Field-theoretical derivation of the CP-violating Boltzmann equation for electroweak baryogenesis



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Collaboration: Motoi Endo (KEK), Tenta Tsuji (KEK)

Based on arXiv:2607.28321 (to be appeared from JHEP)

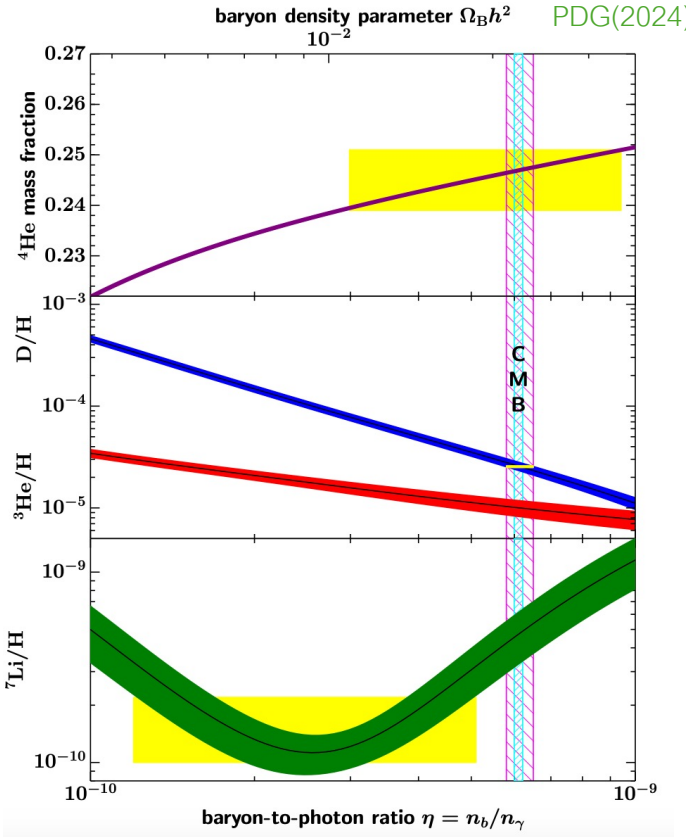
DISCRETE, 2026/11/14

Baryon asymmetry of the Universe

- Success of the standard model and physics beyond the SM
- Baryon asymmetry of the Universe

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} \qquad \eta_B = (6.12 \pm 0.04) \times 10^{-10}$$ Planck (2018)

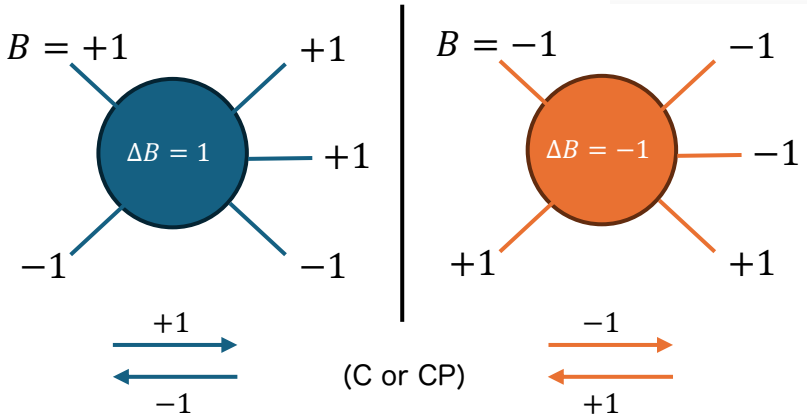
- Sakharov's three conditions for baryogenesis Sakharov (1967)



PDG(2024)

① Baryon # violation
② C and CP violation
③ Departure from thermal equilibrium

No baryon asymmetry remains
if the lengths of the arrows are the same.

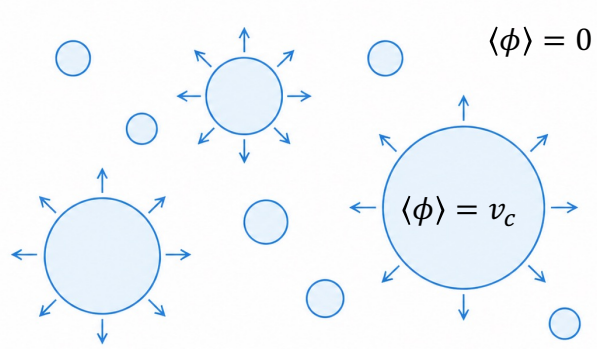


Electroweak baryogenesis

- **Electroweak baryogenesis** Kuzmin, Rubakov, and Shaposhnikov (1985)

  ① EW sphaleron (baryon # violation)
  ② EW theory with CP violation (C and CP violation)
  ③ EW first order phase transition (departure from thermal equilibrium)

First order phase transition at $T \simeq T_c$

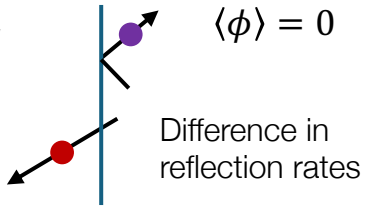


- **Baryon number generation around expanding bubble walls**

1. CP violation

Chirality: Left ● Anti-Left ●

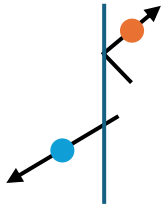
Charge: +1 -1



Difference in reflection rates

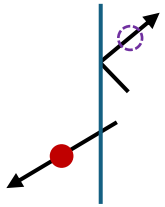
Chirality: Right ● Anti-Right ●

Charge: +1 -1

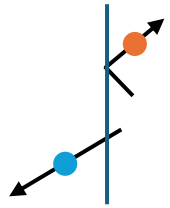


Total charge = 0 (CPT theorem)

2. C violation & B violation



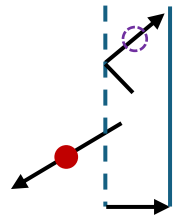
Sphaleron changing left number in the symmetric phase



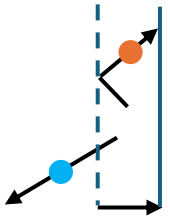
Left↔Right interactions wash out the asymmetry

Total charge = +1

3. Freeze out



Sphaleron decoupled in the broken phase

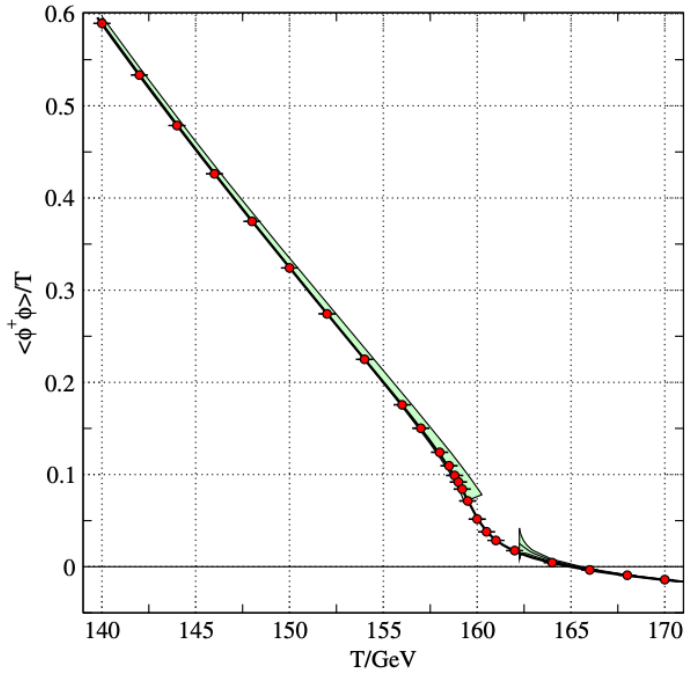


Total charge = +1

Failure in the SM

- **EW phase transition is a crossover in the SM**

Kajantie et al., PRL (1996); D'Onofrio and Rummukainen, PRD (2016);



