GOOFy "symmetries" - understanding RG fixed points - *introduction and generic approach*

Bohdan Grzadkowski

University of Warsaw



DISCRETE 2026,
November 13th 2026,
Faculty of Mathematics, Physics and Informatics,
Comenius University, Bratislava

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- P. M. Ferreira, B.G., O. M. Ogreid and P. Osland, “New symmetries of the two-Higgs-doublet model,” Eur. Phys. J. C **84** (2024) no.3, 234, arXiv:2306.02410,
- P.M. Ferreira, B.G. and O.M. Ogreid, "Imaginary scaling invariance of the one-loop effective potential", *Phys. Rev. D* **113**, 075036, arXiv:2506.21145,
- B.G. and O. M. Ogreid, “GOOFy - a systematic approach,” JHEP **07** (2026) 038, arXiv:2602.20849.

Selected GOOFy literature

1. P. M. Ferreira, B.G., O. M. Ogreid and P. Osland, "New symmetries of the two-Higgs-doublet model", Eur. Phys. J. C **84** (2024) no.3, 234, e-Print: 2306.02410,
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2. A. Kuncinas, P. Osland, M.N. Rebelo, "Systematic analysis of 3HDM symmetries", Phys.Rev.D 113 (2026) 11, 115054, e-Print:2512.07657,
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The Two-Higgs Doublet Model (2HDM) in the bilinear notation

$$
\begin{aligned}
V \;=\;& m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \\
& \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \\
& \left\{ \frac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\} ,
\end{aligned}
$$

where m_{12}^2 and $\lambda_{5,6,7}$ might be complex. $\Phi_{1,2}$ are $SU(2)$ doublets.

An alternative notation uses four gauge-invariant bilinears constructed from the doublets (Velhinho 1994, Nagel 2004, Ivanov 2005, Maniatis 2006, Nishi 2006):

$$
\begin{aligned}
r_0 &\equiv \tfrac{1}{2}\left(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2\right), \\
r_1 &\equiv \tfrac{1}{2}\left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1\right) = \text{Re}\left(\Phi_1^\dagger \Phi_2\right), \\
r_2 &\equiv -\tfrac{i}{2}\left(\Phi_1^\dagger \Phi_2 - \Phi_2^\dagger \Phi_1\right) = \text{Im}\left(\Phi_1^\dagger \Phi_2\right), \\
r_3 &\equiv \tfrac{1}{2}\left(\Phi_1^\dagger \Phi_1 - \Phi_2^\dagger \Phi_2\right).
\end{aligned}
$$

The Two-Higgs Doublet Model (2HDM) in the bilinear notation

The potential may be written as

$$V = M_\mu \, r^\mu + \Lambda_{\mu\nu} \, r^\mu \, r^\nu \, ,$$

where

$$r^\mu \equiv (r_0 \, , \, r_1 \, , \, r_2 \, , \, r_3) = (r_0 \, , \, \vec{r}) \, ,$$

$$M^\mu \equiv \left(m_{11}^2 + m_{22}^2 \, , \, 2\mathrm{Re}(m_{12}^2) \, , \, -2\mathrm{Im}(m_{12}^2) \, , \, m_{22}^2 - m_{11}^2\right) = (M_0 \, , \, \vec{M}) \, ,$$

$$\Lambda^{\mu\nu} \equiv \begin{pmatrix} \frac{1}{2}(\lambda_1 + \lambda_2) + \lambda_3 & -\mathrm{Re}\,(\lambda_6 + \lambda_7) & \mathrm{Im}\,(\lambda_6 + \lambda_7) & \frac{1}{2}(\lambda_2 - \lambda_1) \\ -\mathrm{Re}\,(\lambda_6 + \lambda_7) & \lambda_4 + \mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_5) & \mathrm{Re}\,(\lambda_6 - \lambda_7) \\ \mathrm{Im}\,(\lambda_6 + \lambda_7) & -\mathrm{Im}\,(\lambda_5) & \lambda_4 - \mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_6 - \lambda_7) \\ \frac{1}{2}(\lambda_2 - \lambda_1) & \mathrm{Re}\,(\lambda_6 - \lambda_7) & -\mathrm{Im}\,(\lambda_6 - \lambda_7) & \frac{1}{2}(\lambda_1 + \lambda_2) - \lambda_3 \end{pmatrix}$$

$$\Lambda^{\mu\nu} \equiv \begin{pmatrix} \Lambda_{00} & \vec{\Lambda} \\ \vec{\Lambda}^T & \Lambda \end{pmatrix}$$

Global symmetries of the 2HDM

- *Higgs-family symmetries*, unitary transformations mix both doublets,

$$\Phi_i \to \Phi_i' = \sum_{j=1}^{2} Y_{ij}\,\Phi_j\,, \qquad Y \in U(2)$$

  e.g. Z_2:

$$\Phi_1 \to \Phi_1 \quad , \quad \Phi_2 \to -\Phi_2\,,$$

  prevents the occurrence of tree-level flavour-changing neutral currents (FCNC).
- *generalized CP (GCP)*, transformations:

$$\Phi_i \to \Phi_i' = \sum_{j=1}^{2} X_{ij}\,\Phi_j^*\,, \qquad X \in U(2)$$

  e.g. “standard" CP transformation (CP1):

$$\Phi_i \to \Phi_i^*$$

Global symmetries of the 2HDM

In the bilinear formalism, symmetries are represented by rotations in the 3-dimensional space defined by the vector $\vec{r}$:

$$\vec{r} \to \vec{r}' = S\,\vec{r},$$

where $S \in O(3)$.

$$S_{Z_2} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad S_{CP1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

CP2: $\Phi_1 \to \Phi_2^*, \quad \Phi_2 \to -\Phi_1^*$

$$S_{CP2} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

A parity transformation about the three axes.

