Non-anomalous axions: lessons from the Majoron



Antonio Herrero-Brocal

IFIC (CSIC – Universitat de València)

DISCRETE 2026 · based on Phys.Lett.B 882 (2026) 141045 [arXiv:2602.22311]





The original question

Original motivation of this work: *which symmetry is associated with the Majoron?*

The original question

Original motivation of this work: *which symmetry is associated with the Majoron?*

- Majoron models impose $U(1)_L$: J is “the Goldstone of lepton number”

The original question

Original motivation of this work: *which symmetry is associated with the Majoron?*

- Majoron models impose $U(1)_L$: J is "the Goldstone of lepton number"
- But $\mathcal{L}$ also conserves B, and ϕ carries no baryon number
- **$\Rightarrow \langle\phi\rangle$ breaks every combination $\alpha L + \beta B$ with $\alpha \neq 0$, and leaves B unbroken**

The original question

Original motivation of this work: *which symmetry is associated with the Majoron?*

- Majoron models impose $U(1)_L$: J is "the Goldstone of lepton number"
- But $\mathcal{L}$ also conserves B, and ϕ carries no baryon number
- **$\Rightarrow \langle\phi\rangle$ breaks every combination $\alpha L + \beta B$ with $\alpha \neq 0$, and leaves B unbroken**
- Typical choices in the literature: L or B − L. Is it just a choice?

The original question

Original motivation of this work: *which symmetry is associated with the Majoron?*

- Majoron models impose $U(1)_L$: J is "the Goldstone of lepton number"
- But $\mathcal{L}$ also conserves B, and ϕ carries no baryon number
- **$\Rightarrow \langle\phi\rangle$ breaks every combination $\alpha L + \beta B$ with $\alpha \neq 0$, and leaves B unbroken**
- Typical choices in the literature: L or B − L. Is it just a choice?

→ **It is not a convention: L and B − L are physically very different symmetries**

L or B – L?

$$\partial_\mu j^\mu_B = \partial_\mu j^\mu_L = \frac{N_g}{32\pi^2}\left(g^2\, W\widetilde{W} - g'^2\, B\widetilde{B}\right)$$

L **Lepton number**

Anomalous: mixed anomaly with $SU(2)_L \times U(1)_Y$

⇒ EW instantons break $U(1)_L$ to an exact $\mathbb{Z}_N$

⇒ J is a pseudo-Goldstone (pNGB)



L or B – L?

$$\partial_\mu j^\mu_B = \partial_\mu j^\mu_L = \frac{N_g}{32\pi^2}\left(g^2\, W\widetilde{W} - g'^2\, B\widetilde{B}\right) \qquad \partial_\mu j^\mu_{B-L} = 0$$

L — Lepton number

Anomalous: mixed anomaly with $SU(2)_L \times U(1)_Y$

⇒ EW instantons break $U(1)_L$ to an exact $\mathbb{Z}_N$

⇒ J is a pseudo-Goldstone (pNGB)

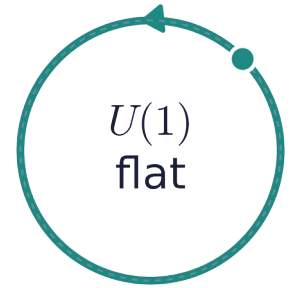


B–L — B – L

Free of gauge anomalies

⇒ an exact U(1), even non-perturbatively

⇒ J is an exact Nambu–Goldstone boson



→ **Gauge-anomaly-free ≠ fully anomaly-free: gravitational and $(B - L)^3$ anomalies need v_R**

Why it matters

m Mass

If L: $m_J \neq 0$ from EW instantons

A Anomalous terms

If L:

$$J W\widetilde{W}, \qquad J B\widetilde{B}$$

fixed by the anomaly: an ALP

DW Domain walls

If L: SSB of $\mathbb{Z}_N \Rightarrow$ N vacua $\Rightarrow$ domain walls that can overclose the Universe

Lazarides, Reig, Shafi, Srivastava, Valle '19 · Heeck, Patel '19 · Brune '22, '25 · Berbig '25

Recently debated in the literature

Not just a label: the answer fixes the Majoron mass, its couplings and its cosmology

Why it matters

m Mass

If L: $m_J \neq 0$ from EW instantons

If B − L: $m_J = 0$ exactly

A Anomalous terms

If L:

$$J W\widetilde{W}, \qquad J B\widetilde{B}$$

fixed by the anomaly: an ALP

If B − L: no anomalous terms

DW Domain walls

If L: SSB of $\mathbb{Z}_N \Rightarrow$ N vacua $\Rightarrow$ domain walls that can overclose the Universe