- **CP violation in the CKM matrix is insufficient**

Gavela et al. (1994);
Huet and Sather (1995);

Jarlskog invariant $J_{CP} = \mathrm{Im}[V_{ud}V_{us}^*V_{cs}V_{cd}^*] = 3.12\times10^{-5}$ PDG (2024)

CP violating observables need to be proportional to

$$\Delta_{CP} = J_{CP} \times (m_t^2 - m_u^2)(m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_b^2 - m_d^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2).$$

Need a scale $T_c \sim 100$ GeV, Δ_{CP} to be dimensionless: $\eta_B \sim \frac{\Delta_{CP}}{T_c^{12}} \sim 10^{-20}$ → much smaller than the observed value $\eta_B \sim 10^{-10}$

In addition, decoherence due to QCD effects before collision suppresses it more (discussion in QM). Huet and Sather (1995)

- **Various extensions of the SM:**

SM + $SU(2)_L$ doublet scalar, SM + $SU(2)_L$ singlet scalar, …

New physics near the EW scale → many observables in collider, flavor, EDMs, GWs,…

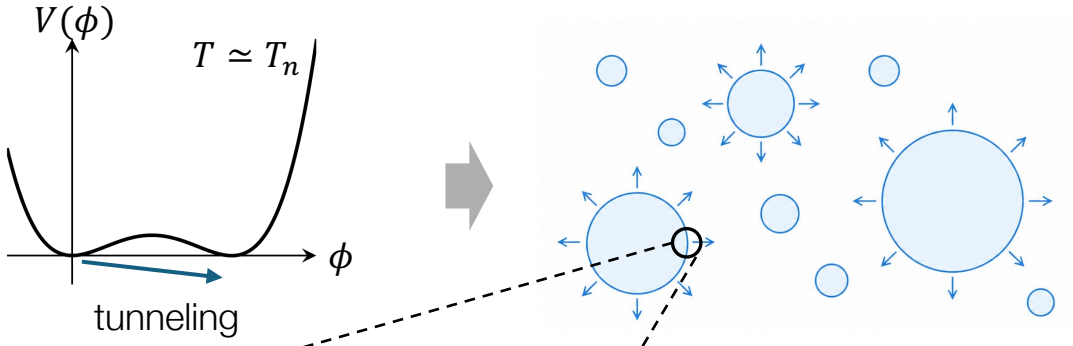
How to calculate baryon asymmetry?

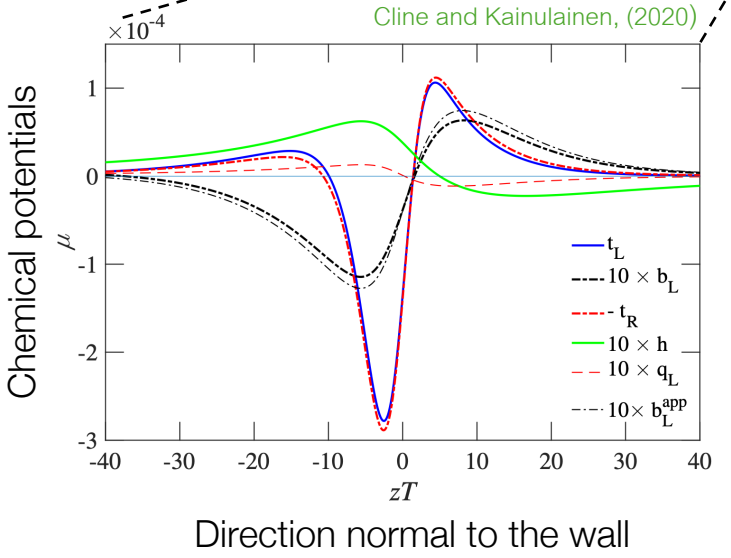
Let's break down the problem:

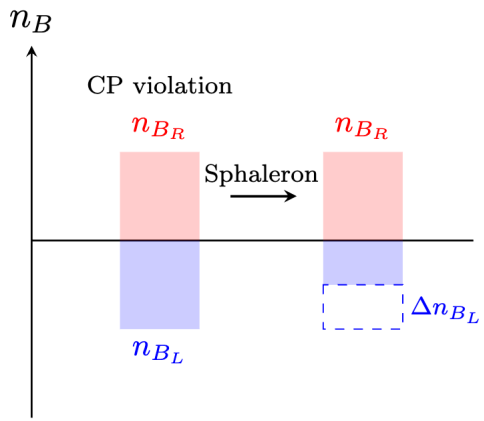
1. Calculate the EW first order phase transition, the bubble profile, and its dynamics

2. Evaluate the chiral charge asymmetry around the wall generated by the CP violation

3. Calculate the total baryon charge generated by the EW sphaleron ($\Gamma_{sph} << \Gamma_{scattering}$)



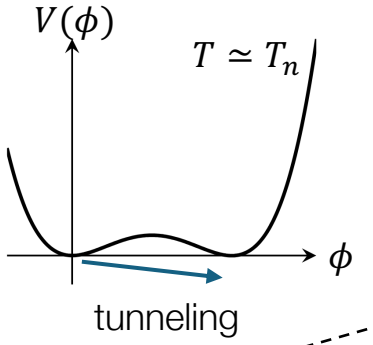


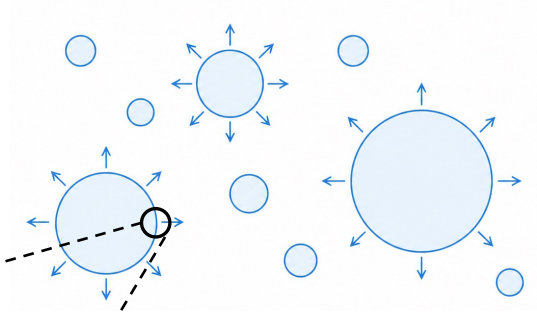


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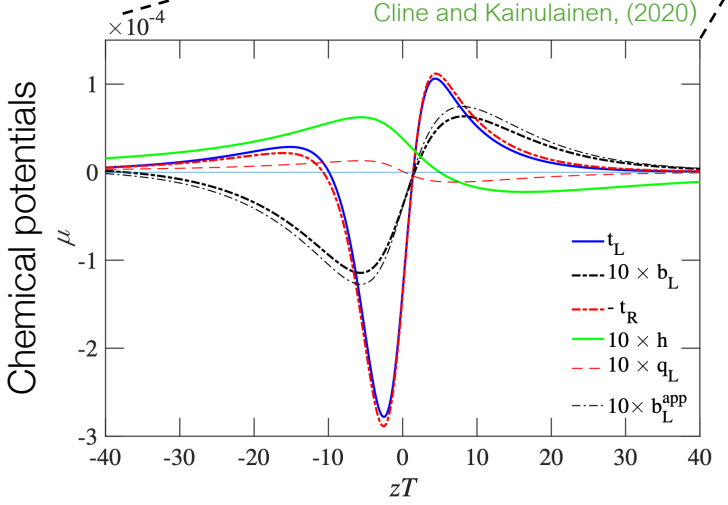