Global symmetries of the 2HDM

S	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
CP1			real					real	real	real
Z_2			0						0	0
U(1)			0					0	0	0
CP2		m_{11}^2	0		λ_1					$-\lambda_6$
CP3		m_{11}^2	0		λ_1			λ_{134}	0	0
$SO(3)$		m_{11}^2	0		λ_1		$\lambda_1 - \lambda_3$	0	0	0

Table 1: Relations between 2HDM scalar potential parameters for each of the six symmetries discussed, $\lambda_{134} \equiv \lambda_1 - \lambda_3 - \lambda_4$.

$$
\begin{aligned}
V \;=\; & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \\
& \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \\
& \left\{ \frac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \left[\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)\right] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\} ,
\end{aligned}
$$

Running of parameters of the 2HDM

The 1-loop β-functions for the quadratic couplings

$$\beta_{m_{11}^2} = 3\lambda_1 m_{11}^2 + (2\lambda_3 + \lambda_4) m_{22}^2 - 3\left(\lambda_6^* m_{12}^2 + \text{h.c.}\right) - \frac{1}{4}(9g^2 + 3g'^2) m_{11}^2 + \beta_{m_{11}^2}^F,$$

$$\beta_{m_{22}^2} = (2\lambda_3 + \lambda_4) m_{11}^2 + 3\lambda_2 m_{22}^2 - 3\left(\lambda_7^* m_{12}^2 + \text{h.c.}\right) - \frac{1}{4}(9g^2 + 3g'^2) m_{22}^2 + \beta_{m_{22}^2}^F,$$

$$\beta_{m_{12}^2} = -3\left(\lambda_6 m_{11}^2 + \lambda_7 m_{22}^2\right) + (\lambda_3 + 2\lambda_4) m_{12}^2 + 3\lambda_5 {m_{12}^2}^* - \frac{1}{4}(9g^2 + 3g'^2) m_{12}^2 + \beta_{m_{12}^2}^F,$$

Running of parameters of the 2HDM

and 1-loop β functions for the quartic ones,

$$
\begin{aligned}
\beta_{\lambda_1} &= 6\lambda_1^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + |\lambda_5|^2 + 12\,|\lambda_6|^2 \\
&\quad + \frac{3}{8}(3g^4 + g'^4 + 2g^2g'^2) - \frac{3}{2}\lambda_1(3g^2 + g'^2) + \beta^F_{\lambda_1}\,, \\
\beta_{\lambda_2} &= 6\lambda_2^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + |\lambda_5|^2 + 12\,|\lambda_7|^2 \\
&\quad + \frac{3}{8}(3g^4 + g'^4 + 2g^2g'^2) - \frac{3}{2}\lambda_2(3g^2 + g'^2) + \beta^F_{\lambda_2}\,, \\
\beta_{\lambda_3} &= (\lambda_1 + \lambda_2)(3\lambda_3 + \lambda_4) + 2\lambda_3^2 + \lambda_4^2 + |\lambda_5|^2 + 2\left(|\lambda_6|^2 + |\lambda_7|^2\right) + 8\,\mathrm{Re}\left(\lambda_6\lambda_7^*\right) \\
&\quad + \frac{3}{8}(3g^4 + g'^4 - 2g^2g'^2) - \frac{3}{2}\lambda_3(3g^2 + g'^2) + \beta^F_{\lambda_3}\,, \\
\beta_{\lambda_4} &= (\lambda_1 + \lambda_2)\,\lambda_4 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 4\,|\lambda_5|^2 + 5\left(|\lambda_6|^2 + |\lambda_7|^2\right) + 2\,\mathrm{Re}\left(\lambda_6\lambda_7^*\right) \\
&\quad + \frac{3}{2}g^2g'^2 - \frac{3}{2}\lambda_4(3g^2 + g'^2) + \beta^F_{\lambda_4}\,, \\
\beta_{\lambda_5} &= (\lambda_1 + \lambda_2 + 4\lambda_3 + 6\lambda_4)\,\lambda_5 + 5\left(\lambda_6^2 + \lambda_7^2\right) + 2\lambda_6\lambda_7 \\
&\quad - \frac{3}{2}\lambda_5(3g^2 + g'^2) + \beta^F_{\lambda_5}\,, \\
\beta_{\lambda_6} &= (6\lambda_1 + 3\lambda_3 + 4\lambda_4)\,\lambda_6 + (3\lambda_3 + 2\lambda_4)\,\lambda_7 + 5\lambda_5\lambda_6^* + \lambda_5\lambda_7^* \\
&\quad - \frac{3}{2}\lambda_6(3g^2 + g'^2) + \beta^F_{\lambda_6}\,, \\
\beta_{\lambda_7} &= (6\lambda_2 + 3\lambda_3 + 4\lambda_4)\,\lambda_7 + (3\lambda_3 + 2\lambda_4)\,\lambda_6 + 5\lambda_5\lambda_7^* + \lambda_5\lambda_6^* \\
&\quad - \frac{3}{2}\lambda_7(3g^2 + g'^2) + \beta^F_{\lambda_7}\,,
\end{aligned}
$$

where the β_x^F terms contain all contributions coming from fermions.