If B − L: flat vacuum manifold: no walls

Lazarides, Reig, Shafi, Srivastava, Valle '19 · Heeck, Patel '19 · Brune '22, '25 · Berbig '25

Recently debated in the literature

Why it matters

m Mass

If L: $m_J \neq 0$ from EW instantons

If B − L: $m_J = 0$ exactly

A Anomalous terms

If L:

$$J W\widetilde{W}, \qquad J B\widetilde{B}$$

fixed by the anomaly: an ALP

If B − L: no anomalous terms

DW Domain walls

If L: SSB of $\mathbb{Z}_N \Rightarrow$ N vacua $\Rightarrow$ domain walls that can overclose the Universe

If B − L: flat vacuum manifold: no walls

Lazarides, Reig, Shafi, Srivastava, Valle '19 · Heeck, Patel '19 · Brune '22, '25 · Berbig '25

Recently debated in the literature

→ **Not just a label: the answer fixes the Majoron mass, its couplings and its cosmology**

Proof of B – L nature

1 First argument: Goldstone's theorem

Goldstone's theorem: *spontaneous breaking of an exact continuous symmetry ⇒ an exactly massless NGB*

a B – L is exact

Free of gauge anomalies and broken by the vev of ϕ ⇒ a massless NGB must exist

Proof of B – L nature

1 First argument: Goldstone's theorem

Goldstone's theorem: *spontaneous breaking of an exact continuous symmetry ⇒ an exactly massless NGB*

a B – L is exact

Free of gauge anomalies and broken by the vev of ϕ ⇒ a massless NGB must exist

b If J belonged to L

L is anomalous, so J would get a mass. That would violate Goldstone's theorem

Proof of B – L nature

1 First argument: Goldstone's theorem

Goldstone's theorem: *spontaneous breaking of an exact continuous symmetry ⇒ an exactly massless NGB*

a B – L is exact

Free of gauge anomalies and broken by the vev of ϕ ⇒ a massless NGB must exist

b If J belonged to L

L is anomalous, so J would get a mass. That would violate Goldstone's theorem

c Add QQQL

θ_{EW} becomes physical, but B − L stays exact: a mass for J would still violate the theorem

$$\Delta B = \Delta L = 1, \quad \Delta(B - L) = 0$$

Proof of B − L nature

1 First argument: Goldstone's theorem

Goldstone's theorem: *spontaneous breaking of an exact continuous symmetry ⇒ an exactly massless NGB*

a B − L is exact

Free of gauge anomalies and broken by the vev of ϕ ⇒ a massless NGB must exist

b If J belonged to L

L is anomalous, so J would get a mass. That would violate Goldstone's theorem

c Add QQQL

θ_{EW} becomes physical, but B − L stays exact: a mass for J would still violate the theorem

$$\Delta B = \Delta L = 1, \quad \Delta(B - L) = 0$$

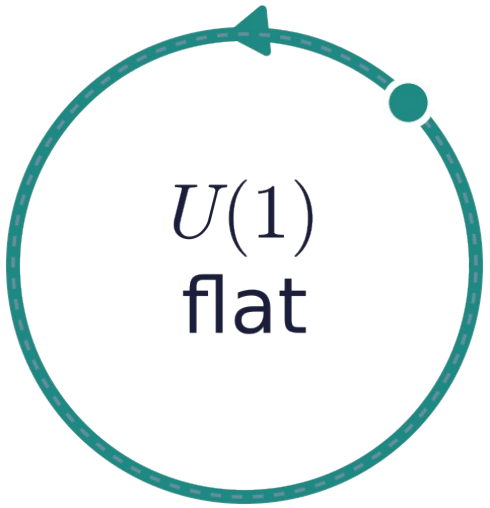
→ **J must be the NGB of the exact symmetry: B − L, not L**

A general statement

If a VEV breaks simultaneously an anomalous symmetry and a symmetry free of gauge anomalies, the Goldstone is associated with the non-anomalous one.

→ **In general: J belongs to whichever combination is anomaly-free, here B − L**

1º Implication: no domain walls for the Majoron



$U(1)_{B-L}$: flat vacuum manifold

- J is the NGB of B − L, which is free of gauge anomalies
- EW instantons do not break B − L ⇒ no $\mathbb{Z}_N$, no degenerate vacua
- This holds even when θ_{EW} is physical, e.g. with the B − L-conserving operator QQQL

- **What matters is explicit B − L breaking, not θ_{EW}**

Only then does J get a potential and a mass, and whether walls appear depends on that explicit breaking, not on the EW anomaly.

