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CZECH TECHNICAL UNIVERSITY IN PRAGUE

Radiative proton decay from Peccei-Quinn symmetry

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Czech Technical University (CTU) in Prague

Presentation based on the work:

Phys.Lett.B 881 (2026) 140918 (arXiv: 2607.02026 [hep-ph])

Proton stability in the Standard Model

Accidental symmetries

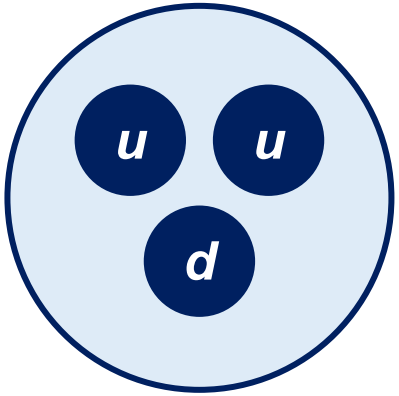
$$\mathrm{U}(1)_B \times \mathrm{U}(1)_{L_e} \times \mathrm{U}(1)_{L_\mu} \times \mathrm{U}(1)_{L_\tau}$$

Proton stability in the Standard Model

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No gauge-invariant operator with $d \leq 4$ violates **baryon (*B*)** or **lepton (*L*) number**

	Fields	$\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$	B	L
Leptons	ℓ_L	$(\mathbf{1}, \mathbf{2}, -1/2)$	0	1
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Quarks	q_L	$(\mathbf{3}, \mathbf{2}, 1/6)$	1/3	0
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Proton

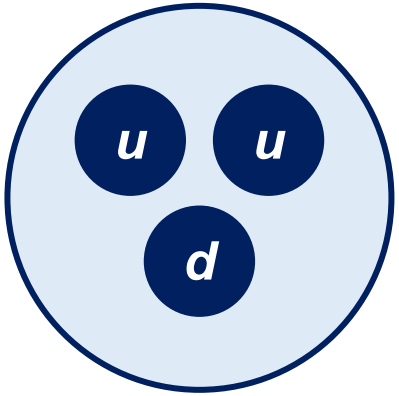
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B* and *L* are anomalous: electroweak instantons/sphalerons violate *B*+*L* (Δ*B* = Δ*L* = 3) but conserve *B*−*L

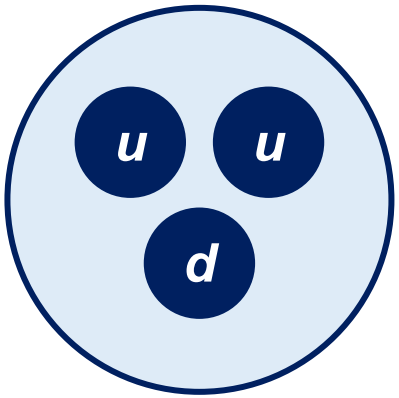
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Proton stability is an accident of the renormalizable SM: *B* and *L* are conserved, so the proton cannot decay

Beyond the Standard Model

Proton decay

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \underbrace{\frac{(\ell_L \Phi)(\ell_L \Phi)}{\Lambda}}_{d=5} + \underbrace{\frac{q\,q\,q\,\ell}{\Lambda^2}}_{d=6} + \cdots$$

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***d* = 5: Weinberg operator**

$$\Delta L = 2 \quad \Rightarrow \quad m_\nu \sim v^2/\Lambda$$

Majorana neutrino masses

Weinberg (1979)

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***d* = 6: Baryon-number violation**

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***B+L* violated by two units ⇒ nucleon decay**

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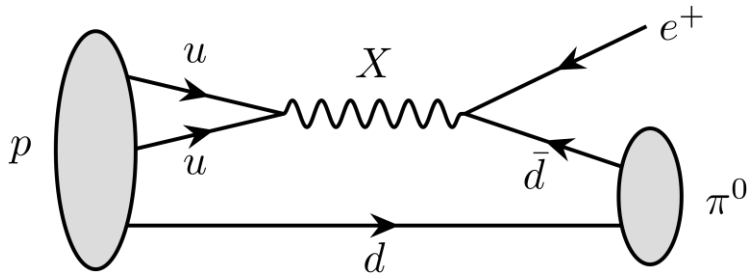
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UV completion: Grand Unified Theories



SU(5): heavy gauge bosons *X* generate *qqqℓ*/M_X^2 operators at tree level

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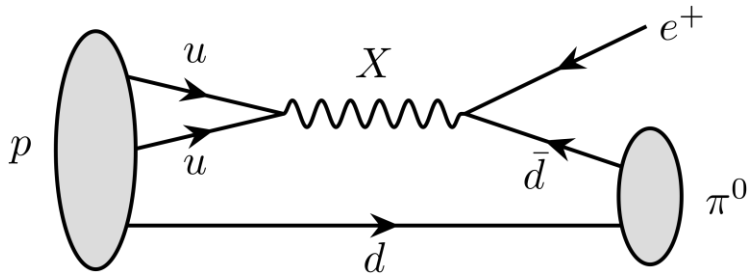
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Golden channel $p \to e^+\pi^0$

$$\tau(p \to e^+\pi^0) > 2.4 \times 10^{34}\ \text{yr}$$

Super-Kamiokande (2020), 90% CL

$$\Rightarrow \quad \Lambda \sim \mathcal{O}(10^{16})\ \text{GeV}$$

Next generation:
Hyper-K (about 10× in lifetime),
DUNE, JUNO

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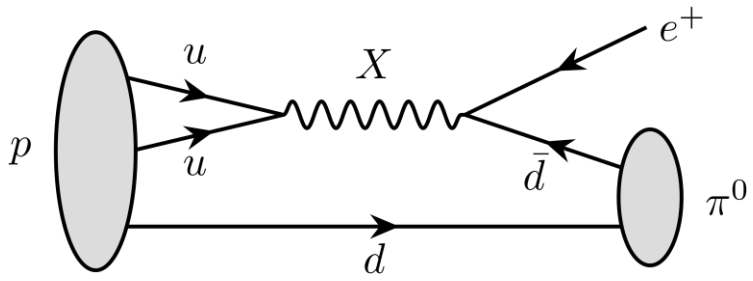
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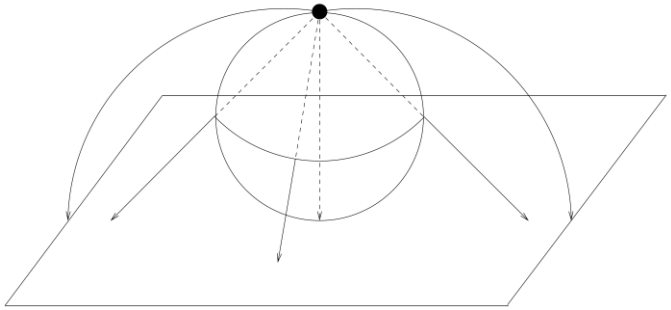
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In the SMEFT proton decay starts at *d* = 6, conserving *B*−*L*, and probes Λ ~ 10^{16} GeV

Beyond the Standard Model

Strong CP problem

Topological



P and CP violation in QCD

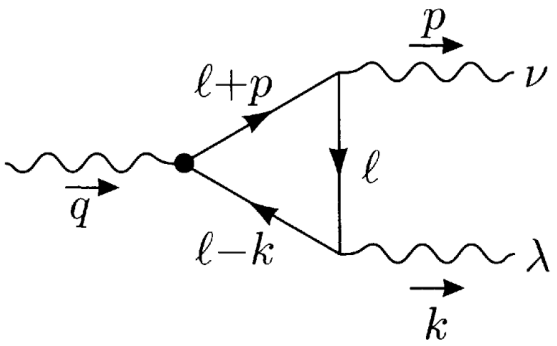
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Strong CP phase

$$\overline{\theta} = \theta_0 + \theta_F$$

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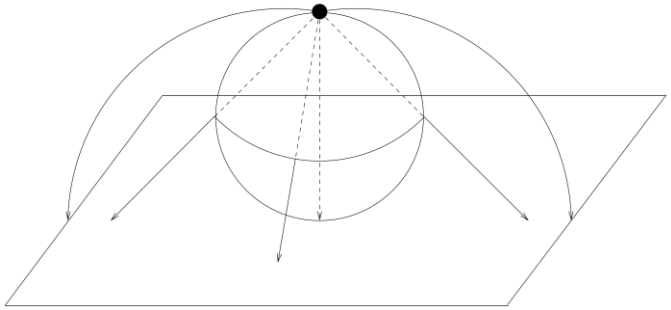
Anomalous chiral rotations



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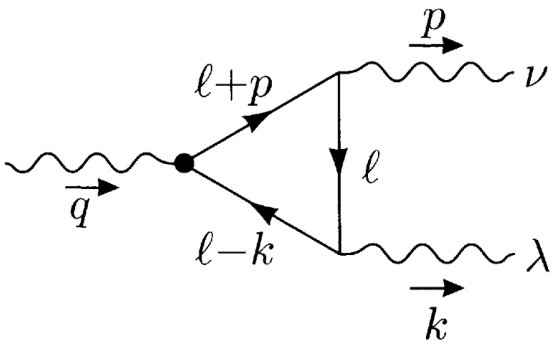
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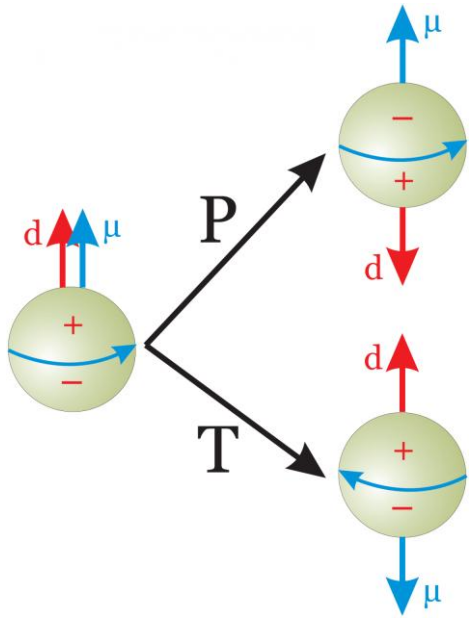
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Neutron electric dipole moment

$$|d_n| < 3\times10^{-26}\ e\cdot\mathrm{cm} \Rightarrow \left|\overline{\theta}\right| < 10^{-10}$$

Baker et al (2006), Pendlebury et al (2015)