Running of parameters of the 2HDM

$$Z_2 \text{ symmetry} \Rightarrow \lambda_6 = \lambda_7 = 0 \Rightarrow \beta_{\lambda_6} = \beta_{\lambda_7} = 0$$

a symmetry-based conditions on λ's are preserved by RGE running at the one-loop order.

For the Z_2 symmetry

$$\beta_{\lambda_5} = \left[\lambda_1 + \lambda_2 + 4\lambda_3 + 6\lambda_4 - \frac{3}{2}\left(3g^2 + g'^2\right)\right]\lambda_5$$

$\lambda_5 = 0$ is *a fixed point* of this RG equation: if at any scale $\lambda_5 = 0$, that coupling will remain zero for all renormalization scales. Such fixed points of RG equations are usually fingerprints of symmetries, and indeed that is the case here: if $\lambda_6 = \lambda_7 = 0$, the extra constraint $\lambda_5 = 0$ takes us from Z_2-symmetric model to $U(1)$-symmetric.

Running of parameters of the 2HDM

We have noticed that

$$\left\{ m_{11}^2 + m_{22}^2 = 0 \;\;,\;\; \lambda_1 - \lambda_2 = 0 \;\;,\;\; \lambda_6 + \lambda_7 = 0 \right\}$$

- constitutes a fixed point of the 1-loop RG equations,
- are basis transformation invariants.

$$\begin{aligned}
\beta_{m_{11}^2+m_{22}^2} &= 3(\lambda_1 m_{11}^2 + \lambda_2 m_{22}^2) + (2\lambda_3 + \lambda_4)(m_{11}^2 + m_{22}^2) \\
&\quad -3\left[(\lambda_6^* + \lambda_7^*) m_{12}^2 + \text{h.c.}\right] - \frac{1}{4}(9g^2 + 3g'^2)(m_{11}^2 + m_{22}^2) \\
\beta_{\lambda_1-\lambda_2} &= 6\left(\lambda_1^2 - \lambda_2^2\right) + 12\left(|\lambda_6|^2 - |\lambda_7|^2\right) - \frac{3}{2}(\lambda_1 - \lambda_2)(3g^2 + g'^2) \\
\beta_{\lambda_6+\lambda_7} &= 6(\lambda_1\lambda_6 + \lambda_2\lambda_7) + (3\lambda_3 + 2\lambda_4)(\lambda_6 + \lambda_7) + 6\lambda_5(\lambda_6^* + \lambda_7^*) \\
&\quad - \frac{3}{2}(\lambda_6 + \lambda_7)(3g^2 + g'^2)
\end{aligned}$$

Running of parameters of the 2HDM

It turns out that

$$\left\{ m_{11}^2 + m_{22}^2 = 0 \;\;,\;\; \lambda_1 - \lambda_2 = 0 \;\;,\;\; \lambda_6 + \lambda_7 = 0 \right\}$$

is also the 2-loop fixed point.

$$\Downarrow$$

Conclusion:
Perhaps there is a symmetry behind the fixed point:

$$\left\{ m_{11}^2 + m_{22}^2 = 0 \;\;,\;\; \lambda_1 - \lambda_2 = 0 \;\;,\;\; \lambda_6 + \lambda_7 = 0 \right\}$$

Basis transformations

Weak-basis transformation, $U(2)$:

$$\begin{pmatrix} \Phi_1' \\ \Phi_2' \end{pmatrix} = \underbrace{e^{i\psi} \begin{pmatrix} \cos\theta & e^{-i\tilde{\xi}} \sin\theta \\ -e^{i\chi} \sin\theta & e^{i(\chi-\tilde{\xi})} \cos\theta \end{pmatrix}}_{U(2)} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$$

$$\mathcal{L}_k = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2)\,,$$

- The Higgs kinetic terms remain invariant.
- Physical observables are basis-independent.

Basis transformations

$$V = M_\mu\, r^\mu + \Lambda_{\mu\nu}\, r^\mu\, r^\nu$$

basis field transformations:

$$\vec{r} \to \vec{r}' = R\,\vec{r} \qquad \text{for} \qquad R_{ij}(U) \equiv \tfrac{1}{2}\mathrm{Tr}\left(U^\dagger \sigma_i U \sigma_j\right) \quad U \in U(2)$$

basis parameter transformations:

$$\vec{M} \to \vec{M}' = R\,\vec{M} \qquad \vec{\Lambda} \to \vec{\Lambda}' = R\,\vec{\Lambda} \qquad \Lambda \to \Lambda' = R\,\Lambda\,R^T$$

whereas r_0, M_0 and Λ_{00} are *basis invariants*.