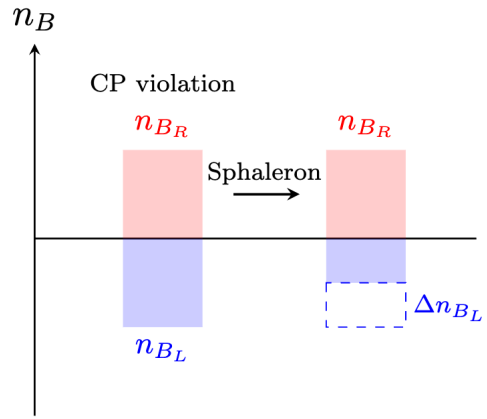
Today's topic: particle transport in the plasma

2. Evaluate the chiral charge asymmetry around the wall generated by the CP violation

Cline and Kainulainen, (2020)



3. Calculate the total baryon charge generated by the EW sphaleron ($\Gamma_{sph} << \Gamma_{scattering}$)



The Boltzmann equation

- **Particle transport in a non-equilibrium system (flat spacetime) can be described by the Boltzmann equation:**

$$\boldsymbol{L}[f_i] = \boldsymbol{C}[f_i, f_a, \dots]$$

Liouville term Collision term

$f_i(k, x)$: distribution function of particle i

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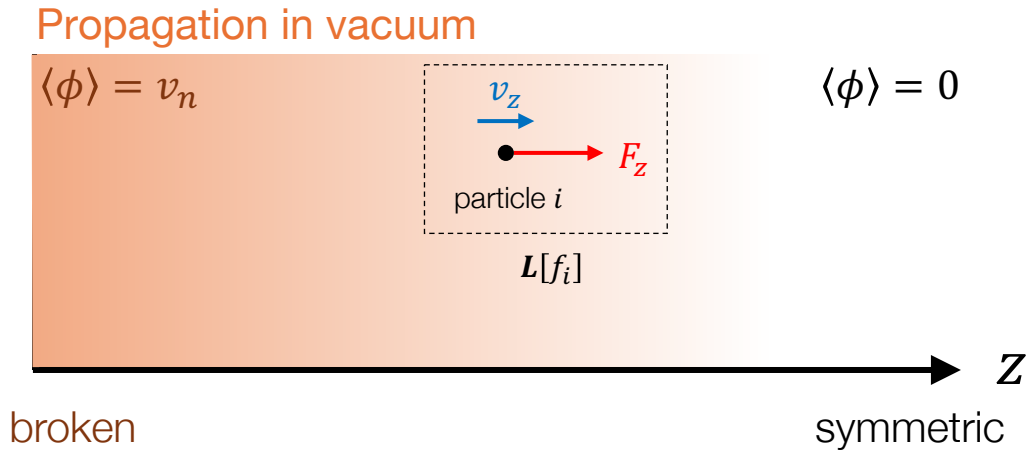
Liouville term Collision term

$f_i(k, x)$: distribution function of particle i

Liouville term: $\boldsymbol{L}[f_i] = (v\,\partial_x + F\,\partial_k)\,f_i$ describing the kinetic evolution of the (stational) distribution.

velocity force

CP violating effects from spacetime-dependent VEV are encoded in velocity & force terms (explain next).



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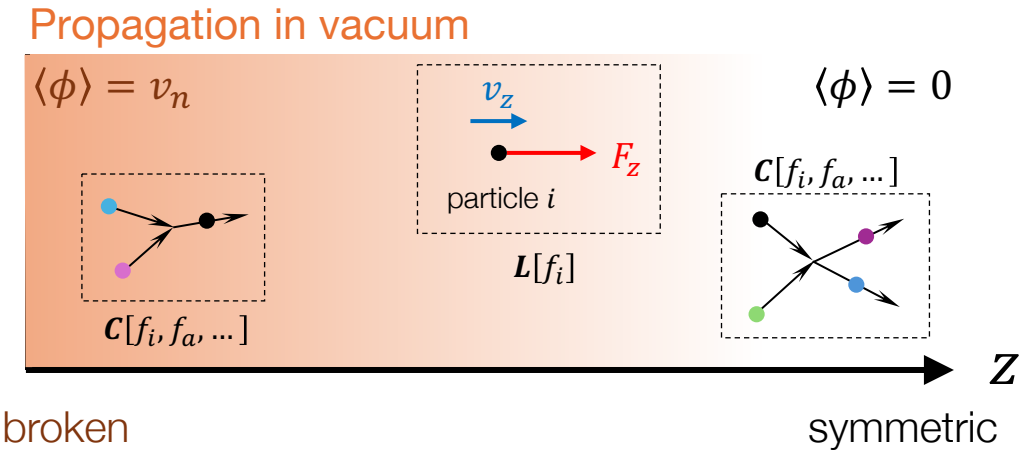
velocity force

CP violating effects from spacetime-dependent VEV are encoded in velocity & force terms (explain next).

Collision term: $\boldsymbol{C}[f_i, f_a, \dots] = \int d\Pi_i \dots |M|^2 (f_b f_c - f_i f_a) + \cdots$

describing various particle scatterings.

Processes: top Yukawa, weak current, Higgs interactions, etc.

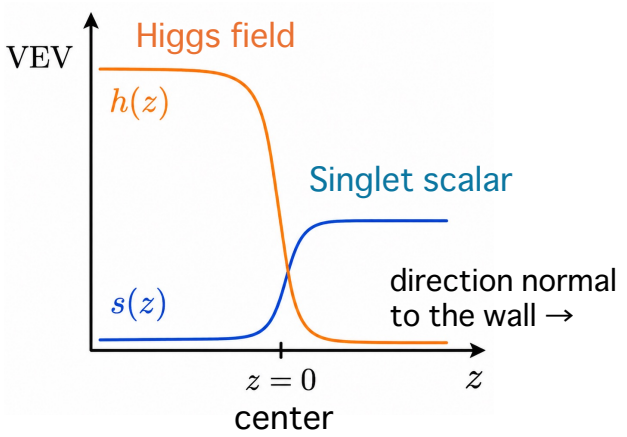


How to encode CP violation from the wall?

- **Complex mass term generated around the bubble wall**

(e.g.) Singlet extension of the SM Cline, Kainulainen, and Smith, (2017);

Effective top quark mass $m_t(z) = y_t\, h(z) \left(1 + i \dfrac{s(z)}{\Lambda}\right)$ CP violation

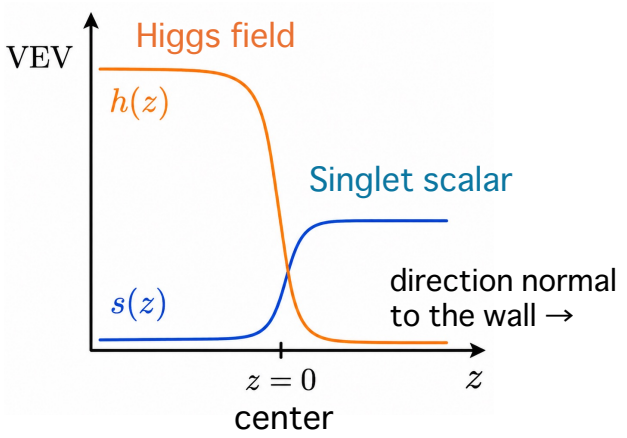


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- **Wall varying effects make dispersion relations different (w/ thick wall)** $(\partial_x \sim L_w^{-1} \ll T)$.