Weak sector CP violation

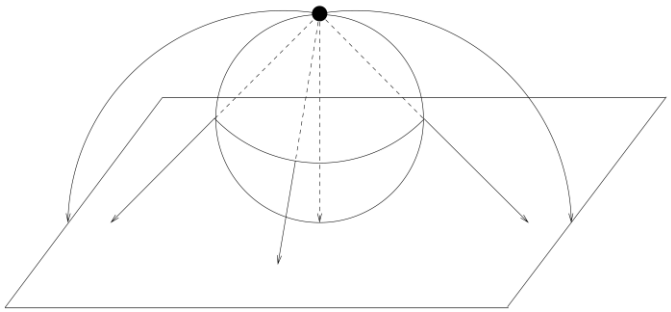
$$\delta \sim \mathcal{O}(1)$$

$$\mathbf{V}_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

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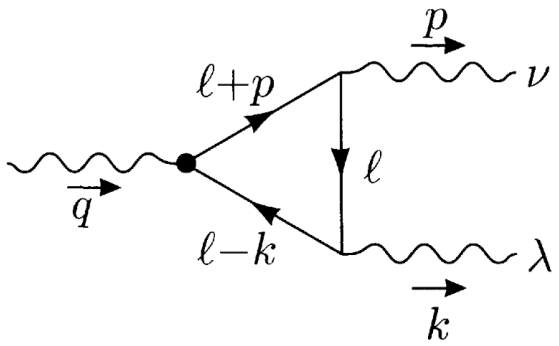
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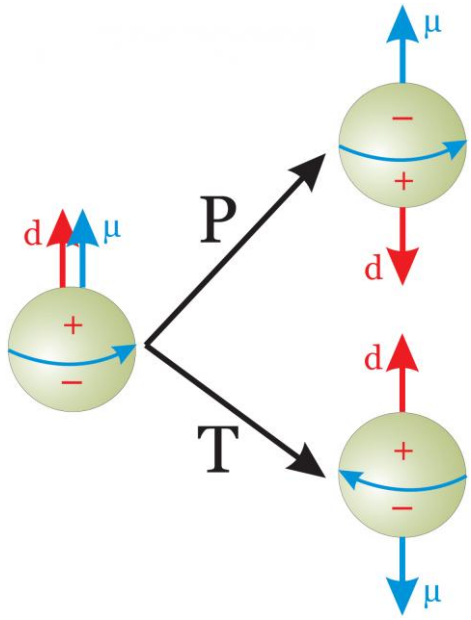
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Why does QCD seem to preserve CP ?

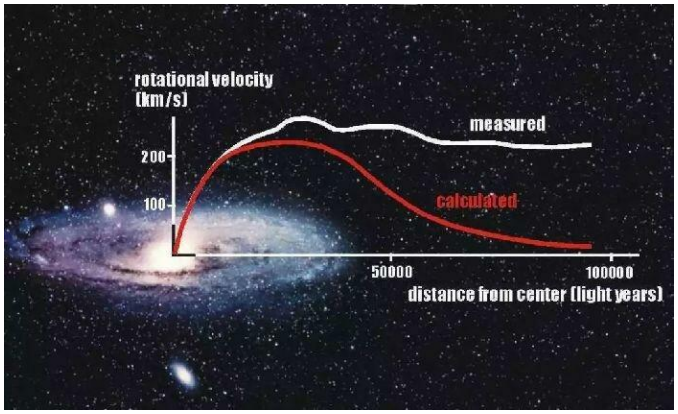
- **Nelson-Barr mechanism** — Nelson, Barr (1984)
- **Axion solution** — Peccei,Quinn (1977), Weinberg (1978), Wilczek (1978)

Beyond the Standard Model

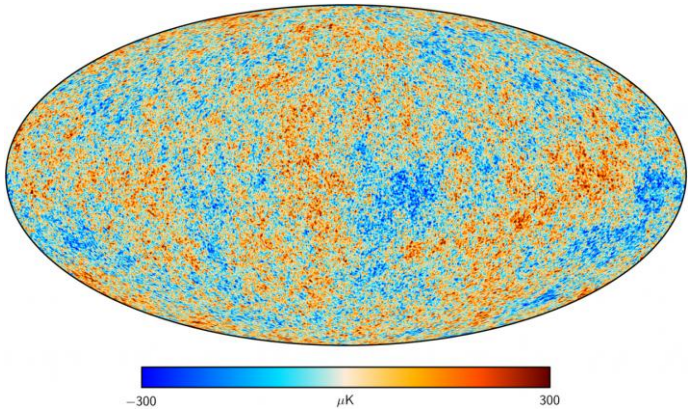
Dark matter

Astrophysical evidence, for example from:

Galactic rotation curves



CMB Anisotropies

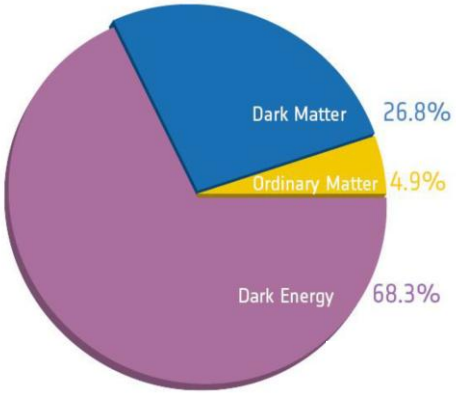


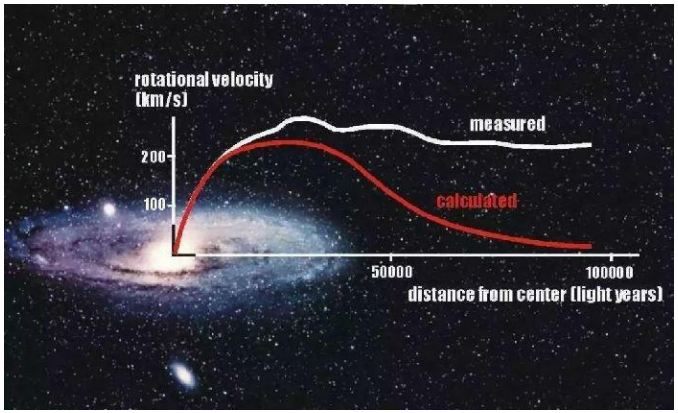
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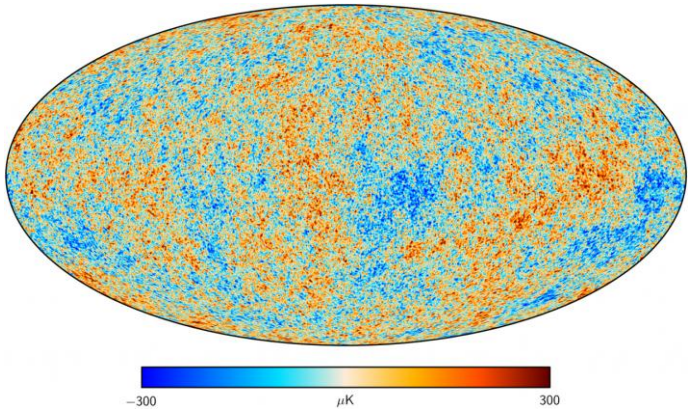
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CMB Anisotropies



Cold Dark Matter relic density

$$\Omega_{\rm CDM} h^2 = 0.1200 \pm 0.0012$$

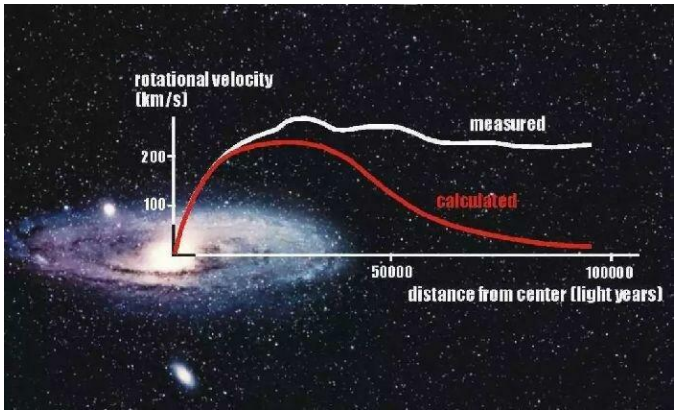
Planck Collab. (2018)

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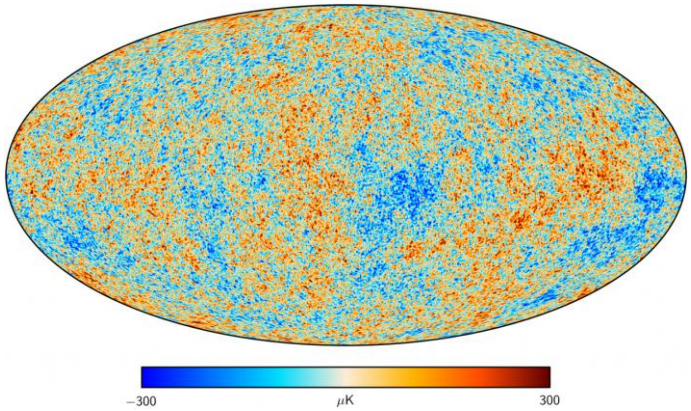
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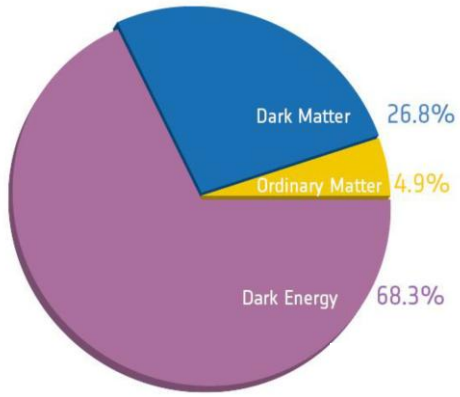
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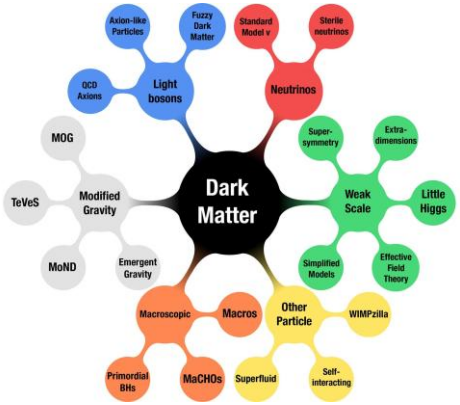




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Properties of a **particle DM candidate: cold, neutral, non-baryonic, stable**

- **Dark-sectors** featuring **weakly interacting massive particle (WIMP) DM** see Kolb & Turner (1990)
- **Axion DM** from **misalignment mechanism** Callan et al. (1978); Gross et al. (1981); Dimopoulos et al. (2008)

Motivation

p

Proton stability

B+*L* is an accidental symmetry of the SM: quarks 1/3, leptons 1

θ

Strong CP problem

Axion solution: a global $U(1)_{PQ}$ with a QCD anomaly

DM

Dark matter

The axion is a natural cold dark matter (DM) candidate

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This work

- Vector-like quarks chirally charged under PQ: **KSVZ-type axion**
- **Axion DM** in pre and post inflationary cosmology
- Residual Z_2 forbids tree-level proton decay
- Proton decay generated **spontaneously** and **at one loop**

B+*L* ≡ PQ with tree-level proton decay: Reig, Srivastava (2019); Ohata, Takeuchi, Tsumura (2021)
One-loop proton decay: Helo, Hirsch, Ota (2019); with a residual Z_4 and WIMP DM: Kumar, Srivastava (2026)