$$M^\mu \equiv \left(m_{11}^2 + m_{22}^2\,,\; 2\mathrm{Re}(m_{12}^2)\,,\; -2\mathrm{Im}(m_{12}^2)\,,\; m_{22}^2 - m_{11}^2\right) = \left(M_0\,,\; \vec{M}\right),$$

$$\Lambda^{\mu\nu} \equiv \begin{pmatrix} \frac{1}{2}(\lambda_1+\lambda_2)+\lambda_3 & -\mathrm{Re}\,(\lambda_6+\lambda_7) & \mathrm{Im}\,(\lambda_6+\lambda_7) & \frac{1}{2}(\lambda_2-\lambda_1) \\ -\mathrm{Re}\,(\lambda_6+\lambda_7) & \lambda_4+\mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_5) & \mathrm{Re}\,(\lambda_6-\lambda_7) \\ \mathrm{Im}\,(\lambda_6+\lambda_7) & -\mathrm{Im}\,(\lambda_5) & \lambda_4-\mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_6-\lambda_7) \\ \frac{1}{2}(\lambda_2-\lambda_1) & \mathrm{Re}\,(\lambda_6-\lambda_7) & -\mathrm{Im}\,(\lambda_6-\lambda_7) & \frac{1}{2}(\lambda_1+\lambda_2)-\lambda_3 \end{pmatrix}.$$

$$\Lambda^{\mu\nu} = \begin{pmatrix} \Lambda_{00} & \vec{\Lambda} \\ \vec{\Lambda}^T & \Lambda \end{pmatrix}$$

All-order results

Basis transformation invariants:

$$I_{1,1} = \Lambda_{00}\,, \qquad I_{1,2} = \mathrm{Tr}\Lambda$$
$$I_{2,1} = \vec{\Lambda}\cdot\vec{\Lambda}\,, \qquad I_{2,2} = \mathrm{Tr}\Lambda^2$$
$$I_{3,1} = \vec{\Lambda}\cdot\Lambda\vec{\Lambda}\,, \qquad I_{3,2} = \mathrm{Tr}\Lambda^3$$
$$I_{4,1} = \vec{\Lambda}\cdot\Lambda^2\vec{\Lambda}\,,$$

To all orders in perturbation theory,

$$\beta_{\vec{\Lambda}} = a_0\,\vec{\Lambda} + a_1\,\Lambda\,\vec{\Lambda} + a_2\,\Lambda^2\,\vec{\Lambda}$$

- $\vec{\Lambda} = \vec{0}$ *is a fixed point to all orders in perturbation theory.*

where the a_i are polynomial expressions involving invariants,

see A.V. Bednyakov, "On three-loop RGE for the Higgs sector of 2HDM", JHEP 11 (2018) 154, e-Print: 1809.04527

All-order results

$$\beta_{\vec{M}} = c_0\,\vec{M} + c_1\,\Lambda\,\vec{M} + c_2\,\Lambda^2\,\vec{M} + c_3\,I_{M3}\,\vec{\Lambda} + c_4\,I_{M4}\,\Lambda\,\vec{\Lambda} + c_5\,I_{M5}\,\Lambda^2\,\vec{\Lambda}$$

- *If* $\vec{\Lambda} = \vec{0}$, *then* $\vec{M} = \vec{0}$ *is a fixed point to all orders*

$$\beta_{M_0} = b_0\,M_0 + b_1\,\vec{\Lambda}\cdot\vec{M} + b_2\,\vec{\Lambda}\cdot\left(\Lambda\vec{M}\right) + b_3\,\vec{\Lambda}\cdot\left(\Lambda^2\vec{M}\right)$$

- *If* $\vec{\Lambda} = \vec{0}$, *then* $M_0 = 0$ *is a fixed point to all orders.*

where the b_i and c_i are polynomial expressions involving invariants, see A.V. Bednyakov

All-order results

Two all-order fixed points of the 2HDM RG equations:

- $\{\vec{M} = \vec{0}\,, \vec{\Lambda} = \vec{0}\}$, $(CP2)$:

$$m_{11}^2 = m_{22}^2 \quad , \quad m_{12}^2 = 0 \quad , \quad \lambda_1 = \lambda_2 \quad , \quad \lambda_6 = -\lambda_7 \,.$$

- $\{M_0 = 0\,, \vec{\Lambda} = \vec{0}\}$:

$$M_0 \equiv m_{11}^2 + m_{22}^2 = 0 \quad , \quad \lambda_1 = \lambda_2 \quad , \quad \lambda_6 = -\lambda_7 \,.$$

Who are they?

These are the conditions mentioned before and are basis invariant, so they are *not* a basis change of the previous ones.