$\omega = \omega_0 \pm \delta\omega$ $\omega_0 = \sqrt{\boldsymbol{k}^2 + |m|^2}$ $\delta\omega \propto |m(z)| \times \partial_z \theta(z)$

↑ Particle vs Anti-particle ↑ Phase of the mass

Dispersion relation (Hamiltonian) determines kinetics in particle picture
→ Liouville term is determined.

$$\boldsymbol{L}[f_i] = [(v \pm \delta v)\, \partial_x + (F \pm \delta F)\, \partial_k]\, f_i$$

↑ CPV difference ↑ CPV difference

From QM: Cline, Joyce, Kainulainen, (1998) (2000);
Fromme and Huber, (2007);

From QFT: Kainulainen, Prokopec, Schmidt, Weinstock, (2001) (2002);
Prokopec, Schmidt, Weinstock, (2004);
Kainulainen, (2021); Endo, YM, Tsuji, arXiv:2607.28321

Missing effects in the Boltzmann equation

- The Kadanoff-Baym (KB) equation: Describing the evolution of propagators

  Schwinger (1961); Keldysh (1965); Kadanoff and Baym (1962); and more

- The Boltzmann equation can be derived from the KB equation, but…

  Kainulainen, Prokopec, Schmidt, Weinstock, (2001) (2002); Prokopec, Schmidt, Weinstock, (2004);

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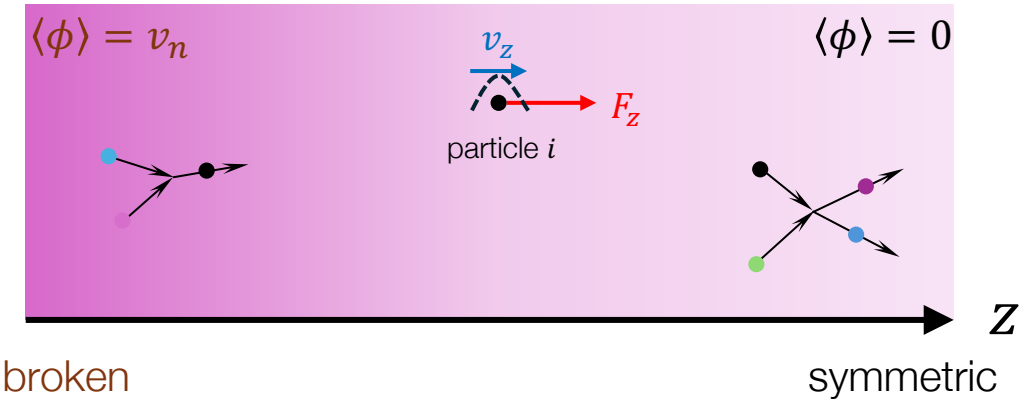
  - Dispersion relation is not for that in thermal plasma (neglecting plasma interaction in the derivation).
  - Plasma interactions are included only in the collision term.

    Thermal correction or damping? cf.) Thermal corrections (single fermion): Kainulainen, (2021)

    How can we recover a particle picture?
    What kind of approximations are needed?
    Bosons? Fermions? Multi-flavor cases?

- **Our approach: Starting from the full KB equation**

Propagation in thermal plasma & vacuum



Our strategy

- **Systematic derivation of the Boltzmann equation**

  ① Algebraically solve the full KB equations and obtain a formal solution

  ② Apply approximations commonly or implicitly used in the EWBG community

  ③ By using the quasiparticle approximation, identify the on-shell part of the solutions

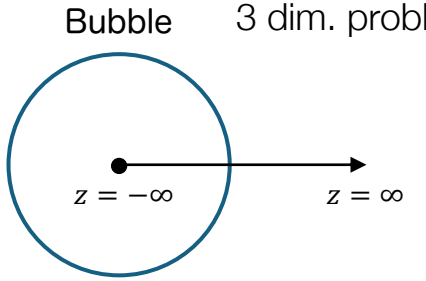
  ④ Substitute the on-shell solution into the kinetic equation

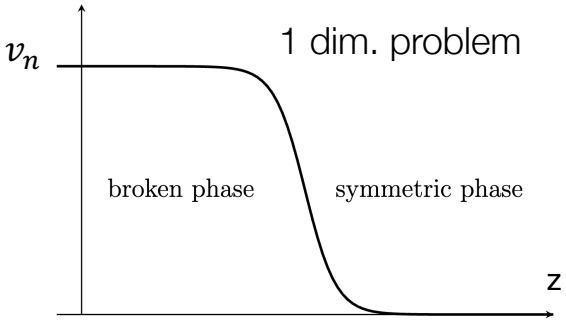
Approximations conventionally used

- **Assume the bubble wall is static and planar in the wall-comoving frame**

Spin along the z axis is approximately conserved in the EW plasma.

→ (spinors) = (spin) × (chiral) in an appropriate frame





- **Assume the wall is thick ($\partial_z \ll k_z \sim T$)** (typically, $L_w T \sim O(10)$, but model-dependent)

Derivative expansion: $e^{-i\,\partial_z \partial_{k_z}} = 1 + \left(-i\,\partial_z \partial_{k_z}\right) + \frac{1}{2}\left(-i\,\partial_z \partial_{k_z}\right)^2 + \cdots$ (perturbative treatment for non-local effects)

We algebraically obtain the solutions order by order of the derivative: $G = \sum_0^\infty G^{[n]}$

Quasiparticle approximation

- **Spectrum of the system under the quasiparticle approximation:** $\Gamma \ll \Omega^2$ $\quad \Omega^2 = k^2 - m^2$ (free field)

0th order in derivative

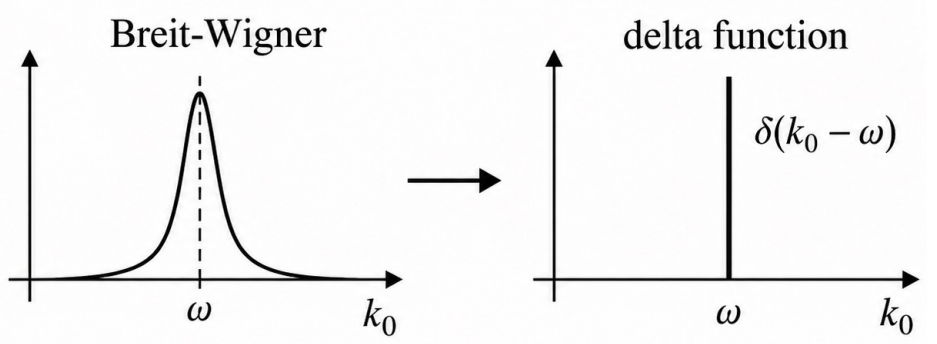
$$\mathcal{A}^{[0]} \sim \frac{\Gamma}{\Omega_0^4 + \Gamma^2} \to \delta(\Omega_0^2)$$

Breit-Wigner distribution

1st order in derivative (fermionic case)

$$\mathcal{A}^{[1]} \sim \delta\Omega^2 \times \frac{2\Gamma\,\Omega_0^2}{(\Omega_0^4 + \Gamma^2)^2}$$

$$\to \delta\Omega^2 \times \delta'(\Omega_0^2)$$ ($\delta\Omega^2$ are CP-odd)