One symmetry links the axion (strong CP + DM) to proton decay

Framework

Vector-like quarks

Colored scalars

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				Case A	Case B	Case C	
Fermion	$\Psi_L; \Psi_R$	$\mathbf{R}_\Psi$	$\mathbf{n}$	Y_Ψ	Y_Ψ	Y_Ψ	$5/6; -1/6 \to -$
	$F_L; F_R$	$\mathbf{R}_F$	$\mathbf{n}$	$-Y_\Psi - 1/3$	$-Y_\Psi - 1/3$	$-Y_\Psi - 4/3$	$1/2; -1/2 \to -$
Scalar	S	$\mathbf{R}_S$	$\mathbf{n}$	$-Y_\Psi + 1/3$	$-Y_\Psi - 2/3$	$-Y_\Psi - 2/3$	$-1/6 \to -$
	χ	$\mathbf{R}_\chi$	$\mathbf{n}$	$Y_\Psi - 2/3$	$Y_\Psi - 2/3$	$Y_\Psi + 1/3$	$1/2 \to -$

$\mathbf{R}_\Psi$	$\mathbf{R}_F$	$\mathbf{R}_S$	$\mathbf{R}_\chi$	N/n
$\mathbf{1}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	1
$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	1
$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{3}$	2
$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{8}$	$\mathbf{3}$	2
$\bar{\mathbf{3}}$	$\mathbf{6}$	$\mathbf{8}$	$\bar{\mathbf{6}}$	6
$\mathbf{6}$	$\bar{\mathbf{3}}$	$\mathbf{8}$	$\mathbf{3}$	6
$\mathbf{3}$	$\mathbf{8}$	$\mathbf{3}$	$\mathbf{8}$	7
$\mathbf{3}$	$\mathbf{8}$	$\bar{\mathbf{6}}$	$\mathbf{8}$	7
$\mathbf{8}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	7
$\mathbf{8}$	$\mathbf{3}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	7
$\bar{\mathbf{6}}$	$\mathbf{8}$	$\mathbf{3}$	$\mathbf{8}$	11
$\mathbf{8}$	$\bar{\mathbf{6}}$	$\bar{\mathbf{3}}$	$\mathbf{6}$	11

Framework

Vector-like quarks

Colored scalars

	Fields	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$			$\mathrm{U}(1)_{\mathrm{PQ}} \equiv \mathrm{U}(1)_{B+L} \to \mathcal{Z}_2$
				Case A	Case B	Case C	
Fermion	$\Psi_L; \Psi_R$	$\mathbf{R}_\Psi$	$\mathbf{n}$	Y_Ψ	Y_Ψ	Y_Ψ	$5/6; -1/6 \to -$
	$F_L; F_R$	$\mathbf{R}_F$	$\mathbf{n}$	$-Y_\Psi - 1/3$	$-Y_\Psi - 1/3$	$-Y_\Psi - 4/3$	$1/2; -1/2 \to -$
Scalar	S	$\mathbf{R}_S$	$\mathbf{n}$	$-Y_\Psi + 1/3$	$-Y_\Psi - 2/3$	$-Y_\Psi - 2/3$	$-1/6 \to -$
	χ	$\mathbf{R}_\chi$	$\mathbf{n}$	$Y_\Psi - 2/3$	$Y_\Psi - 2/3$	$Y_\Psi + 1/3$	$1/2 \to -$

$\mathbf{R}_\Psi$	$\mathbf{R}_F$	$\mathbf{R}_S$	$\mathbf{R}_\chi$	N/n
$\mathbf{1}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	1
$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	1
$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{3}$	2
$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{8}$	$\mathbf{3}$	2
$\bar{\mathbf{3}}$	$\mathbf{6}$	$\mathbf{8}$	$\bar{\mathbf{6}}$	6
$\mathbf{6}$	$\bar{\mathbf{3}}$	$\mathbf{8}$	$\mathbf{3}$	6
$\mathbf{3}$	$\mathbf{8}$	$\mathbf{3}$	$\mathbf{8}$	7
$\mathbf{3}$	$\mathbf{8}$	$\bar{\mathbf{6}}$	$\mathbf{8}$	7
$\mathbf{8}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	7
$\mathbf{8}$	$\mathbf{3}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	7
$\bar{\mathbf{6}}$	$\mathbf{8}$	$\mathbf{3}$	$\mathbf{8}$	11
$\mathbf{8}$	$\bar{\mathbf{6}}$	$\bar{\mathbf{3}}$	$\mathbf{6}$	11

Framework

Vector-like quarks

Colored scalars

	Fields	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$ Case A	$\mathrm{U}(1)_Y$ Case B	$\mathrm{U}(1)_Y$ Case C	$\mathrm{U}(1)_{\mathrm{PQ}} \equiv \mathrm{U}(1)_{B+L} \to \mathcal{Z}_2$
Fermion	$\Psi_L; \Psi_R$	$\mathbf{R}_\Psi$	$\mathbf{n}$	Y_Ψ	Y_Ψ	Y_Ψ	$5/6; -1/6 \to -$
	$F_L; F_R$	$\mathbf{R}_F$	$\mathbf{n}$	$-Y_\Psi - 1/3$	$-Y_\Psi - 1/3$	$-Y_\Psi - 4/3$	$1/2; -1/2 \to -$
Scalar	S	$\mathbf{R}_S$	$\mathbf{n}$	$-Y_\Psi + 1/3$	$-Y_\Psi - 2/3$	$-Y_\Psi - 2/3$	$-1/6 \to -$
	χ	$\mathbf{R}_\chi$	$\mathbf{n}$	$Y_\Psi - 2/3$	$Y_\Psi - 2/3$	$Y_\Psi + 1/3$	$1/2 \to -$

Strong CP problem

Dynamical solution

Peccei,Quinn (1977),

Weinberg (1978), Wilczek (1978)

$$\left(\frac{a}{f_a} - \bar{\theta}\right) \frac{\alpha_s}{8\pi} G\tilde{G} \qquad \bar{\theta}_{\text{eff}} = \bar{\theta} + \left\langle \frac{a}{f_a} \right\rangle = 0$$

$\mathbf{R}_\Psi$	$\mathbf{R}_F$	$\mathbf{R}_S$	$\mathbf{R}_\chi$	N/n
$\mathbf{1}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	1
$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	1
$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{3}$	2
$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{8}$	$\mathbf{3}$	2
$\bar{\mathbf{3}}$	$\mathbf{6}$	$\mathbf{8}$	$\bar{\mathbf{6}}$	6
$\mathbf{6}$	$\bar{\mathbf{3}}$	$\mathbf{8}$	$\mathbf{3}$	6
$\mathbf{3}$	$\mathbf{8}$	$\mathbf{3}$	$\mathbf{8}$	7
$\mathbf{3}$	$\mathbf{8}$	$\bar{\mathbf{6}}$	$\mathbf{8}$	7
$\mathbf{8}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	7
$\mathbf{8}$	$\mathbf{3}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	7
$\bar{\mathbf{6}}$	$\mathbf{8}$	$\mathbf{3}$	$\mathbf{8}$	11
$\mathbf{8}$	$\bar{\mathbf{6}}$	$\bar{\mathbf{3}}$	$\mathbf{6}$	11

Framework

Vector-like quarks

Colored scalars

	Fields	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$ Case A	$\mathrm{U}(1)_Y$ Case B	$\mathrm{U}(1)_Y$ Case C	$\mathrm{U}(1)_{\mathrm{PQ}} \equiv \mathrm{U}(1)_{B+L} \to \mathcal{Z}_2$
Fermion	$\Psi_L; \Psi_R$	$\mathbf{R}_\Psi$	$\mathbf{n}$	Y_Ψ	Y_Ψ	Y_Ψ	$5/6; -1/6 \to -$
	$F_L; F_R$	$\mathbf{R}_F$	$\mathbf{n}$	$-Y_\Psi - 1/3$	$-Y_\Psi - 1/3$	$-Y_\Psi - 4/3$	$1/2; -1/2 \to -$
Scalar	S	$\mathbf{R}_S$	$\mathbf{n}$	$-Y_\Psi + 1/3$	$-Y_\Psi - 2/3$	$-Y_\Psi - 2/3$	$-1/6 \to -$
	χ	$\mathbf{R}_\chi$	$\mathbf{n}$	$Y_\Psi - 2/3$	$Y_\Psi - 2/3$	$Y_\Psi + 1/3$	$1/2 \to -$

Strong CP problem

Dynamical solution

Peccei,Quinn (1977),

Weinberg (1978), Wilczek (1978)

$$\left(\frac{a}{f_a} - \bar{\theta}\right) \frac{\alpha_s}{8\pi} G\tilde{G} \qquad \bar{\theta}_{\text{eff}} = \bar{\theta} + \left\langle \frac{a}{f_a} \right\rangle = 0$$

Complex scalar singlet

$$\sigma = \frac{v_\sigma + \rho}{\sqrt{2}} e^{ia_\sigma / v_\sigma}$$

$\mathbf{R}_\Psi$	$\mathbf{R}_F$	$\mathbf{R}_S$	$\mathbf{R}_\chi$	N/n
$\mathbf{1}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	1
$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	1
$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{3}$	2
$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{8}$	$\mathbf{3}$	2
$\bar{\mathbf{3}}$	$\mathbf{6}$	$\mathbf{8}$	$\bar{\mathbf{6}}$	6
$\mathbf{6}$	$\bar{\mathbf{3}}$	$\mathbf{8}$	$\mathbf{3}$	6
$\mathbf{3}$	$\mathbf{8}$	$\mathbf{3}$	$\mathbf{8}$	7
$\mathbf{3}$	$\mathbf{8}$	$\bar{\mathbf{6}}$	$\mathbf{8}$	7
$\mathbf{8}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	7
$\mathbf{8}$	$\mathbf{3}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	7
$\bar{\mathbf{6}}$	$\mathbf{8}$	$\mathbf{3}$	$\mathbf{8}$	11
$\mathbf{8}$	$\bar{\mathbf{6}}$	$\bar{\mathbf{3}}$	$\mathbf{6}$	11

Framework

Vector-like quarks

KSVZ (1979, 1980)

Colored scalars

	Fields	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$			$\mathrm{U}(1)_{\mathrm{PQ}} \equiv \mathrm{U}(1)_{B+L} \to \mathcal{Z}_2$
				Case A	Case B	Case C	
Fermion	$\Psi_L; \Psi_R$	$\mathbf{R}_\Psi$	$\mathbf{n}$	Y_Ψ	Y_Ψ	Y_Ψ	$5/6; -1/6 \to -$
	$F_L; F_R$	$\mathbf{R}_F$	$\mathbf{n}$	$-Y_\Psi - 1/3$	$-Y_\Psi - 1/3$	$-Y_\Psi - 4/3$	$1/2; -1/2 \to -$
Scalar	S	$\mathbf{R}_S$	$\mathbf{n}$	$-Y_\Psi + 1/3$	$-Y_\Psi - 2/3$	$-Y_\Psi - 2/3$	$-1/6 \to -$
	χ	$\mathbf{R}_\chi$	$\mathbf{n}$	$Y_\Psi - 2/3$	$Y_\Psi - 2/3$	$Y_\Psi + 1/3$	$1/2 \to -$