The r_0 symmetry

$$V = M_0\, r_0 + \Lambda_{00}\, r_0^2 - \vec{M}\cdot\vec{r} - 2\left(\vec{\Lambda}\cdot\vec{r}\right) r_0 + \vec{r}\cdot(\Lambda\,\vec{r})$$

- $\{\vec{M} = \vec{0}\,,\ \vec{\Lambda} = \vec{0}\}$. These are exactly the CP2 ($\vec{r} \to -\vec{r}$).
- $\{M_0 = 0\,,\ \vec{\Lambda} = \vec{0}\}$ These are new, perhaps $r_0 \xrightarrow{?} -r_0$

The r_0 symmetry

$$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix},$$

The transformation

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{pmatrix}$$

implies

$$r_0 \rightarrow -r_0 \qquad r_i \rightarrow +r_i$$

The r_0 symmetry

$$\Phi_1 \to -\Phi_2^* \quad \Phi_1^\dagger \to \Phi_2^T,$$
$$\Phi_2 \to \Phi_1^*, \quad \Phi_2^\dagger \to -\Phi_1^T.$$

- Higgs kinetic terms

$$\mathcal{L}_k = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2),$$

where

$$D^\mu = \partial^\mu + \frac{ig}{2}\sigma_i W_i^\mu + i\frac{g'}{2}B^\mu,$$

$\mathcal{L}_k$ remains invariant if the above transformation of $\Phi_{1,2}$ is supplemented by

$$\partial_\mu \to -i\partial_\mu, \qquad (x^\mu \to ix^\mu),$$
$$B_\mu \to iB_\mu,$$
$$W_{1\mu} \to iW_{1\mu}, \quad W_{2\mu} \to -iW_{2\mu}, \quad W_{3\mu} \to iW_{3\mu}.$$

The r_0 symmetry

- Gauge kinetic terms

$$\mathcal{L}^B = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W_{i\mu\nu} W_i^{\mu\nu},$$

where $B^{\mu\nu} = \partial^\nu B^\mu - \partial^\mu B^\nu$ and $W_i^{\mu\nu} = \partial^\nu W_i^\mu - \partial^\mu W_i^\nu + g\epsilon_{ijk} W_j^\mu W_k^\nu$.
Under r_0 transformation

$$B^{\mu\nu} \to B^{\mu\nu},$$
$$W_1^{\mu\nu} \to W_1^{\mu\nu}, \quad W_2^{\mu\nu} \to -W_2^{\mu\nu}, \quad W_3^{\mu\nu} \to W_3^{\mu\nu}$$

GOOFy systematic approach

Scalars

C-inspired scalar GOOFy transformations:

$$x^\mu \to i x^\mu, \qquad \Phi \to X_\phi \Phi^\star, \qquad \Phi^\dagger \to \Phi^T \bar{X}_\phi{}^\dagger,$$

where X_ϕ and $\bar{X}_\phi$ are both unitary $N_\phi \times N_\phi$ matrices. Requiring

$$\mathcal{L}_{\text{kin}} = \left(\partial_\mu \Phi^\dagger\right)\left(\partial^\mu \Phi\right)$$

to be invariant under the above transformation implies $\bar{X}_\phi = -X_\phi$. Then

$$x^\mu \to i x^\mu, \qquad \Phi \to X_\phi \Phi^\star, \qquad \Phi^\dagger \to -\Phi^T X_\phi^\dagger$$

will constitute the GOOFy transformation.

GOOFy systematic approach

Scalar potential

$$V(\Phi) = \Phi^\dagger M^2 \Phi$$

Hermiticity implies that M^2 is a Hermitian $N_\phi \times N_\phi$ matrix.
Goofy-invariance of the $\Phi^\dagger\Phi$ mass terms requires

$$M^2 = -X_\phi^T \left(M^2\right)^T X_\phi^\star$$

Therefore

$$\mathrm{Tr}\, M^2 = 0$$

This requirement forbids $\Phi^\dagger M^2 \Phi$ mass term in the case of a single complex scalar field ($N_\phi = 1$). For $N_\phi \geq 2$, this type of mass terms is allowed.

GOOFy systematic approach

Fermions

For fermions we also start with C-inspired transformation:

$$x^\mu \to i x^\mu, \qquad \Psi \to -X_\psi \gamma_0 C \Psi^\star, \qquad \bar{\Psi} \to -\Psi^T C^{-1} \bar{X}_\psi^\dagger \tag{2}$$

where $\Psi^T \equiv (\psi_1, \cdots, \psi_{N_\psi})$ denotes a multiplet of N_ψ fermionic fields while X_ψ and $\bar{X}_\psi$ are unitary $N_\psi \times N_\psi$ matrices. Again, we allow for independent field transformations of ψ and $\bar{\psi}$. As before GOOFy symmetries are defined by demanding invariance of

$$\mathcal{L}_{\text{kin}} = i\bar{\Psi}\gamma^\mu \partial_\mu \Psi$$

under (2), which in turn implies $\bar{X}_\psi = -iX_\psi$ i.e.

$$x^\mu \to i x^\mu, \qquad \Psi \to -X_\psi \gamma_0 C \Psi^\star, \qquad \bar{\Psi} \to -\Psi^T C^{-1} i X_\psi^\dagger$$

which will therefore constitute the GOOFy transformation for fermionic fields.