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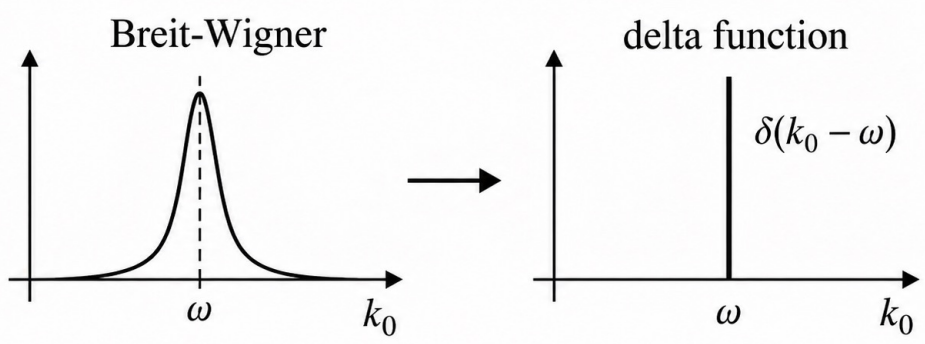
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$$\to \delta\Omega^2 \times \delta'(\Omega_0^2) \quad (\delta\Omega^2 \text{ are CP-odd})$$



- **Modification of the dispersion relation from ∂_z can be understood as a shift of pole position.**

0th + 1st contributions: $\delta(\Omega_0^2) + \delta\Omega^2 \times \delta'(\Omega_0^2) = \delta(\Omega_0^2 + \delta\Omega^2) + \mathcal{O}(\partial_z^2)$.

Pole position is determined by $\Omega_0^2 \pm \delta\Omega^2 = 0 \to k^0 = \omega_0 \pm \delta\omega$ $\quad \to \quad \boldsymbol{L}[f_i] = [(v \pm \delta v)\,\partial_x + (F \pm \delta F)\,\partial_k]\,f_i$

Thermal corrections are included in Ω^2s, and the solutions consistent to the previous ones.

- **We obtain the Liouville term w/ thermal correction and the collision term in a consistent way.**

Summary and Outlook

- **We gave a systematic way to obtain the CPV Boltzmann equation for EWBG.**
- **We algebraically solved the full KB equation order by order in the derivative expansion and applied approximations to the solutions.**

  Static- and planar-wall approximation, quasiparticle approximation, etc.

- **We obtained the CP-violating Boltzmann equation with thermal corrections.**

Future work

- Including flavor coherent effects
- Phenomenological applications
- …

Supplement

How to solve the Boltzmann equation?

- **Moment expansion**

  Taking momentum averages (two lowest moments)

  0th $\int d^3p \times$ (Boltzmann)

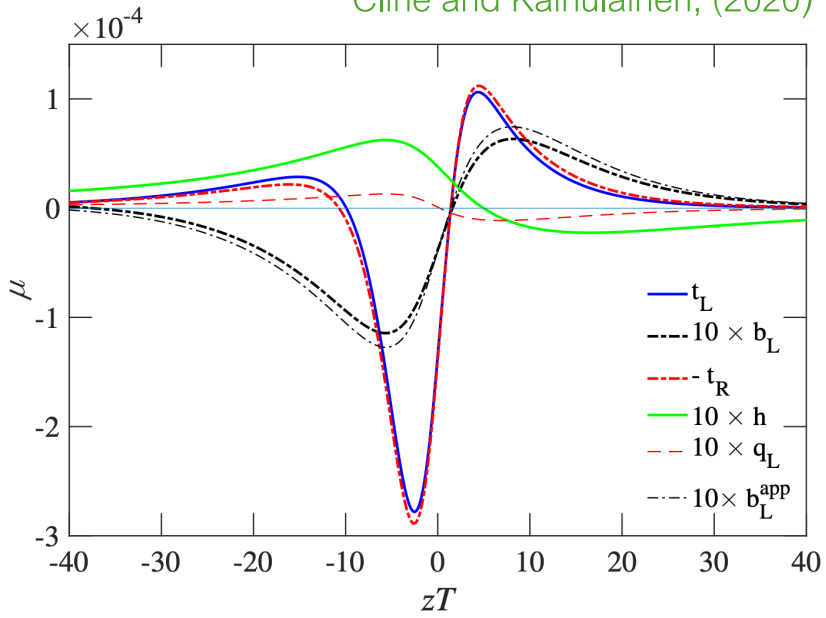
  1st $\int d^3p\ (p_z/\omega) \times$ (Boltzmann)

- **Transport equation for chemical potentials**

  $C_1\,\mu''(z) + C_2\,\mu'(z) + C_3\,\mu(z) = S_{\mathrm{CPV}}$ + collisions

  where, $\mu(z) = \mu_\psi - \mu_{\bar{\psi}}$ and C_i are functions of $z, T, m,$ and wall velocity

Cline and Kainulainen, (2020)



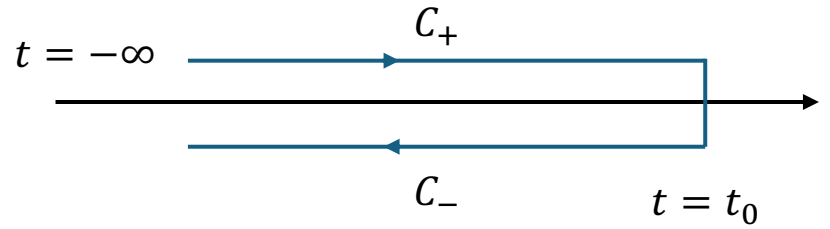
- **Computing the baryon number generated by weak sphalerons from μ_{B_L}**

- **Recent developments in higher-order moment expansions** Kainulainen and Venkatesan (2024)

Closed-time-path formalism

Schwinger (1961); Keldysh (1965);
Kadanoff and Baym (1962); and more

- **Time ordered 2×2 propagators on the closed-time-path**



(e.g.) bosonic propagators (u, v are the spacetime variables)

$$iG^{++}(u,v) \equiv iG^{t}(u,v) = \langle \mathrm{T}[\phi(u)\phi^{\dagger}(v)] \rangle, \quad : \text{time ordered propagator on } C_+$$

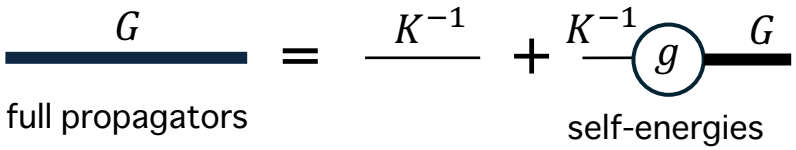
$$iG^{+-}(u,v) \equiv iG^{<}(u,v) = \langle \phi^{\dagger}(v)\phi(u) \rangle,$$

$$iG^{-+}(u,v) \equiv iG^{>}(u,v) = \langle \phi(u)\phi^{\dagger}(v) \rangle, \quad : \text{Wightman functions } G^{\lambda} \ (\lambda = <, >)$$

$$iG^{--}(u,v) \equiv iG^{\bar{t}}(u,v) = \langle \bar{\mathrm{T}}[\phi(u)\phi^{\dagger}(v)] \rangle. \quad : \text{time ordered propagator on } C_-$$

- **Schwinger-Dyson equation on the path**

$$K(u)\, iG^{ab}(u,v) = ia\, \delta^{ab}\delta^{4}(u-v) + \sum_{c} c \int d^{4}w \ g^{ac}(u,w) iG^{cb}(w,v), \qquad (a,b,c = \pm)$$



The Kadanoff-Baym equations

- **The Kadanoff-Baym equations** $(u, v) \to (u - v, (u + v)/2)$ (the relative coordinate is Fourier transformed)

  relative: r average: x → momentum: k

$$\mathcal{D}(k,x)\, G^{\lambda} - e^{-i\diamond}\left(\mathcal{M} + g_{\mathcal{H}}, G^{\lambda}\right) - e^{-i\diamond}\left(g^{\lambda}, G_{\mathcal{H}}\right) = \frac{1}{2}\left(e^{-i\diamond}(g^{>}, G^{<}) - e^{-i\diamond}(g^{<}, G^{>})\right) \qquad (\lambda = <, >)$$

kinetic term mass (VEV) self-energies (plasma)

- **Effects from the wall (varying VEV) are encoded in the mass term:** $\mathcal{M} = \mathcal{M}(x)$

- **The other interactions (plasma) are encoded in the self-energy** g.

