\sin\theta_{\eta\eta'}\right) +$$
$$+ \frac{K_{a3}}{2\sqrt{3}}\frac{M_{\eta'}^2 + m_a^2}{M_{\eta'}^2 - m_a^2}(\sqrt{2}\cos\theta_{\eta\eta'} + \sin\theta_{\eta\eta'})\frac{\Delta}{M_{\eta'}^2 - M_\pi^2}$$
$$\theta_{a\eta} = \frac{F}{f_a}\frac{\sqrt{6}kM_\pi^2}{M_\eta^2 - m_a^2}\sin\theta_{\eta\eta'} + \frac{M_\eta^2 + m_a^2}{M_\eta^2 - m_a^2}\left(\frac{K_{a8}}{2}\cos\theta_{\eta\eta'} - \frac{K_{a0}}{2}\sin\theta_{\eta\eta'}\right) +$$
$$+ \frac{K_{a3}}{2\sqrt{3}}\frac{M_\eta^2 + m_a^2}{M_\eta^2 - m_a^2}(\cos\theta_{\eta\eta'} - \sqrt{2}\sin\theta_{\eta\eta'})\frac{\Delta}{M_\eta^2 - M_\pi^2}$$

ALP-Meson Mixing Angles

$$\theta_{a\pi} = \frac{K_{a3}}{2}\frac{M_\pi^2 + m_a^2}{M_\pi^2 - m_a^2} + \frac{1}{M_\pi^2 - m_a^2}\frac{\Delta}{\sqrt{3}}\left(\sqrt{2}\frac{K_{a0}}{2} + \frac{K_{a8}}{2}\right) +$$

$$+ \frac{1}{\sqrt{3}}\frac{(M_\eta^2 + m_a^2)\Delta}{(M_\pi^2 - m_a^2)(M_\eta^2 - M_\pi^2)}(\cos\theta - \sqrt{2}\sin\theta)\times$$

$$\times\left(\frac{K_{a0}}{2}\sin\theta - \frac{K_{a8}}{2}\cos\theta - \sqrt{6}\frac{F}{f_a}kM_\pi^2\frac{\sin\theta}{M_\eta^2 + m_a^2}\right) -$$

$$- \frac{1}{\sqrt{3}}\frac{(M_{\eta'}^2 + m_a^2)\Delta}{(M_\pi^2 - m_a^2)(M_{\eta'}^2 - M_\pi^2)}(\sqrt{2}\cos\theta + \sin\theta)\times$$

$$\times\left(\frac{K_{a8}}{2}\sin\theta + \frac{K_{a0}}{2}\cos\theta - \sqrt{6}\frac{F}{f_a}kM_\pi^2\frac{\cos\theta}{M_{\eta'}^2 + m_a^2}\right)$$

ALP-Meson Mixing Angles

$$\gamma_{a\eta'} = -\sin(\theta_{\eta\eta'})(\theta_{a\eta} + \theta_{\pi\eta}\theta_{a\pi}) + \cos(\theta_{\eta\eta'})(\theta_{a\eta'} + \theta_{\pi\eta'}\theta_{a\pi}) - \frac{K_{a0}}{2}$$

$$\gamma_{a\pi} = \theta_{a\pi} - (\theta_{\pi\eta'}\theta_{a\eta'} + \theta_{\pi\eta}\theta_{a\eta}) - \frac{K_{a3}}{2}$$

$$\gamma_{a\eta} = \sin(\theta_{\eta\eta'})(\theta_{a\eta'} + \theta_{\pi\eta'}\theta_{a\pi}) + \cos(\theta_{\eta\eta'})(\theta_{a\eta} + \theta_{\pi\eta}\theta_{a\pi}) - \frac{K_{a8}}{2}$$