GOOFy systematic approach

Yukawa interactions

We are going to consider the following Yukawa interactions:

$$-\mathcal{L}_Y = \bar{\Psi}\Gamma\Phi\Psi + \bar{\Psi}\Phi^\dagger\Gamma^\dagger\Psi \equiv \bar{\psi}_a(\Gamma_i)_{ab}\phi_i\psi_b + \bar{\psi}_a(\phi_i)^\star(\Gamma_i)^\star_{ba}\psi_b.$$

Requiring invariance of $\mathcal{L}_Y$ under GOOFy transformation

$$x^\mu \to ix^\mu, \qquad \Psi \to -X_\psi\gamma_0 C\Psi^\star, \qquad \bar{\Psi} \to -\Psi^T C^{-1} iX_\psi^\dagger,$$
$$\Phi \to X_\phi\Phi^\star, \qquad \Phi^\dagger \to -\Phi^T X_\phi^\dagger,$$

implies the following condition:

$$\Gamma_i^\star = i\, X_\psi^T \Gamma_j X_\psi^\star\, (X_\phi)_{ji}$$

These equations constitute a fixed point under the running of the RG if and only if they are order by order scale invariant, i.e.

$$\beta^{(n)}(\Gamma_i^\star) = i\, X_\psi^T\, \beta^{(n)}(\Gamma_j)\, X_\psi^\star\, (X_\phi)_{ji}$$

GOOFy-invariant Standard Model

$$\mathrm{Tr}\, M^2 = 0 \Rightarrow M_h^2 = 0$$

Either zero, or degenerate fermion masses, or $\mathrm{Im}\, \Gamma = 0$

$$\Downarrow$$

no CP violation

GOOFy-invariant 2HDM

Adopting the generic strategy (described and tested earlier in various contexts), the GOOFy transformation for 2HDM shall be:

$$\Phi \equiv \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$$

$$\begin{aligned}
\Phi &\to X_\phi \Phi^\star, & \Phi^\dagger &\to -\Phi^T X_\phi^\dagger, \\
q_L &\to -X_\alpha \gamma^0 C q_L^\star, & \bar{q}_L &\to -i q_L^T C^{-1} X_\alpha^\dagger, \\
d_R &\to -X_\beta \gamma^0 C d_R^\star, & \bar{d}_R &\to -i d_R^T C^{-1} X_\beta^\dagger, \\
u_R &\to -X_\gamma \gamma^0 C u_R^\star, & \bar{u}_R &\to -i u_R^T C^{-1} X_\gamma^\dagger.
\end{aligned}$$

GOOFy-invariant 2HDM

The most general scalar potential for the 2HDM reads

$$
\begin{aligned}
V \;=\; & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.}c.] \\
& + \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\
& + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\
& + \left\{ \frac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)] \Phi_1^\dagger \Phi_2 + \text{h.}c. \right\}
\end{aligned}
$$

where, other than m_{12}^2 and $\lambda_{5,6,7}$, all parameters are real.

The generic Yukawa Lagrangian for this model is as follows

$$
-\mathcal{L}_Y \;=\; \bar{q}_L \Gamma_a \Phi_a d_R + \bar{q}_L \Delta_a \tilde{\Phi}_a u_R + \bar{d}_R \Gamma_a^\dagger \Phi_a^\dagger q_L + \bar{u}_R \Delta_a^\dagger (\tilde{\Phi}_a)^\dagger q_L,
$$

with $a = 1, 2$.

GOOFy-invariant 2HDM

$$\Phi^\dagger M^2 \Phi - \text{invariance} \quad \Rightarrow \quad X_\phi = \begin{pmatrix} 0 & e^{i\theta_1} \\ e^{i\theta_2} & 0 \end{pmatrix}$$

$$\text{basis choice} \quad \Rightarrow \quad X_\psi = \begin{pmatrix} \cos\theta_\psi & \sin\theta_\psi & 0 \\ -\sin\theta_\psi & \cos\theta_\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

for $\psi = \alpha, \beta$ and γ.

GOOFy-invariant 2HDM

Strategy for 2HDM

- $$\Gamma_1 = -iX_\alpha^T \Gamma_2^\star X_\beta^\star e^{-i\theta_2}, \qquad \Gamma_2 = -iX_\alpha^T \Gamma_1^\star X_\beta^\star e^{-i\theta_1} \quad (\star)$$
- Find the determinant (in terms of $\Delta\theta \equiv \theta_1 - \theta_2, \alpha, \beta, \gamma$) of the resulting coefficient matrix for $(\Gamma_{1,2})_{ij}$ and solve for its zeros (constraints/solutions for $\Delta\theta, \alpha, \beta, \gamma$).
- For $\Delta\theta$ obtained above check invariance of the potential and find relations between parameters that must be satisfied for the invariance. For 2HDM $\Delta\theta$'s obtained from determinants being zero allow for invariant potential.
- Adopt $\Delta\theta, \alpha, \beta, \gamma$ obtained above in the equations for Yukawa's $(*)$ to determine final form of the Yukawa matrices.

GOOFy-invariant 2HDM

The determinant D is then the product of the determinants of the four blocks,

$$D = D_1 D_2 D_3 D_4$$

where

$$\begin{aligned}
D_1 &= 256 \sin^2\left(\alpha + \beta - \frac{\Delta\theta}{2}\right) \sin^2\left(\alpha - \beta + \frac{\Delta\theta}{2}\right) \\
&\quad \times \sin^2\left(\alpha - \beta - \frac{\Delta\theta}{2}\right) \sin^2\left(\alpha + \beta + \frac{\Delta\theta}{2}\right), \\
D_2 &= 16 \sin^2\left(\alpha - \frac{\Delta\theta}{2}\right) \sin^2\left(\alpha + \frac{\Delta\theta}{2}\right), \qquad (3) \\
D_3 &= 16 \sin^2\left(\beta - \frac{\Delta\theta}{2}\right) \sin^2\left(\beta + \frac{\Delta\theta}{2}\right), \\
D_4 &= 4 \sin^2\left(\frac{\Delta\theta}{2}\right).
\end{aligned}$$

GOOFy-invariant 2HDM

The following solutions allow for all quark masses to be non-zero.