→ **Answer to the debate: without explicit B − L breaking, J is massless and there are no domain walls**

2º Implication: duality for B – L explicitly broken

Example: only B broken explicitly, $\mathcal{L}_B = \mathcal{L}_{\text{inv}} + c_L\,\mathcal{O}_L$

$$m_J^2 \;\propto\; c_L\; e^{-8\pi^2/g_W^2}$$

needs both: explicit B – L breaking (c_L) and the L anomaly (g_W)

2º Implication: duality for B – L explicitly broken

Example: only B broken explicitly, $\mathcal{L}_B = \mathcal{L}_{\text{inv}} + c_L\,\mathcal{O}_L$

$$m_J^2 \;\propto\; c_L\, e^{-8\pi^2/g_W^2}$$

needs both: explicit B − L breaking (c_L) and the L anomaly (g_W)

$c_L \to 0$

B − L becomes exact ⇒ J is its NGB ⇒ massless

2º Implication: duality for B – L explicitly broken

Example: only B broken explicitly, $\mathcal{L}_B = \mathcal{L}_{\text{inv}} + c_L\,\mathcal{O}_L$

$$m_J^2 \;\propto\; c_L\; e^{-8\pi^2/g_W^2}$$

needs both: explicit B − L breaking (c_L) and the L anomaly (g_W)

$c_L \to 0$

B − L becomes exact ⇒ J is its NGB ⇒ massless

$g_W \to 0$

the L anomaly disappears ⇒ our result: J is the NGB of the (now non-anomalous) L ⇒ massless

2º Implication: duality for B – L explicitly broken

Example: only B broken explicitly, $\mathcal{L}_B = \mathcal{L}_{\text{inv}} + c_L\,\mathcal{O}_L$

$$m_J^2 \;\propto\; c_L\, e^{-8\pi^2/g_W^2}$$

needs both: explicit B − L breaking (c_L) and the L anomaly (g_W)

$c_L \to 0$ B − L becomes exact ⇒ J is its NGB ⇒ massless

$g_W \to 0$ the L anomaly disappears ⇒ our result: J is the NGB of the (now non-anomalous) L ⇒ massless

→ **Two equivalent descriptions of the same mass: explicit breaking of B − L, or the anomaly of L**

2º Implication: the QCD axion paralelism

- PQ and $U(1)_A$ are both $SU(3)_c$-anomalous
- Their anomaly-free combination is explicitly broken only by m_q

2º Implication: the QCD axion paralelism

- PQ and $U(1)_A$ are both $SU(3)_c$-anomalous
- Their anomaly-free combination is explicitly broken only by m_q

$$\mathcal{L}_{\rm QCD} \supset \sum_f \bar{q}_f \left(i\not{D} - m_f\right) q_f + \frac{\theta g_s^2}{32\pi^2} G\widetilde{G}$$

↓

$$q_f \to e^{i\frac{\theta}{2N_f}\gamma_5} q_f$$

$$\mathcal{L}_{\rm QCD} \supset \sum_f \bar{q}_f \left(i\not{D} - m_f\, e^{i\theta/N_f}\right) q_f$$

2º Implication: the QCD axion paralelism

- PQ and $U(1)_A$ are both $SU(3)_c$-anomalous
- Their anomaly-free combination is explicitly broken only by m_q
- **Same duality as for J: m_a needs both m_q and the anomaly**

$$\mathcal{L}_{\rm QCD} \supset \sum_f \bar{q}_f \left(i\not{D} - m_f\right) q_f + \frac{\theta g_s^2}{32\pi^2} G\widetilde{G}$$

$$\downarrow \qquad q_f \to e^{i\frac{\theta}{2N_f}\gamma_5} q_f$$

$$\mathcal{L}_{\rm QCD} \supset \sum_f \bar{q}_f \left(i\not{D} - m_f\, e^{i\theta/N_f}\right) q_f$$

2º Implication: the QCD axion paralelism

- PQ and $U(1)_A$ are both $SU(3)_c$-anomalous
- Their anomaly-free combination is explicitly broken only by m_q
- **Same duality as for J: m_a needs both m_q and the anomaly**

$$\mathcal{L}_{\rm QCD} \supset \sum_f \bar{q}_f \left(i\not{D} - m_f\right) q_f + \frac{\theta g_s^2}{32\pi^2} G\widetilde{G}$$

$$\downarrow \qquad q_f \to e^{i\frac{\theta}{2N_f}\gamma_5} q_f$$

$$\mathcal{L}_{\rm QCD} \supset \sum_f \bar{q}_f \left(i\not{D} - m_f\, e^{i\theta/N_f}\right) q_f$$

$$m_a^2 f_a^2 \simeq m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2} \xrightarrow{m_q \to 0} 0$$