  $g_{\mathcal{H}}$: Hermitian part ~ thermal mass
  $g_{\mathcal{A}}$: Anti-Hermitian part ~ thermal width

- **Dependence on** x **is expressed by the diamond operator :** $\diamond\,(A, B) = \frac{1}{2}(\partial_x A \cdot \partial_k B - \partial_k A \cdot \partial_x B)$

Conventional derivation (n=0)

Kainulainen, Prokopec, Schmidt, and Weinstock, (2001), (2022);
Prokopec, Schmidt, and Weinstock, (2004);

- **(KB equation) = (Hermitian part) + (anti-Hermitian part)**

  Kinetic equation Constraint equation (Note: this correspondence depends on the notation for G.)

- **Exercise: constraint equation in free field theory**

  The constraint equation for a free fermionic field is schematically given by $\left(k_\mu \gamma^\mu - m_R - i\, m_I\, \gamma^5\right) G = 0.$

  $\rightarrow \;\; G \sim \; n \times \delta(\Omega^2)$ where $\Omega^2 = k^2 - |m|^2,$ and n is a non-singular function.

  $\rightarrow$ on the mass-shell, n relates to the distribution f : $G \sim \langle a^\dagger a \rangle \sim f\, \delta(k^0 - \omega_0), \qquad \omega_0 = \sqrt{|\boldsymbol{k}|^2 - |m|^2}$

- **Kinetic equation that describes the evolution of $n \sim f$:** $\quad i\not{\partial}_x G = g^> G^< - g^< G^>$

  Substituting $G \sim \; n \times \delta(\Omega^2)$ and integrating over $k^0 > 0$,

$$\int_0^\infty dk^0\, i\not{\partial}_x\, G \;= \int_0^\infty dk^0\, (g^> G^< - g^< G^>) \quad \rightarrow \quad (v_0 \partial_x + F_0 \partial_k)\, f = \boldsymbol{C}[f, f_a, \dots]$$

Liouville term: $\boldsymbol{L}[f]$

Conventional derivation (n=1)

Kainulainen, Prokopec, Schmidt, and Weinstock, (2001), (2022);
Prokopec, Schmidt, and Weinstock, (2004);

- **Taking into account CPV effects from a slowly varying VEV** $(\partial_x \ll k \sim T)$

The constraint equation at 1st order in derivative: $\left(k_\mu \gamma^\mu - m_R - i\, m_I\, \gamma^5 + \mathcal{O}(\partial_x m)\right) G = 0.$

∂_x correction w/ CPV

$\rightarrow\ G \sim\ n \times \delta(\Omega^2 + \delta\Omega^2)$ where $\Omega^2 = k^2 - |m|^2,\ \delta\Omega^2 = \mathcal{O}(\partial_x m).$

→ modifying dispersion relation $k^0 = \omega_0 \pm \delta\omega$

+: particle
- : anti-particle

- **Substituting the solution into the kinetic equation to determine the evolution of** $n \sim f$**.**

Due to the gradient correction to the mass shell, a CP-violating difference appears:

$$\int_0^\infty dk^0\, i \not{\partial}_x\, G\ = \int_0^\infty dk^0\, (g^> G^< - g^< G^>)\ \ \rightarrow\ \ (v\partial_x + F\partial_k)\, f = \boldsymbol{C}[f, f_a, \ldots]$$

Liouville term: $\boldsymbol{L}[f]$

where $v = v_0 \pm \delta v\ ,\ F = F_0 \pm \delta F.$

Quasiparticle approximation

- **Spectrum of the system $\mathcal{A}$ under the quasiparticle approximation: $\Gamma \ll \Omega^2$** $\quad \Omega^2 = k^2 - m^2$ (free field)

0th order in derivative

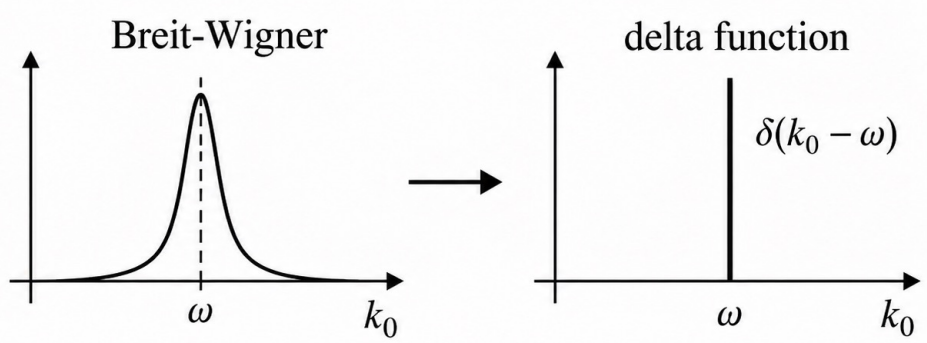
$$\mathcal{A}^{[0]} \sim \frac{\Gamma}{\Omega_0^4 + \Gamma^2} \to \delta(\Omega_0^2)$$

Breit-Wigner distribution

1st order in derivative (fermionic case)

$$\mathcal{A}^{[1]} \sim \Omega_1^2 \times \frac{2\Gamma\,\Omega_0^2}{(\Omega_0^4 + \Gamma^2)^2}$$

$$\to\ \Omega_1^2 \times \delta'(\Omega_0^2)$$



Modification of the on-shell condition from ∂_z can be understood by $\delta(x) + \delta x\, \delta'(x) = \delta(x + \delta x) + \mathcal{O}(\partial_z^2)$.

- **The corrections from the Hermitian self-energy $g_{\mathcal{H}}$ are included in Ω_0^2 and Ω_1^2**

For example, $\Omega_0^2 = \left(k^0 - g_{\mathcal{H}}^0\right)^2 - \left(sk_z + g_{\mathcal{H}}^3\right)^2 - \left(m_R + g_{\mathcal{H}}^1\right)^2 - \left(m_I + g_{\mathcal{H}}^2\right)^2$ (fermionic case)

Derivative corrections included in Ω_1^2 are CP-odd, $\Omega_{1,\mathrm{CP}}^2 = -\Omega_1^2$.

- **With a decomposition of g, the Wightman functions also have an on-shell part: $G \sim n\,\mathcal{A}$.**

Deriving the Boltzmann equation

- **The kinetic equation for spin-s and flavor-i fermion is written in terms of the Poisson bracket:**

$\mathrm{Tr}[P_s P_i (i\not{\partial}_x G)] \propto \{\Omega^2_{s,i,\mathrm{eff}}, n\}_{z,k_z}\ \delta(\Omega^2_{s,i,\mathrm{eff}})$ → describing flow in the phase space w/ the Hamiltonian $\Omega^2_{s,i,\mathrm{eff}}$

(P_s, P_i : projection matrices) → the quasiparticle picture emerges.