Solution A: $(D_1 = D_2 = D_3 = D_4 = 0)$, $\Delta\theta = \alpha = \beta = 0$,

$$\Gamma_1 = \begin{pmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \end{pmatrix}, \quad \Gamma_2 = -ie^{-i\theta_1} \begin{pmatrix} z_{11}^\star & z_{12}^\star & z_{13}^\star \\ z_{21}^\star & z_{22}^\star & z_{23}^\star \\ z_{31}^\star & z_{32}^\star & z_{33}^\star \end{pmatrix}$$

Solution D-1: $(D_2 \neq 0,\ D_3 \neq 0,\ D_1 = D_4 = 0)$, $\Delta\theta = 0,\ \beta = \alpha = \frac{\pi}{2}$,

$$\Gamma_1 = \begin{pmatrix} z_{11} & z_{12} & 0 \\ z_{21} & z_{22} & 0 \\ 0 & 0 & z_{33} \end{pmatrix}, \quad \Gamma_2 = -ie^{-i\theta_1} \begin{pmatrix} z_{22}^\star & -z_{21}^\star & 0 \\ -z_{12}^\star & z_{11}^\star & 0 \\ 0 & 0 & z_{33}^\star \end{pmatrix}$$

Solution D-2: $(D_2 \neq 0,\ D_3 \neq 0,\ D_1 = D_4 = 0)$, $\Delta\theta = 0,\ \beta = \alpha,\ \alpha \in (0, \frac{\pi}{2})$,

$$\Gamma_1 = \begin{pmatrix} z_{11} & z_{12} & 0 \\ -z_{12} & z_{11} & 0 \\ 0 & 0 & z_{33} \end{pmatrix}, \quad \Gamma_2 = -ie^{-i\theta_1} \begin{pmatrix} z_{11}^\star & z_{12}^\star & 0 \\ -z_{12}^\star & z_{11}^\star & 0 \\ 0 & 0 & z_{33}^\star \end{pmatrix}$$

GOOFy-invariant 2HDM

Requiring GOOFy invariance of the 2HDM potential one finds that only four options survive that correspond to different choices of $\Delta\theta \equiv \theta_1 - \theta_2$ and different relations between potential parameters, i.e.:

$$\Delta\theta = 0: \quad m_{11}^2 = m_{22}^2 = 0, \quad m_{12}^2 = 0, \quad \lambda_2 = \lambda_1, \quad \lambda_7 = \lambda_6,$$

$$\Delta\theta = \pi: \quad m_{11}^2 = m_{22}^2 = 0, \quad \lambda_2 = \lambda_1, \quad \lambda_7 = -\lambda_6,$$

$$\Delta\theta = \frac{2\pi}{3}: \quad m_{11}^2 = m_{22}^2 = 0, \quad m_{12}^2 = 0, \quad \lambda_2 = \lambda_1, \quad \lambda_5 = \lambda_6 = \lambda_7 = 0,$$

$$\Delta\theta = \frac{4\pi}{3}: \quad m_{11}^2 = m_{22}^2 = 0, \quad m_{12}^2 = 0, \quad \lambda_2 = \lambda_1, \quad \lambda_5 = \lambda_6 = \lambda_7 = 0.$$

Summary and conclusions

- A class of C-inspired GOOFy transformations of scalars and fermions has been defined and discussed.
- In all cases considered, the constraints implied by GOOFy were RG stable at least up to 2 loops (adopting PyR@TE 3 code).
- We have observed all-order stability for the realistic solution $\Delta\theta = 0$:

$$\Delta\theta = 0: \quad m_{11}^2 + m_{22}^2 = 0, \quad m_{12}^2 = 0, \quad \lambda_2 = \lambda_1, \quad \lambda_7 = \lambda_6$$

- Regardless if GOOFy is a genuine symmetry, resulting relations between parameters are RG-stable.
- Mass term in the SM is vanishing by GOOFy invariance.
- 2HDM might be consistent with GOOFy both in bosonic sector and in Yukawa interactions.

Summary and conclusions

- The constraints could be seen as emerging from the following "GOOFy/r_0 symmetry":

$$x^\mu \to i x^\mu,$$
$$\Phi \to X_\phi \Phi^\star, \qquad \Phi^\dagger \to -\Phi^T X_\phi^\dagger, \qquad A_\mu \to i A_\mu$$
$$\Psi \to -X_\psi \gamma_0 C \Psi^\star, \qquad \bar{\Psi} \to -\Psi^T C^{-1} i X_\psi^\dagger$$

Interesting all-order fixed points:

$$\Delta\theta = 0: \quad m_{11}^2 + m_{22}^2 = 0, \quad m_{12}^2 = 0, \quad \lambda_2 = \lambda_1, \quad \lambda_7 = \lambda_6,$$
$$\Delta\theta = \pi: \quad m_{11}^2 + m_{22}^2 = 0, \quad \lambda_2 = \lambda_1, \quad \lambda_7 = -\lambda_6$$

- Application: finding RG stable relations between parameters.