Strong CP problem

Dynamical solution

Peccei,Quinn (1977),

Weinberg (1978), Wilczek (1978)

$$\left(\frac{a}{f_a} - \bar{\theta}\right) \frac{\alpha_s}{8\pi} G\tilde{G} \qquad \bar{\theta}_{\mathrm{eff}} = \bar{\theta} + \left\langle \frac{a}{f_a} \right\rangle = 0$$

Complex scalar singlet

$$\sigma = \frac{v_\sigma + \rho}{\sqrt{2}} e^{i a_\sigma / v_\sigma}$$

Axion decay constant

$$f_a = \frac{f_{\mathrm{PQ}}}{N} = \frac{v_\sigma}{\sqrt{2} N}$$

Color-anomaly factor

$$N = 2\, n \left[T(\mathbf{R}_\Psi) + T(\mathbf{R}_F) \right]$$

$\mathbf{R}_\Psi$	$\mathbf{R}_F$	$\mathbf{R}_S$	$\mathbf{R}_\chi$	N/n
$\mathbf{1}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	1
$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	1
$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{3}$	2
$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{8}$	$\mathbf{3}$	2
$\bar{\mathbf{3}}$	$\mathbf{6}$	$\mathbf{8}$	$\bar{\mathbf{6}}$	6
$\mathbf{6}$	$\bar{\mathbf{3}}$	$\mathbf{8}$	$\mathbf{3}$	6
$\mathbf{3}$	$\mathbf{8}$	$\mathbf{3}$	$\mathbf{8}$	7
$\mathbf{3}$	$\mathbf{8}$	$\bar{\mathbf{6}}$	$\mathbf{8}$	7
$\mathbf{8}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	7
$\mathbf{8}$	$\mathbf{3}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	7
$\bar{\mathbf{6}}$	$\mathbf{8}$	$\mathbf{3}$	$\mathbf{8}$	11
$\mathbf{8}$	$\bar{\mathbf{6}}$	$\bar{\mathbf{3}}$	$\mathbf{6}$	11

Framework

Vector-like quarks

KSVZ (1979, 1980)

Colored scalars

	Fields	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$ Case A	$\mathrm{U}(1)_Y$ Case B	$\mathrm{U}(1)_Y$ Case C	$\mathrm{U}(1)_{PQ} \equiv \mathrm{U}(1)_{B+L} \to \mathcal{Z}_2$
Fermion	$\Psi_L; \Psi_R$	$\mathbf{R}_\Psi$	$\mathbf{n}$	Y_Ψ	Y_Ψ	Y_Ψ	$5/6; -1/6 \to -$
	$F_L; F_R$	$\mathbf{R}_F$	$\mathbf{n}$	$-Y_\Psi - 1/3$	$-Y_\Psi - 1/3$	$-Y_\Psi - 4/3$	$1/2; -1/2 \to -$
Scalar	S	$\mathbf{R}_S$	$\mathbf{n}$	$-Y_\Psi + 1/3$	$-Y_\Psi - 2/3$	$-Y_\Psi - 2/3$	$-1/6 \to -$
	χ	$\mathbf{R}_\chi$	$\mathbf{n}$	$Y_\Psi - 2/3$	$Y_\Psi - 2/3$	$Y_\Psi + 1/3$	$1/2 \to -$

Strong CP problem

Dynamical solution

Peccei,Quinn (1977),

Weinberg (1978), Wilczek (1978)

$$\left(\frac{a}{f_a} - \bar{\theta}\right) \frac{\alpha_s}{8\pi} G\tilde{G} \qquad \bar{\theta}_{\text{eff}} = \bar{\theta} + \left\langle \frac{a}{f_a} \right\rangle = 0$$

Complex scalar singlet

$$\sigma = \frac{v_\sigma + \rho}{\sqrt{2}} e^{i a_\sigma / v_\sigma}$$

Axion decay constant

$$f_a = \frac{f_{\text{PQ}}}{N} = \frac{v_\sigma}{\sqrt{2}N}$$

Color-anomaly factor

$$N = 2\,n\,[T(\mathbf{R}_\Psi) + T(\mathbf{R}_F)]$$

QCD axion mass relation

$$m_a = 5.70(7)\left(\frac{10^{12}\ \text{GeV}}{f_a}\right)\ \mu\text{eV}$$

Cortona et al.(2016)

$\mathbf{R}_\Psi$	$\mathbf{R}_F$	$\mathbf{R}_S$	$\mathbf{R}_\chi$	N/n
$\mathbf{1}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	1
$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	1
$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{3}$	2
$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{8}$	$\mathbf{3}$	2
$\bar{\mathbf{3}}$	$\mathbf{6}$	$\mathbf{8}$	$\bar{\mathbf{6}}$	6
$\mathbf{6}$	$\bar{\mathbf{3}}$	$\mathbf{8}$	$\mathbf{3}$	6
$\mathbf{3}$	$\mathbf{8}$	$\mathbf{3}$	$\mathbf{8}$	7
$\mathbf{3}$	$\mathbf{8}$	$\bar{\mathbf{6}}$	$\mathbf{8}$	7
$\mathbf{8}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	7
$\mathbf{8}$	$\mathbf{3}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	7
$\bar{\mathbf{6}}$	$\mathbf{8}$	$\mathbf{3}$	$\mathbf{8}$	11
$\mathbf{8}$	$\bar{\mathbf{6}}$	$\bar{\mathbf{3}}$	$\mathbf{6}$	11

Spontaneous radiative proton decay

$$\mathcal{L}_{\rm A} \supset \mathbf{Y}_1 \overline{d_R^c} S \Psi_R + \mathbf{Y}_2 \overline{u_R} \chi^\dagger \Psi_L + \mathbf{Y}_3 \overline{u_R^c} S^\dagger F_R + \mathbf{Y}_4 \overline{e_R} \chi F_L + \text{H.c.}$$

$$\mathcal{L}_{\rm B} \supset \mathbf{Y}_1 \overline{u_R^c} S \Psi_R + \mathbf{Y}_2 \overline{u_R} \chi^\dagger \Psi_L + \mathbf{Y}_3 \overline{d_R^c} S^\dagger F_R + \mathbf{Y}_4 \overline{e_R} \chi F_L + \text{H.c.}$$

$$\mathcal{L}_{\rm C} \supset \mathbf{Y}_1 \overline{u_R^c} S \Psi_R + \mathbf{Y}_2 \overline{d_R} \chi^\dagger \Psi_L + \mathbf{Y}_3 \overline{u_R^c} S^\dagger F_R + \mathbf{Y}_4 \overline{e_R} \chi F_L + \text{H.c.}$$

Chiral PQ charges: masses from PQ breaking

$$\mathcal{L}_{\rm PQ} = y_\Psi \, \overline{\Psi_L} \, \sigma \, \Psi_R + y_F \, \overline{F_L} \, \sigma \, F_R + \text{H.c.}$$

$$M_{\Psi,F} = y_{\Psi,F} \, f_{\rm PQ} \,, \qquad M^2_{S,\chi} \supset \lambda_{\sigma S, \sigma\chi} \, f^2_{\rm PQ}$$

Spontaneous radiative proton decay

$$\mathcal{L}_{\rm A} \supset \mathbf{Y}_1 \overline{d_R^c} S \Psi_R + \mathbf{Y}_2 \overline{u_R} \chi^\dagger \Psi_L + \mathbf{Y}_3 \overline{u_R^c} S^\dagger F_R + \mathbf{Y}_4 \overline{e_R} \chi F_L + \text{H.c.}$$

$$\mathcal{L}_{\rm B} \supset \mathbf{Y}_1 \overline{u_R^c} S \Psi_R + \mathbf{Y}_2 \overline{u_R} \chi^\dagger \Psi_L + \mathbf{Y}_3 \overline{d_R^c} S^\dagger F_R + \mathbf{Y}_4 \overline{e_R} \chi F_L + \text{H.c.}$$

$$\mathcal{L}_{\rm C} \supset \mathbf{Y}_1 \overline{u_R^c} S \Psi_R + \mathbf{Y}_2 \overline{d_R} \chi^\dagger \Psi_L + \mathbf{Y}_3 \overline{u_R^c} S^\dagger F_R + \mathbf{Y}_4 \overline{e_R} \chi F_L + \text{H.c.}$$

Chiral PQ charges: masses from PQ breaking

$$\mathcal{L}_{\rm PQ} = y_\Psi \, \overline{\Psi_L} \, \sigma \, \Psi_R + y_F \, \overline{F_L} \, \sigma \, F_R + \text{H.c.}$$

$$M_{\Psi,F} = y_{\Psi,F} \, f_{\rm PQ} \,, \qquad M^2_{S,\chi} \supset \lambda_{\sigma S, \sigma\chi} \, f^2_{\rm PQ}$$

$$\underbrace{u_R \, u_R \, d_R \, e_R}_{\rm PQ = +2} \, \underbrace{\sigma^{*2}}_{-2} \xrightarrow{\langle \sigma \rangle = f_{\rm PQ}} u_R \, u_R \, d_R \, e_R$$

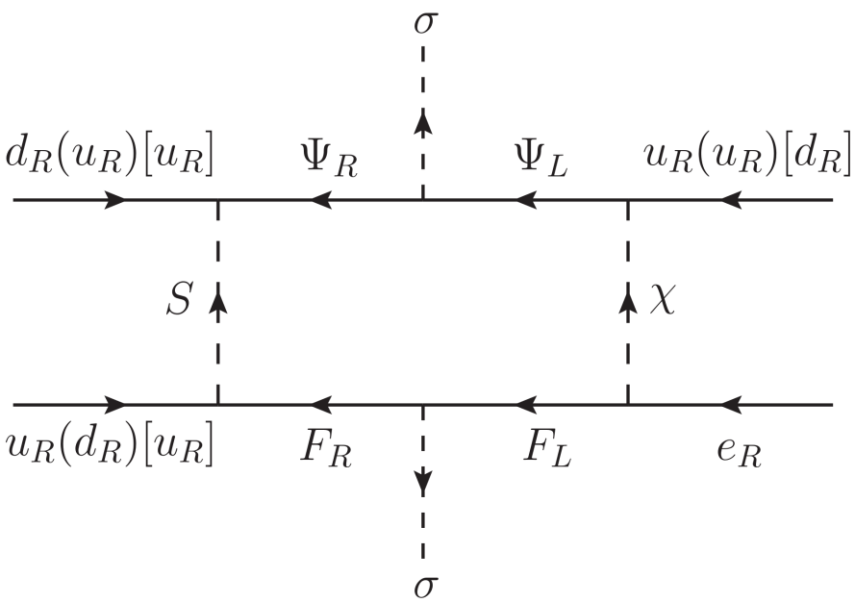
Spontaneous radiative proton decay

$$\mathcal{L}_{\rm A} \supset \mathbf{Y}_1 \overline{d_R^c} S \Psi_R + \mathbf{Y}_2 \overline{u_R} \chi^\dagger \Psi_L + \mathbf{Y}_3 \overline{u_R^c} S^\dagger F_R + \mathbf{Y}_4 \overline{e_R} \chi F_L + \text{H.c.}$$

$$\mathcal{L}_{\rm B} \supset \mathbf{Y}_1 \overline{u_R^c} S \Psi_R + \mathbf{Y}_2 \overline{u_R} \chi^\dagger \Psi_L + \mathbf{Y}_3 \overline{d_R^c} S^\dagger F_R + \mathbf{Y}_4 \overline{e_R} \chi F_L + \text{H.c.}$$

$$\mathcal{L}_{\rm C} \supset \mathbf{Y}_1 \overline{u_R^c} S \Psi_R + \mathbf{Y}_2 \overline{d_R} \chi^\dagger \Psi_L + \mathbf{Y}_3 \overline{u_R^c} S^\dagger F_R + \mathbf{Y}_4 \overline{e_R} \chi F_L + \text{H.c.}$$