The effective on-shell function is written as $\Omega^2_{s,i,\mathrm{eff}} = \Omega_0^2 + \delta\Omega^2$,

where $\delta\Omega^2 = -\frac{1}{K^0}\left(m_I'\left(K_z\partial_{k_z}M_R - M_R\partial_{k_z}K_z\right) - m_R'\left(K_z\partial_{k_z}M_I - M_I\partial_{k_z}K_z\right)\right)$, and $K^0 = k^0 - g^0_{\mathcal{H}}$, $M_R = m_R + g^1_{\mathcal{H}}$, $K_z = sk_z + g^3_{\mathcal{H}}$, $M_I = m_I + g^2_{\mathcal{H}}$.

- **At the energy $k^0 = +\omega_{s,i}$, which solves $\Omega^2_{s,i,\mathrm{eff}}\,(k^0) = 0$, we finally have**

Liouville term (w/ thermal correction): $\boldsymbol{L}_{s,i}\left[f_{s,i}\right] = \underbrace{\partial_{k_z}\omega_{s,i}}_{\text{velocity}}\,\partial_z f_{s,i} \underbrace{-\,\partial_z\omega_{s,i}}_{\text{force}}\,\partial_{k_z} f_{s,i}$, cf.) canonical equation

Collision term (conventional form): $\boldsymbol{C}_{s,i}\left[f_{s,i}, \ldots\right] = \int_0^\infty \frac{dk^0}{2\pi}\,\mathrm{Tr}[P_s P_i (g^> G^< - g^< G^>)]$

Restoring on-shell structure

- **Quasiparticle approximation in the narrow width limit**

  e.g., for the bosonic case, at zeroth order in the derivative expansion, the spectral function takes a Breit-Wigner form as given by

  (cf.) spectral function

  $$G_{\mathcal{A}} \equiv \frac{1}{2i}(G^a - G^r) = \frac{i}{2}(G^> - G^<) \equiv \mathscr{A},$$

  $$\hat{\mathscr{A}}_\phi^{[0]} = \sum_i \frac{\overline{\Pi}_{\mathcal{A}}}{\Omega_i^4 + \overline{\Pi}_{\mathcal{A}}^2} P_i \qquad P_i \hat{\mathscr{A}}_\phi^{[0]} \to \pi\,\mathrm{sign}(k^0)\,\delta(\Omega_i^2) \qquad P_i : \text{projection to i-th flavor}$$

- **Prescription such that the Wightman functions have the on-shell structure**

  In the absence of wall-gradients, the system should be in local thermal equilibrium ($\eta = \pm 1$)

  $$g^\lambda = \frac{2\eta}{i} n^\lambda_{\mathrm{leq}} g_{\mathcal{A}} \qquad n^<_{\mathrm{leq}} = \frac{1}{e^{\beta k\cdot u} - \eta} \qquad n^>_{\mathrm{leq}} = n^<_{\mathrm{leq}} + \eta$$

  Distribution will be modified by the wall-gradient $\quad n^\lambda_{\mathrm{leq}} \to n^\lambda(k,x)$

  e.g., for bosonic case, the zeroth order Wightman functions are $\quad P_i i\hat{\Delta}^\lambda_{\mathrm{shell}} = 2\pi n_i^\lambda \mathrm{sign}(k^0)\delta(\Omega_i^2)$.

Discussions on VEV-insertion approximation

VEV insertion approximation (VIA)

- **Transport equation proposed by Riotto in 1996-1998**

Left-handed current: $j_L^\mu(u) \equiv \langle \bar{\psi}(u)\gamma^\mu P_L \psi(u)\rangle = -\mathrm{Tr}\left[\gamma^\mu P_L\, iS^<(u,u)\right]$

$S^{<,>}$: Wightman function for a fermion

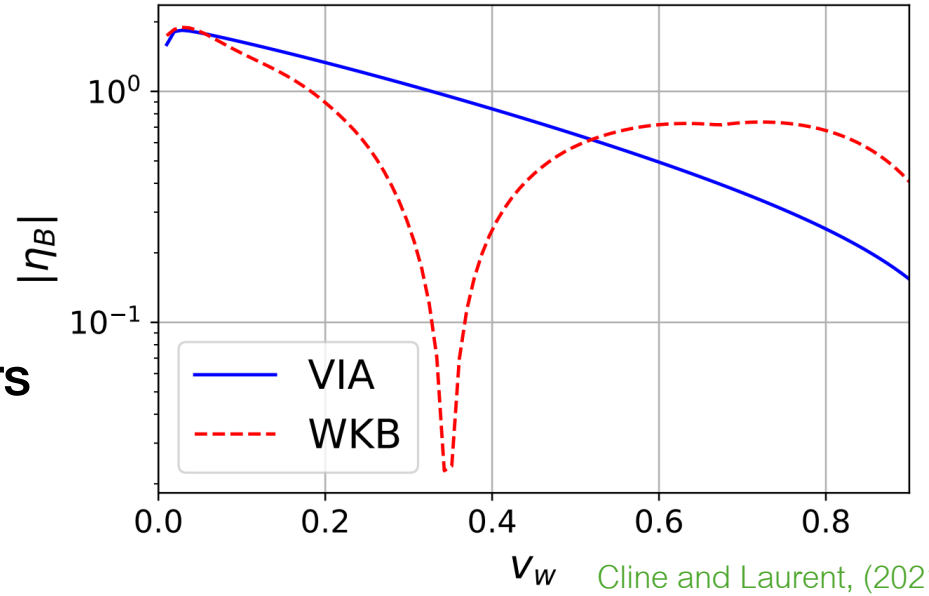
$\Sigma^{<,>}$: corresponding self-energy

Integrating the kinetic equation w.r.t. the momentum, the current divergence equation is written as

$$\partial_\mu j_L^\mu = \partial_t j_L^0 + \boldsymbol{\nabla}\cdot\boldsymbol{j}_L = S^{\not{CP}}_{L,\mathrm{VIA}} + \mathcal{C}_L .$$

Using Fick's law $\boldsymbol{j}_L = -D_L \boldsymbol{\nabla} j_L^0$, we obtain the transport equation for number density j_L^0.

- **The baryon asymmetry predicted by the VIA method differs from that predicted by the SC (WKB) approach.**



Cline and Laurent, (2021)

Cline and Kainulainen, (2020); Cline and Laurent, (2021);

Basler, Biermann, Mühlleitner, and Müller, (2023);

Problems with the VIA method

① Discussion by Kainulainen (2021)

- **The conventional VIA source:** $S^{\not CP}_{L,\mathrm{VIA}}(u) = 2|m(u)|^2\partial_\mu\theta(u)\int d^4w\,\Theta(u^0-w^0)\,(u-w)^\mu \times I_{\mathrm{VIA}}(u,w) + \mathcal{O}(\partial_u^2)$

  Non-zero chiral current source: $S^{\not CP}_{R,\mathrm{VIA}} = -S^{\not CP}_{L,\mathrm{VIA}}$

- **Pinch singularity from I_{VIA}**

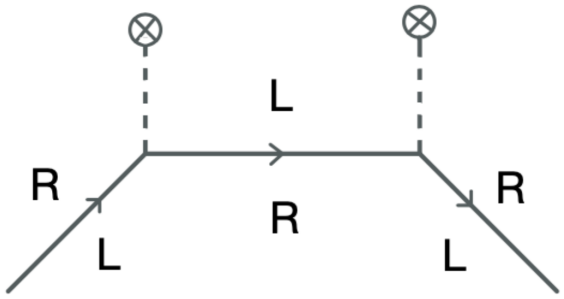
  The integrand is $I_{\mathrm{VIA}}(u,w) = \mathrm{Im}\Big[\mathrm{Tr}\left[iS^{>}_{0,L}(u,w)iS^{<}_{0,R}(w,u) - iS^{<}_{0,L}(u,w)iS^{>}_{0,R}(w,u)\right]\Big]$.

  $$I_{\mathrm{VIA}} \sim \int \frac{d^4k}{(2\pi)^4}k^2(\mathrm{sign}(k^0))^2\delta(k^2)^2$$ → an ill-defined distribution with a singularity

- **It is regularized by replacing S_0 with a thermal propagator, but it is still scheme dependent.**

What was the problem?