Backup slides

Phenomenology of the r_0 symmetry (semisymmetry)

The set of 11 independent physical parameters of 2HDM:

$$\mathcal{P} \equiv \{M_{H^\pm}^2, M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3, q\}$$

The kinetic Lagrangian:

$$\mathcal{L}_k = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2)$$

$$\text{Coefficient}\left(\mathcal{L}_k, Z^\mu \left[H_j \overleftrightarrow{\partial_\mu} H_i\right]\right) = \frac{g}{2v \cos\theta_W} \epsilon_{ijk} e_k$$

$$\text{Coefficient}\left(\mathcal{L}_k, H_i Z^\mu Z^\nu\right) = \frac{g^2}{4 \cos^2\theta_W} e_i \, g_{\mu\nu}$$

$$\text{Coefficient}\left(\mathcal{L}_k, H_i W^{+\mu} W^{-\nu}\right) = \frac{g^2}{2} e_i \, g_{\mu\nu}$$

$$q_i \equiv \text{Coefficient}(V, H_i H^+ H^-)$$

$$q \equiv \text{Coefficient}(V, H^+ H^+ H^- H^-)$$

Phenomenology of the r_0 symmetry (semisymmetry)

CP-sensitive invariants in the bilinear notation

$$I_1 = \left(\vec{M} \times \vec{\Lambda}\right) \cdot \left(\Lambda \vec{M}\right)$$
$$I_2 = \left(\vec{M} \times \vec{\Lambda}\right) \cdot \left(\Lambda \vec{\Lambda}\right)$$
$$I_3 = \left[\vec{M} \times \left(\Lambda \vec{M}\right)\right] \cdot \left(\Lambda^2 \vec{M}\right)$$
$$I_4 = \left[\vec{\Lambda} \times \left(\Lambda \vec{\Lambda}\right)\right] \cdot \left(\Lambda^2 \vec{\Lambda}\right)$$

Since the r_0 symmetry implies $\vec{\Lambda} = \vec{0}$ the invariants $I_{1,2,4}$ are automatically zero. However

$$I_3 = -16\lambda_5\, m_{11}^2\, \mathrm{Im}(m_{12}^2)\, \mathrm{Re}(m_{12}^2)\, \left[(\lambda_1 - \lambda_3 - \lambda_4)^2 - \lambda_5^2\right] \neq 0$$

explicit violation of CP

Phenomenology of the r_0 symmetry (semisymmetry)

Stationary-point equations:

$$\begin{aligned} m_{11}^2 &= \frac{1}{2}\lambda_1\left(v_2^2 - v_1^2\right), \\ \mathrm{Re}\, m_{12}^2 &= \frac{1}{2} v_1 v_2 \cos\xi \left(\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5\right), \\ \mathrm{Im}\, m_{12}^2 &= -\frac{1}{2} v_1 v_2 \sin\xi \left(\lambda_1 + \lambda_3 + \lambda_4 - \lambda_5\right). \end{aligned}$$

The neutral sector rotation matrix is then given by

$$R = \begin{pmatrix} \frac{v_2 \cos\xi}{v} & \frac{v_1 \cos\xi}{v} & -\sin\xi \\ -\frac{v_1}{v} & \frac{v_2}{v} & 0 \\ \frac{v_2 \sin\xi}{v} & \frac{v_1 \sin\xi}{v} & \cos\xi \end{pmatrix},$$

yielding masses

$$M_1^2 = \frac{1}{2} v^2 \left(\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5\right), \quad M_2^2 = \lambda_1 v^2,$$

$$M_3^2 = \frac{1}{2} v^2 \left(\lambda_1 + \lambda_3 + \lambda_4 - \lambda_5\right), \quad M_{H^\pm}^2 = \frac{1}{2}\left(\lambda_1 + \lambda_3\right) v^2$$

No decoupling limit!

Phenomenology of the r_0 symmetry (semisymmetry)

Assuming that M_2 is the SM-like Higgs boson, we obtain from unitarity and boundedness-from-below constraints:

$$\begin{aligned} M_{H^\pm} &\leq 711 \text{ GeV}, \\ M_3 &\leq 712 \text{ GeV}, \\ M_1 &\leq 711 \text{ GeV} \end{aligned}$$

Input parameters:

$$\mathcal{P} \equiv \{M_{H^\pm}^2, M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3, q\}$$

Constraints implied by the r_0 symmetry:

$$v^2(e_1 q_2 - e_2 q_1) + e_1 e_2 (M_2^2 - M_1^2) = 0, \quad v^2(e_1 q_3 - e_3 q_1) + e_1 e_3 (M_3^2 - M_1^2) = 0,$$

$$v^2(e_2 q_3 - e_3 q_2) + e_2 e_3 (M_3^2 - M_2^2) = 0, \quad q = \frac{1}{2v^4}(e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2),$$

$$M_{H^\pm}^2 = \frac{1}{2}(e_1 q_1 + e_2 q_2 + e_3 q_3) + \frac{1}{2v^2}(e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2),$$