Case A (B) [C]



Chiral PQ charges: masses from PQ breaking

$$\mathcal{L}_{\rm PQ} = y_\Psi \, \overline{\Psi_L} \, \sigma \, \Psi_R + y_F \, \overline{F_L} \, \sigma \, F_R + \text{H.c.}$$

$$M_{\Psi,F} = y_{\Psi,F} \, f_{\rm PQ} \,, \qquad M^2_{S,\chi} \supset \lambda_{\sigma S, \sigma\chi} \, f^2_{\rm PQ}$$

One-loop box diagram

$$\underbrace{u_R \, u_R \, d_R \, e_R}_{\rm PQ = +2} \, \underbrace{\sigma^{*2}}_{-2} \xrightarrow{\langle\sigma\rangle = f_{\rm PQ}} u_R \, u_R \, d_R \, e_R$$

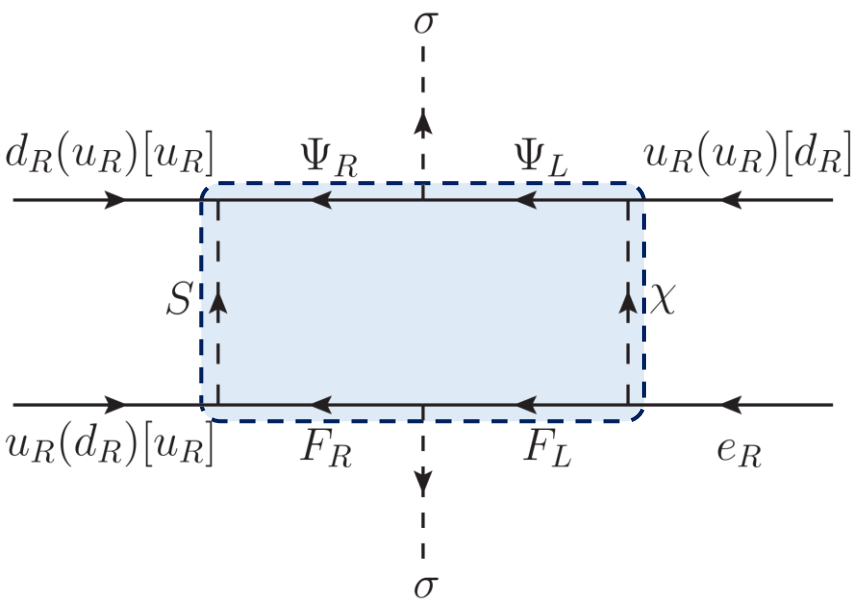
Spontaneous radiative proton decay

$$\mathcal{L}_{\rm A} \supset \mathbf{Y}_1 \overline{d_R^c} S \Psi_R + \mathbf{Y}_2 \overline{u_R} \chi^\dagger \Psi_L + \mathbf{Y}_3 \overline{u_R^c} S^\dagger F_R + \mathbf{Y}_4 \overline{e_R} \chi F_L + \text{H.c.}$$

$$\mathcal{L}_{\rm B} \supset \mathbf{Y}_1 \overline{u_R^c} S \Psi_R + \mathbf{Y}_2 \overline{u_R} \chi^\dagger \Psi_L + \mathbf{Y}_3 \overline{d_R^c} S^\dagger F_R + \mathbf{Y}_4 \overline{e_R} \chi F_L + \text{H.c.}$$

$$\mathcal{L}_{\rm C} \supset \mathbf{Y}_1 \overline{u_R^c} S \Psi_R + \mathbf{Y}_2 \overline{d_R} \chi^\dagger \Psi_L + \mathbf{Y}_3 \overline{u_R^c} S^\dagger F_R + \mathbf{Y}_4 \overline{e_R} \chi F_L + \text{H.c.}$$

Case A (B) [C]



VLQs and colored scalars in the loops

Chiral PQ charges: masses from PQ breaking

$$\mathcal{L}_{\rm PQ} = y_\Psi \, \overline{\Psi_L} \, \sigma \, \Psi_R + y_F \, \overline{F_L} \, \sigma \, F_R + \text{H.c.}$$

$$M_{\Psi,F} = y_{\Psi,F} \, f_{\rm PQ} \,, \qquad M^2_{S,\chi} \supset \lambda_{\sigma S, \sigma \chi} \, f^2_{\rm PQ}$$

One-loop box diagram

$$\underbrace{u_R \, u_R \, d_R \, e_R}_{\rm PQ = +2} \, \underbrace{\sigma^{*2}}_{-2} \xrightarrow{\langle \sigma \rangle = f_{\rm PQ}} u_R \, u_R \, d_R \, e_R$$

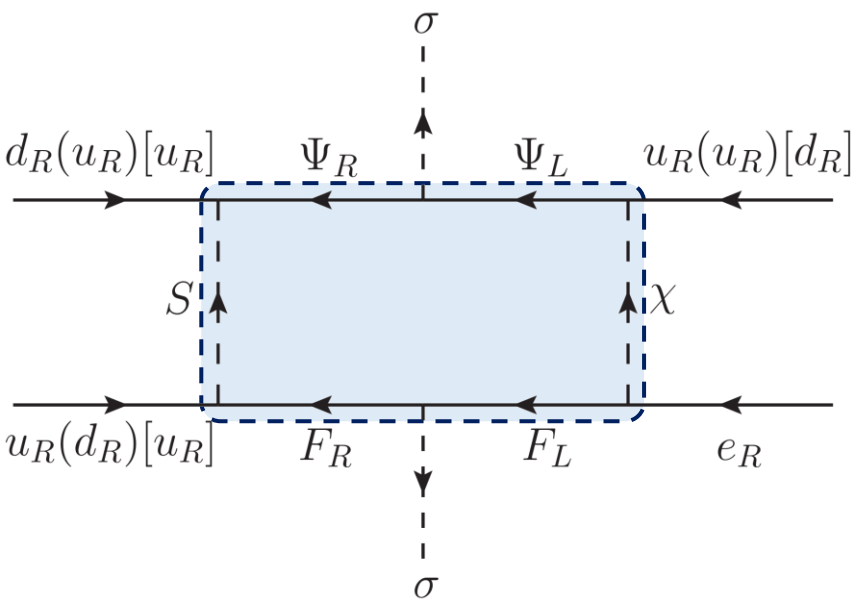
Spontaneous radiative proton decay

$$\mathcal{L}_{\rm A} \supset \mathbf{Y}_1 \overline{d_R^c} S \Psi_R + \mathbf{Y}_2 \overline{u_R} \chi^\dagger \Psi_L + \mathbf{Y}_3 \overline{u_R^c} S^\dagger F_R + \mathbf{Y}_4 \overline{e_R} \chi F_L + \text{H.c.}$$

$$\mathcal{L}_{\rm B} \supset \mathbf{Y}_1 \overline{u_R^c} S \Psi_R + \mathbf{Y}_2 \overline{u_R} \chi^\dagger \Psi_L + \mathbf{Y}_3 \overline{d_R^c} S^\dagger F_R + \mathbf{Y}_4 \overline{e_R} \chi F_L + \text{H.c.}$$

$$\mathcal{L}_{\rm C} \supset \mathbf{Y}_1 \overline{u_R^c} S \Psi_R + \mathbf{Y}_2 \overline{d_R} \chi^\dagger \Psi_L + \mathbf{Y}_3 \overline{u_R^c} S^\dagger F_R + \mathbf{Y}_4 \overline{e_R} \chi F_L + \text{H.c.}$$

Case A (B) [C]



VLQs and colored scalars in the loops

Chiral PQ charges: masses from PQ breaking

$$\mathcal{L}_{\rm PQ} = y_\Psi \, \overline{\Psi_L} \, \sigma \, \Psi_R + y_F \, \overline{F_L} \, \sigma \, F_R + \text{H.c.}$$

$$M_{\Psi,F} = y_{\Psi,F} \, f_{\rm PQ} \,, \qquad M^2_{S,\chi} \supset \lambda_{\sigma S, \sigma\chi} \, f^2_{\rm PQ}$$

One-loop box diagram

$$\underbrace{u_R \, u_R \, d_R \, e_R}_{\rm PQ = +2} \, \underbrace{\sigma^{*2}}_{-2} \xrightarrow{\langle\sigma\rangle = f_{\rm PQ}} u_R \, u_R \, d_R \, e_R$$

Residual Z_2 parity

SM fields and σ even; **Ψ, *F*, *S*, χ odd**
⇒ No proton decay at tree level

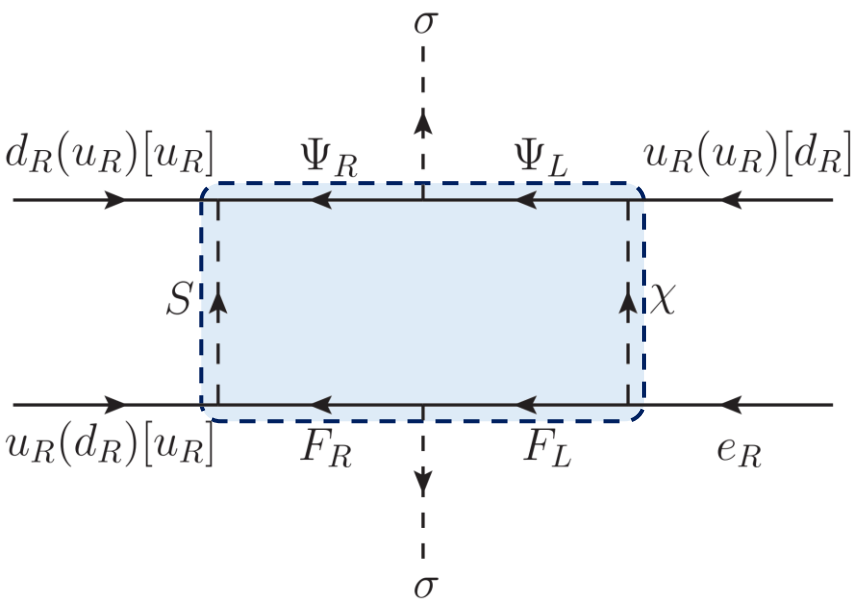
Spontaneous radiative proton decay

$$\mathcal{L}_{\rm A} \supset \mathbf{Y}_1 \overline{d_R^c} S \Psi_R + \mathbf{Y}_2 \overline{u_R} \chi^\dagger \Psi_L + \mathbf{Y}_3 \overline{u_R^c} S^\dagger F_R + \mathbf{Y}_4 \overline{e_R} \chi F_L + \text{H.c.}$$

$$\mathcal{L}_{\rm B} \supset \mathbf{Y}_1 \overline{u_R^c} S \Psi_R + \mathbf{Y}_2 \overline{u_R} \chi^\dagger \Psi_L + \mathbf{Y}_3 \overline{d_R^c} S^\dagger F_R + \mathbf{Y}_4 \overline{e_R} \chi F_L + \text{H.c.}$$

$$\mathcal{L}_{\rm C} \supset \mathbf{Y}_1 \overline{u_R^c} S \Psi_R + \mathbf{Y}_2 \overline{d_R} \chi^\dagger \Psi_L + \mathbf{Y}_3 \overline{u_R^c} S^\dagger F_R + \mathbf{Y}_4 \overline{e_R} \chi F_L + \text{H.c.}$$