① Discussion by Kainulainen (2021)



- **One-particle-reducible diagrams**

The divergence equation is

$$\partial_\mu j^\mu_L(u) = -\lim_{v\to u}\mathrm{Tr}\left[(m(u)P_R - m^*(v)P_L)\,S^<(u,v)\right] - 2\int d^3\boldsymbol{w}\int_{-\infty}^{u^0} dw^0\,\mathrm{Re}\left[\mathrm{Tr}\left[P_L\left(\Sigma^>(u,w)S^<(w,u) - \Sigma^<(u,w)S^>(w,u)\right)\right]\right]$$

Riotto omitted the explicit local mass contribution
and instead incorporated the VEV effects through non-1PI mass-insertion diagrams in the collision kernel.

$$\mathrm{Tr}\left[P_L\left(\Sigma^>(u,w)S^<(w,u) - \Sigma^<(u,w)S^>(w,u)\right)\right] \simeq \mathrm{Tr}\left[\Sigma^>_{L,\mathrm{VIA}}(u,w)S^<_{0,L}(w,u) - \Sigma^<_{L,\mathrm{VIA}}(u,w)S^>_{0,L}(w,u)\right]$$

where $i\Sigma^\lambda_{L,\mathrm{VIA}}(u,w) = [-im(u)]\,iS^\lambda_{0,R}(u,w)\,[-im^*(w)]$ at leading order.

- **In the SC (and our) approach, the local mass effect is resummed into the propagators, and derivative corrections are consistently taken into account by the derivative expansion.**

VIA cancellation

② Discussion by Postma, van de Vis, and White (2022)

- **Current divergence equation:** $\partial_{x,\mu} j_L^{\mu,\lambda}(x) = S^{\lambda}_{\mathrm{F},LL}$ Lesser current: $j_L^{\mu,<}(x) = \langle \overline{\psi_L}(v)\gamma^\mu \psi_L(u)\rangle\Big|_{u=v=x}$

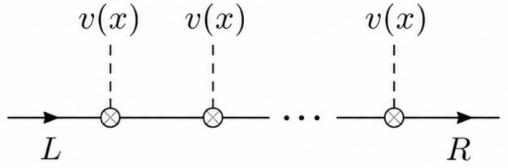
  Here, a Wigner transformation is performed, but an equivalent formula can be obtained from Riotto's formula.

  Current divergence: $$\partial_{x,\mu} j_L^{\mu,\lambda}(x) = -\int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\left\{\frac{i}{2}\not{\partial}_x, P_L S^{\lambda}\right\}\right]$$

  Source: $$S^{\lambda}_{\mathrm{F},LL} = -\int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\left(\mathcal{I}^{\lambda}\Big|_{\diamond^0}\right)_{LL}\right]$$

  Integrand: $$\mathcal{I}^{\lambda}\Big|_{\diamond^0} = \left[\mathcal{M} + \Sigma_H,\, S^{\lambda}\right] + \left[\Sigma^{\lambda},\, S_H\right] + \frac{1}{2}\left(\{\Sigma^{>}, S^{<}\} - \{\Sigma^{<}, S^{>}\}\right)$$

- **Substituting the VEV resummed solution at 0th order:** $S^{[0]} =$



  VIA cancellation: $$\underbrace{\left[M + \Sigma_H, S^{\lambda,[0]}\right]}_{\text{mass commutator}} + \underbrace{\left[\Sigma^{\lambda}, S_H^{[0]}\right] + \frac{1}{2}\left(\{\Sigma^{>}, S^{<,[0]}\} - \{\Sigma^{<}, S^{>,[0]}\}\right)}_{\text{collision}} = 0$$

Behind the VIA cancellation

- **(Our 0th order solution) = (the VEV resummed solution of Postma et al.)**

We obtain the following formal solution of the KB equation:

Definition: $\alpha_\pm \equiv \not{k} - M_0 + i\gamma^5 M_5 + \Sigma_H \pm i\Sigma_A$

$$S^{\lambda,[0]} = \alpha_-^{-1}\,\Sigma^\lambda\,\alpha_+^{-1},$$ ← coincides with the VEV resummed solution!

$$S^{\lambda,[1]} = S^{\lambda,[0]}\, i\diamond(\alpha_+,\alpha_+^{-1}) + \alpha_-^{-1}\, i\diamond(\alpha_-, S^{\lambda,[0]}) - \alpha_-^{-1}\, i\diamond(\Sigma^\lambda,\alpha_+^{-1}).$$ ← NLO solution (important for CP violating source)

In particular, the 0th order solution is obtained from $\boxed{\alpha_- S^{\lambda,[0]} - \Sigma^\lambda S^{a,[0]} = 0,}$

Retarded: $S^{r,[0]} = \alpha_-^{-1}$

Advanced: $S^{a,[0]} = \alpha_+^{-1}$.

- **Important rearrangement of the source kernel**

$$\mathcal{I}^\lambda\big|_{\diamond^0} = \left[\mathcal{M}+\Sigma_H,\, S^\lambda\right] + \left[\Sigma^\lambda,\, S_H\right] + \frac{1}{2}\left(\{\Sigma^>, S^<\} - \{\Sigma^<, S^>\}\right)$$
$$= -\alpha_- S^\lambda + \Sigma^\lambda S^a + S^\lambda \alpha_+ - S^r \Sigma^\lambda$$

- **VIA cancellation from our perspective** $\mathcal{I}^\lambda\big|_{\diamond^0} = -\underbrace{\left(\alpha_- S^{\lambda,[0]} - \Sigma^\lambda S^{a,[0]}\right)}_{=0} + \underbrace{\left(S^{\lambda,[0]}\alpha_+ - S^{r,[0]}\Sigma^\lambda\right)}_{=0} = 0$

(h.c. of the 1st term)

Summary around the VIA method

- **In 1996-1998, Riotto formulated transport equations with a CPV source by treating the spacetime-dependent VEV perturbatively through insertions in the self-energy.**
- **Kainulainen pointed out in 2021 that the treatment of the mass term in the VIA is inconsistent.**
  - The local mass term was treated perturbatively through VEV insertions in the self energy, rather than being consistently resummed into the propagators → pinch singularities.
  - The local mass effects should instead be resummed into the propagators, while derivative effects should be consistently incorporated through the derivative expansion, as in the SC approach and in our formulation.
- **Postma et al. showed the cancellation of the VIA source at 0^{th} order in 2022.**
  - We found that the VIA source kernel is built from the equations that determine the 0^{th} order solution.
  - Hence, it vanishes identically for the 0^{th} order solution.