Case A (B) [C]



VLQs and colored scalars in the loops

Chiral PQ charges: masses from PQ breaking

$$\mathcal{L}_{\rm PQ} = y_\Psi \, \overline{\Psi_L} \, \sigma \, \Psi_R + y_F \, \overline{F_L} \, \sigma \, F_R + \text{H.c.}$$

$$M_{\Psi,F} = y_{\Psi,F} \, f_{\rm PQ} \,, \qquad M^2_{S,\chi} \supset \lambda_{\sigma S, \sigma\chi} \, f^2_{\rm PQ}$$

One-loop box diagram

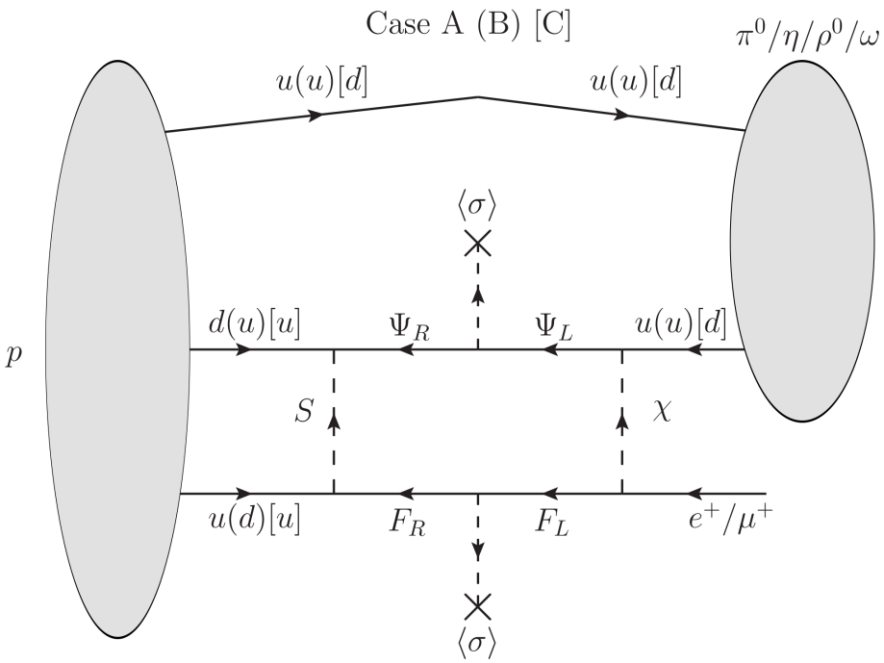
$$\underbrace{u_R \, u_R \, d_R \, e_R}_{\rm PQ=+2} \, \underbrace{\sigma^{*2}}_{-2} \xrightarrow{\langle\sigma\rangle = f_{\rm PQ}} u_R \, u_R \, d_R \, e_R$$

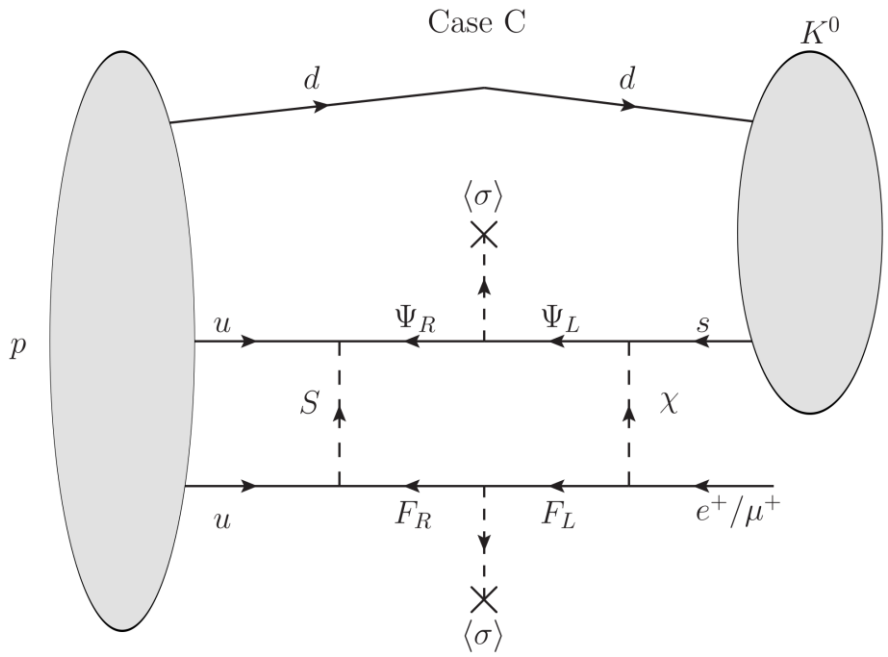
Residual Z_2 parity

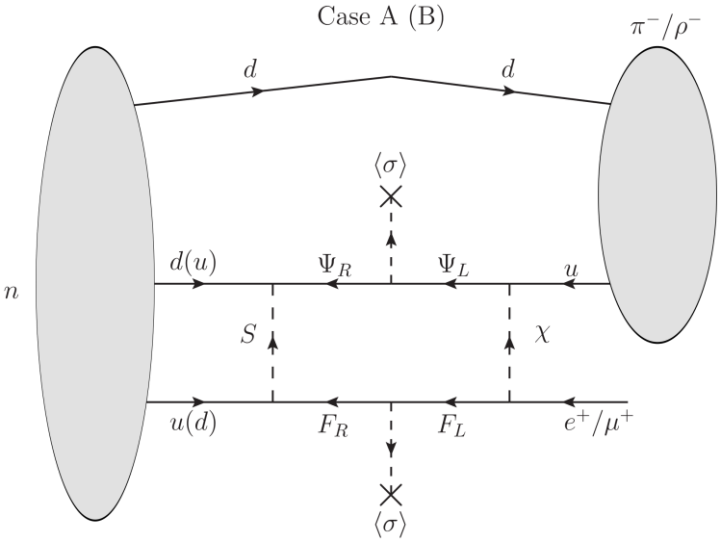
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Proton decay arises spontaneously (PQ breaking) and radiatively (at one loop)

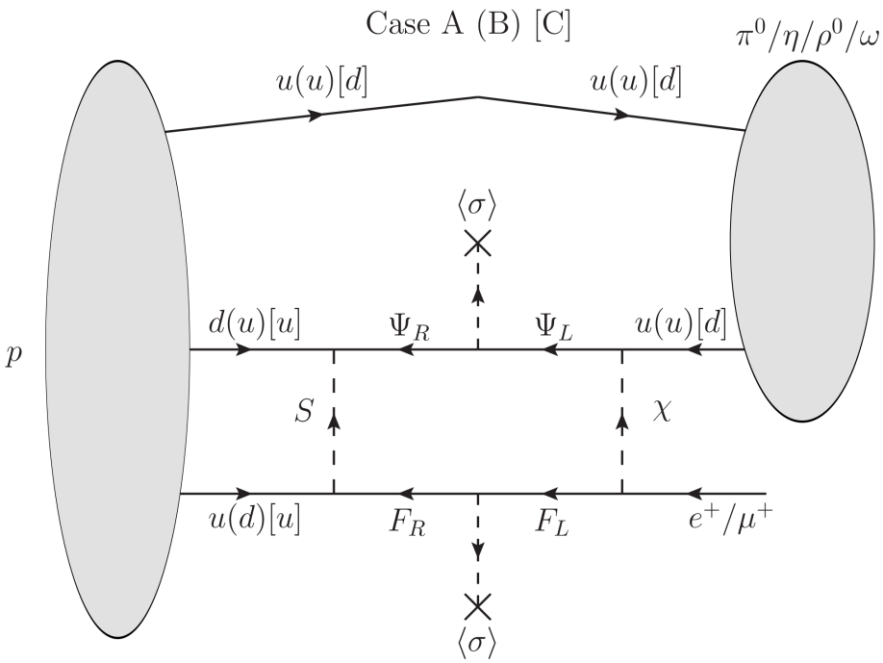
Radiative nucleon decay processes

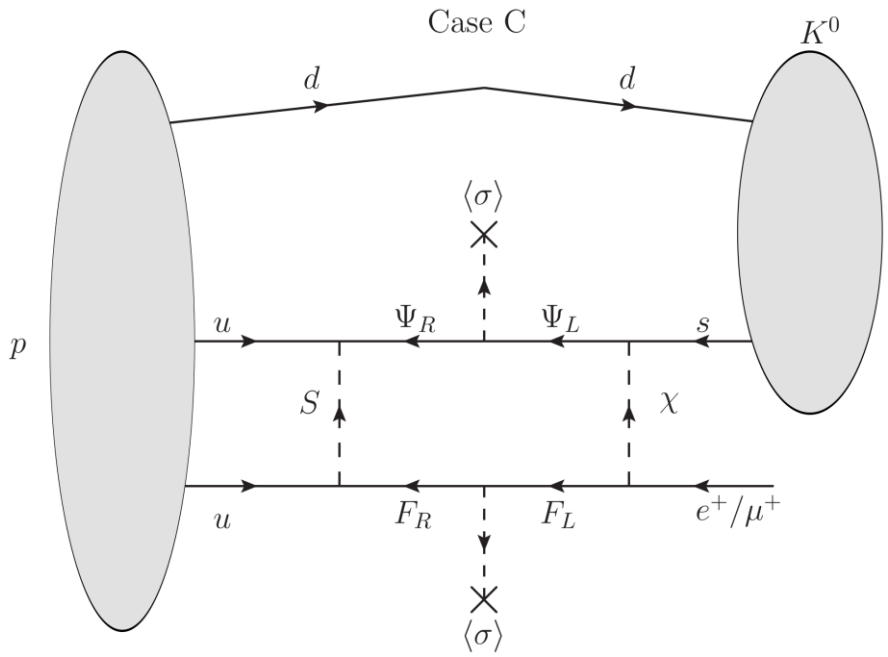


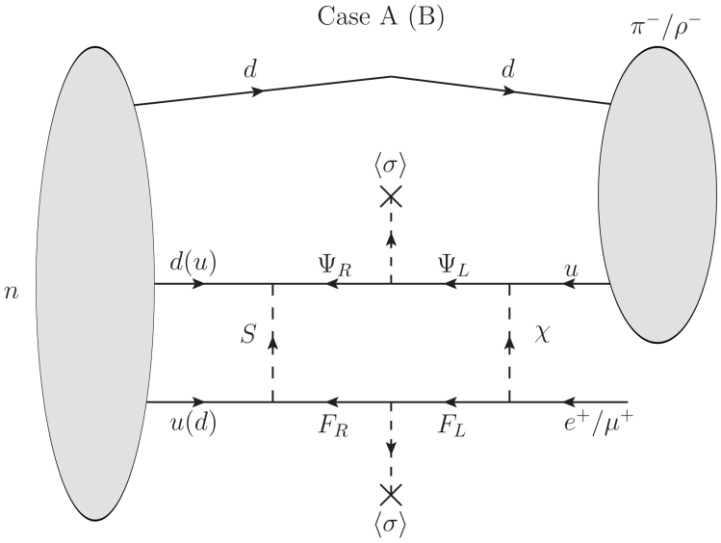




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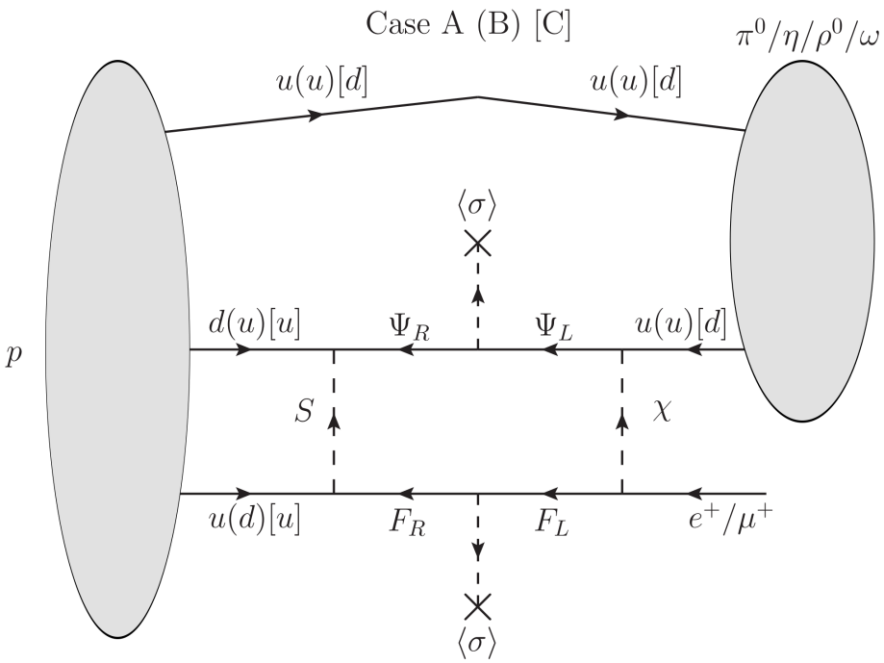


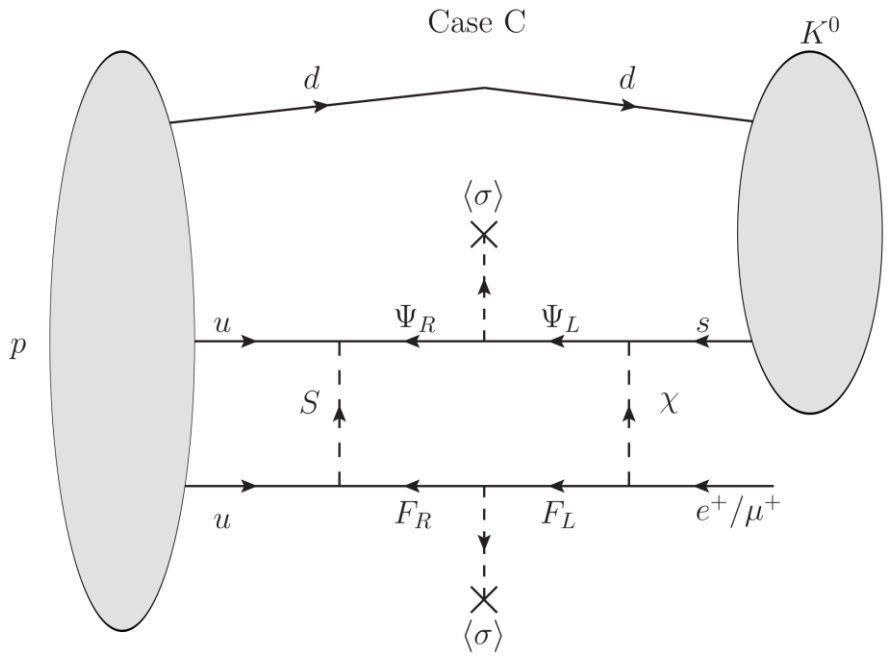
All cases A, B and C generate the nucleon decays:

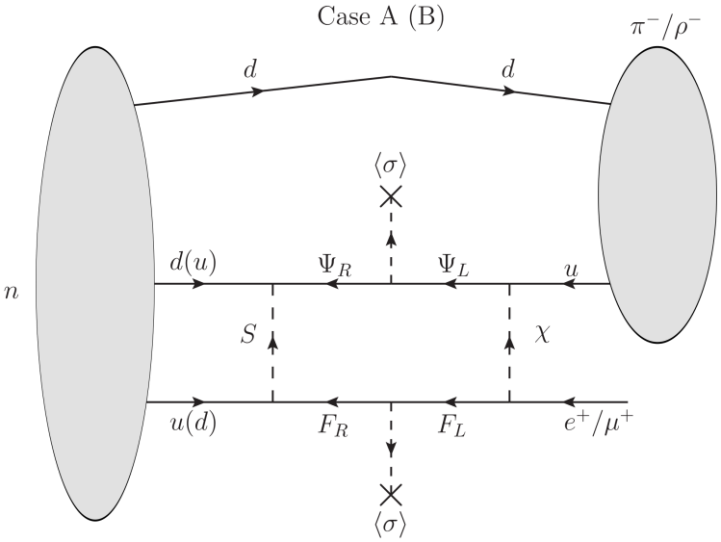
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Radiative nucleon decay processes







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- e^+ modes probe the same $Y_1 Y_2 Y_3 Y_4$ as $p \to e^+\pi^0$
- μ^+ modes: second generation of $\mathbf{Y}_4$
- K^0 modes: second generation of $\mathbf{Y}_1$ (A), $\mathbf{Y}_3$ (B), $\mathbf{Y}_2$ (C)

Helo, Hirsch, Ota (2019); Arbeláez, Helo, Hirsch, Ota (2026)

Proton decay phenomenology

Decay mode	τ ($\times 10^{33}$ yr)
$p \to e^+\pi^0 \,/\, \mu^+\pi^0$	$> 24 \,/\, 14$
$p \to e^+\eta \,/\, \mu^+\eta$	$> 14 \,/\, 7.3$
$p \to e^+\rho^0 \,/\, \mu^+\rho^0$	$> 0.72 \,/\, 0.57$
$p \to e^+\omega \,/\, \mu^+\omega$	$> 1.6 \,/\, 2.8$
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Super-Kamiokande, 90% CL

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Benchmark: Golden channel

Case A, $Y_\Psi = 0$, $\mathbf{n = 1}$: $\quad \mathbf{R}_\Psi = \mathbf{R}_S = \mathbf{3}\,, \quad \mathbf{R}_F = \mathbf{R}_\chi = \mathbf{1}$

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$$\mathcal{C} = -|Y_1 Y_2 Y_3 Y_4|\, M_\Psi M_F\, \mathcal{I}(M_\Psi, M_F, M_S, M_\chi)$$

$$M_{\Psi,F,S,\chi} \equiv M \quad \Rightarrow \quad \mathcal{I}(M) \simeq \frac{1}{16\pi^2}\,\frac{1}{6M^4}$$

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$$\frac{\Gamma(p \to e^+\pi^0 a)}{\Gamma(p \to e^+\pi^0)} \sim \frac{1}{16\pi^2}\left(\frac{m_p}{f_{\rm PQ}}\right)^2 \sim \mathcal{O}(10^{-27})$$

⇒ exotic modes e.g. $p \to e^+\pi^0 a$ are negligible

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For $M \simeq f_{\rm PQ} \simeq 10^{12}$ GeV the Super-K bound requires only $Y_1 Y_2 Y_3 Y_4 \lesssim 10^{-4}$

Axion dark matter in pre-inflationary scenario

Colored scalars

$$S \sim (\mathbf{R}_S, \mathbf{n}, Y_S)$$

$$\chi \sim (\mathbf{R}_\chi, \mathbf{n}, Y_\chi)$$

Vector-like quarks

$$\Psi_{L,R} \sim (\mathbf{R}_\Psi, \mathbf{n}, Y_\Psi)$$

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All Z_2-odd



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Isocurvature fluctuations are constrained by CMB data setting a bound on the inflationary scale

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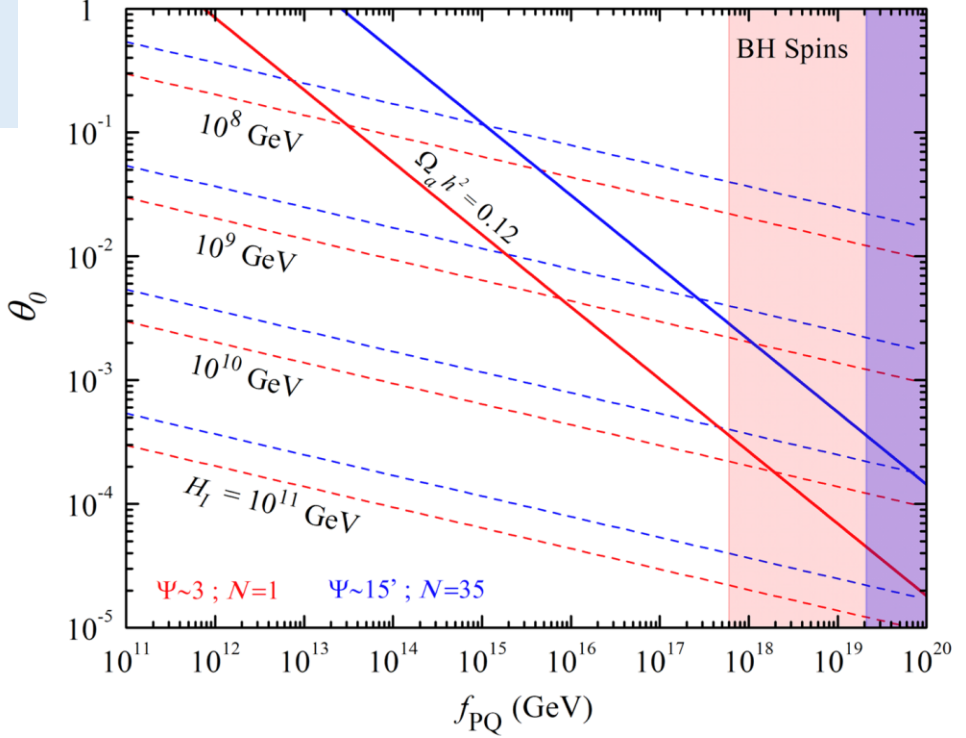
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Di Luzio et al. (2017)

Batra, HBC, Joaquim, Srivastava, Valle, PRL 132, 051801 (2024)



For ϑ_0~O(1), axions account for the **full CDM budget**, provided $f_a \sim 5{\times}10^{11}$ **GeV**, a region under scrutiny at **haloscopes.**

Axion dark matter in post-inflationary scenario

Axion dark matter via the misalignment mechanism in post-inflationary scenario is more predictive.

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Axion strings and domain walls also contribute to $\Omega_a h^2$

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Models free of domain walls: $N_{\rm DW} = N = 1$

Model	SM singlet	$\mathbf{R}_\Psi$	$\mathbf{R}_F$	$\mathbf{R}_S$	$\mathbf{R}_\chi$	E/N
$\mathbf{M}_1$	$\Psi \sim (\mathbf{1},\mathbf{1},0)$	$\mathbf{1}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$2/3\ (32/3)$
$\mathbf{M}_2$	$F \sim (\mathbf{1},\mathbf{1},0)$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$2/3\ (32/3)$
$\mathbf{M}_3$	$\chi \sim (\mathbf{1},\mathbf{1},0)$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$14/3\ (8/3)$

n = 1; *E/N* for case A/B (C)

- Ψ, *F*: one color triplet + one singlet ⇒ *N* = 1
- A **SM-singlet** mediator can be the lightest Z_2-odd state: no colored or charged relic
- *S* ~ (**1**, **1**, 0) gives $N_{\rm DW} = 2$: DW problem

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Models free of domain walls: $N_{\rm DW} = N = 1$

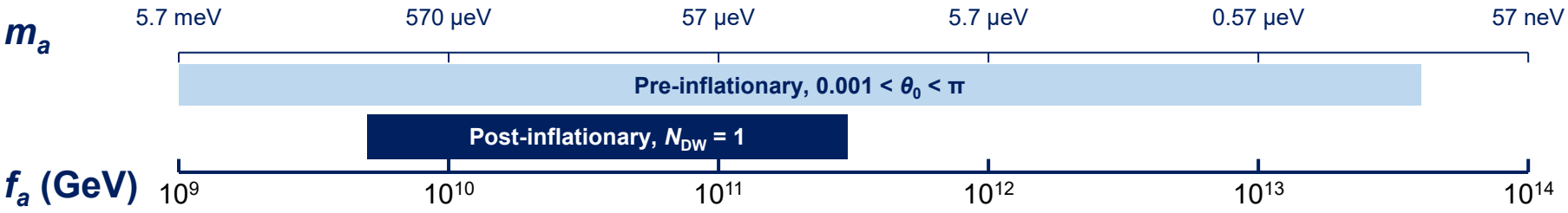
Model	SM singlet	$\mathbf{R}_\Psi$	$\mathbf{R}_F$	$\mathbf{R}_S$	$\mathbf{R}_\chi$	E/N
$\mathbf{M}_1$	$\Psi \sim (\mathbf{1},\mathbf{1},0)$	$\mathbf{1}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$2/3\ (32/3)$
$\mathbf{M}_2$	$F \sim (\mathbf{1},\mathbf{1},0)$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$2/3\ (32/3)$
$\mathbf{M}_3$	$\chi \sim (\mathbf{1},\mathbf{1},0)$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$14/3\ (8/3)$

n = 1; *E/N* for case A/B (C)

- Ψ, *F*: one color triplet + one singlet ⇒ $N = 1$
- A **SM-singlet** mediator can be the lightest Z_2-odd state: no colored or charged relic
- $S \sim (\mathbf{1}, \mathbf{1}, 0)$ gives $N_{\rm DW} = 2$: DW problem

Simulations (Strings + DWs): $f_a \in [5 \times 10^9,\ 3 \times 10^{11}]$ GeV

$\Omega_a h^2 = \Omega_{\rm DM} h^2$



Kawasaki, Saikawa, Sekiguchi (2015); Klaer, Moore (2017); Gorghetto, Hardy, Villadoro (2021); Buschmann et al. (2022); Benabou et al. (2025)

Probing the axion-to-photon coupling

Axion-to-photon coupling

Cortona et al.(2016)

$$g_{a\gamma\gamma} = \frac{\alpha_e}{2\pi f_a}\left[\frac{E}{N} - 1.92(4)\right]$$

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Model dependent contribution for the electromagnetic anomaly factor

Case A/B (Case C): E/N values for $Y_\Psi = 0$				
$SU(3)_c$		$SU(2)_L$		
$\mathbf{R}_\Psi$	$\mathbf{R}_F$	**1**	**2**	**3**
1	**3**	2/3 (32/3)	8/3 (38/3)	6 (16)
3	**1**	2/9 (32/9)	20/9 (50/9)	50/9 (80/9)
$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	1/3 (16/3)	11/6 (41/6)	13/3 (28/3)
$\bar{\mathbf{3}}$	**6**	2/9 (32/9)	35/36 (155/36)	20/9 (50/9)
6	$\bar{\mathbf{3}}$	1/9 (16/9)	31/36 (91/36)	19/9 (34/9)
3	**8**	16/63 (256/63)	131/126 (611/126)	148/63 (388/63)
8	**3**	2/21 (32/21)	37/42 (97/42)	46/21 (76/21)
$\bar{\mathbf{6}}$	**8**	16/99 (256/99)	79/99 (29/9)	184/99 (424/99)
8	$\bar{\mathbf{6}}$	4/33 (64/33)	25/33 (85/33)	20/11 (40/11)

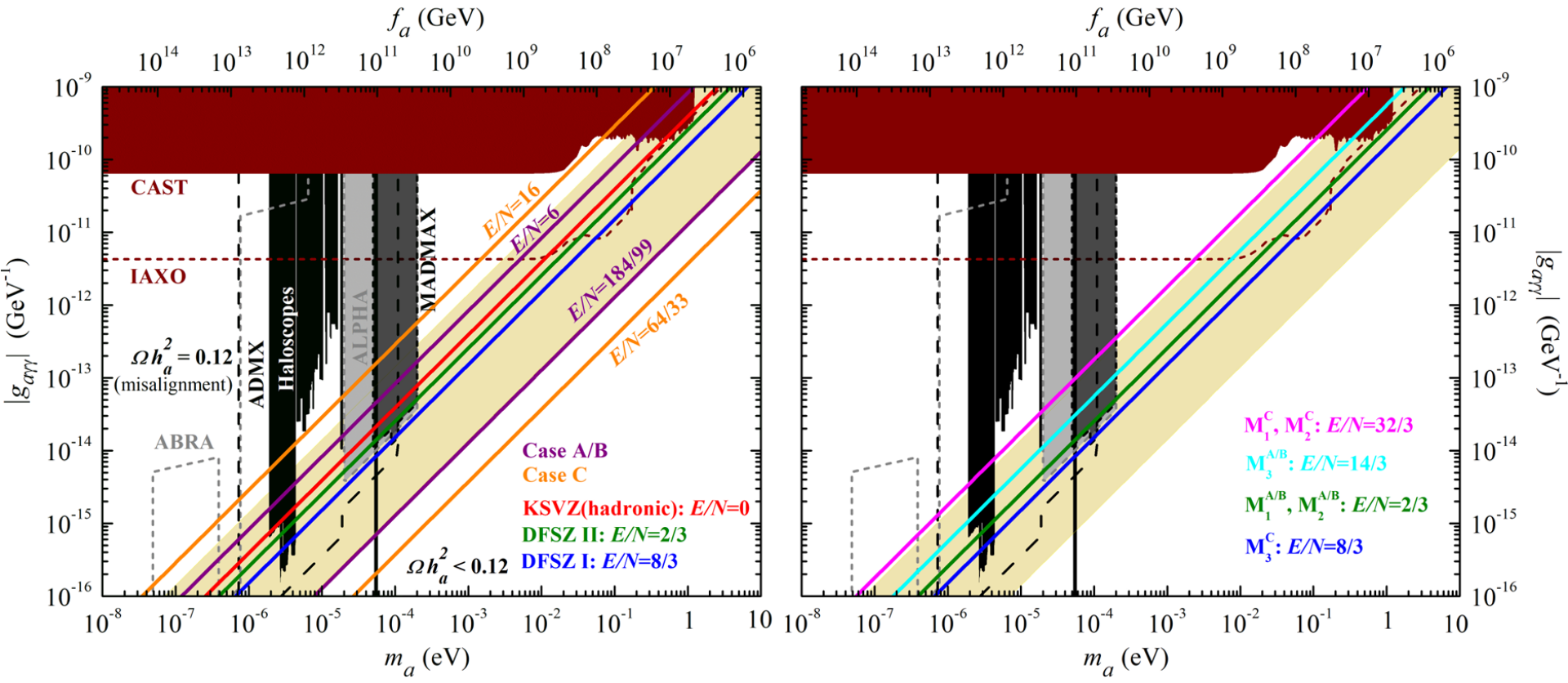
$$E_{\mathrm{A/B}} = 2\sum_{j=0}^{n-1}\left[d(\mathbf{R}_\Psi)\left(j - \frac{n-1}{2} + Y_\Psi\right)^2 + d(\mathbf{R}_F)\left(j - \frac{n-1}{2} - Y_\Psi - \frac{1}{3}\right)^2\right]$$

$$E_{\mathrm{C}} = 2\sum_{j=0}^{n-1}\left[d(\mathbf{R}_\Psi)\left(j - \frac{n-1}{2} + Y_\Psi\right)^2 + d(\mathbf{R}_F)\left(j - \frac{n-1}{2} - Y_\Psi - \frac{4}{3}\right)^2\right]$$

Probing the axion-to-photon coupling

Pre-inflationary axion dark matter

Models free of domain walls ($N_{DW} = 1$)



Axion-to-photon coupling allows to probe the different models at **helioscope** and **haloscope** experiments.

Conclusion

- **Spontaneous radiative proton decay**

$U(1)_{PQ} \equiv U(1)_{B+L} \to Z_2$ **forbids tree-level proton decay.**

The Z_2-odd vector-like quarks and colored scalars generate it **at one loop** after **PQ breaking.**

With mediators at $f_{PQ} \sim 10^{12}$ GeV, the Super-K bound on $p \to e^+\pi^0$ only needs $Y \lesssim 0.1$.

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  The PQ anomaly sector (Ψ, F) gives a **KSVZ-type axion** that solves the strong CP problem.

  Axion can be all of **dark matter** via misalignment mechanism: **pre-inflationary** for $\theta_0 \sim O(1)$ and $f_a \sim 5\times10^{11}$ GeV and **post-inflationary** for the $N_{DW} = 1$ models $f_a \in [5\times10^9, 3\times10^{11}]$ GeV.

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Thank you !