$m_a \to 0$ as $m_q \to 0$, understood without any calculation

→ **The axion is the NGB of the anomaly-free combination, explicitly broken by m_q**

2º Implication: the QCD axion paralelism

The usual argument

- $m_q = 0 \Rightarrow \theta$ can be rotated away
- $\Rightarrow \theta$ is unphysical $\Rightarrow$ no potential
- $\Rightarrow m_a = 0$

Not valid, for the same reason as for J: the mass cannot depend on the breaking of an independent symmetry

2º Implication: the QCD axion paralelism

The usual argument

- $m_q = 0 \Rightarrow \theta$ can be rotated away
- $\Rightarrow \theta$ is unphysical $\Rightarrow$ no potential
- $\Rightarrow m_a = 0$

Not valid, for the same reason as for J: the mass cannot depend on the breaking of an independent symmetry

✗

Counterexample

Add an operator $\mathcal{O}$ that breaks PQ and $U(1)_A$ but preserves their anomaly-free combination:

$$\bar{\theta} \text{ physical} \quad \text{and still} \quad m_a = 0$$

at $m_q = 0$ the anomaly-free symmetry is still exact

→ **$m_a = 0$ because the anomaly-free symmetry is exact, not because θ is unphysical**

3º Implication: an anomalous coupling without anomaly

Non-anomalous symmetry, massless Goldstone: does it have the coupling $\frac{a}{f_a}\frac{g_s^2}{32\pi^2}G\widetilde{G}$? **Naively, no**

3º Implication: an anomalous coupling without anomaly

Non-anomalous symmetry, massless Goldstone: does it have the coupling $\frac{a}{f_a}\frac{g_s^2}{32\pi^2}G\widetilde{G}$? **Naively, no**

QCD + axion. Then, add $\mathcal{O}$ and take $m_q \to 0$

$$\mathcal{L} = \lim_{m_q \to 0} \mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm PQ} + \mathcal{O}$$

- Our result: a is the exact NGB of the non-anomalous symmetry
- Its coupling to gluons does not depend on m_q: it survives
- **An interaction formally identical to the anomalous one, without an anomaly**

3º Implication: an anomalous coupling without anomaly

Non-anomalous symmetry, massless Goldstone: does it have the coupling $\frac{a}{f_a}\frac{g_s^2}{32\pi^2}G\widetilde{G}$? **Naively, no**

QCD + axion. Then, add $\mathcal{O}$ and take $m_q \to 0$

$$\mathcal{L} = \lim_{m_q \to 0} \mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm PQ} + \mathcal{O}$$

- Our result: a is the exact NGB of the non-anomalous symmetry
- Its coupling to gluons does not depend on m_q: it survives
- **An interaction formally identical to the anomalous one, without an anomaly**

Seen as the pNGB of PQ

- The coupling comes from the PQ anomaly
- But then the same anomaly should give a mass
- **Cannot explain a coupling without a mass**

3º Implication: an anomalous coupling without anomaly

Non-anomalous symmetry, massless Goldstone: does it have the coupling $\frac{a}{f_a}\frac{g_s^2}{32\pi^2}G\widetilde{G}$? **Naively, no**

QCD + axion. Then, add $\mathcal{O}$ and take $m_q \to 0$

$$\mathcal{L} = \lim_{m_q \to 0} \mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm PQ} + \mathcal{O}$$

- Our result: a is the exact NGB of the non-anomalous symmetry
- Its coupling to gluons does not depend on m_q: it survives
- **An interaction formally identical to the anomalous one, without an anomaly**

Seen as the pNGB of PQ

- The coupling comes from the PQ anomaly
- But then the same anomaly should give a mass
- **Cannot explain a coupling without a mass**

→ **This coupling is not a fingerprint of an anomaly: non-anomalous NGBs can have it**

Summary

1. When a VEV breaks an anomalous and a gauge-anomaly-free symmetry at once, the NGB belongs to the non-anomalous one.

2. The Majoron is the Goldstone of B − L, not L: exactly massless unless B − L is explicitly broken.

3. No domain-wall problem for the Majoron, even with physical θ_{EW} (e.g. QQQL).

4. Anomalous-like interactions are not a fingerprint of anomalies and non-anomalous axions can have it.

Thank you!

arXiv:2602.22311

Backup: Why the choice of basis matters

Physics cannot depend on the basis…

…but we do not have control when computing non-perturbative effects

- Instanton-induced potentials and masses
- The θ dependence of physical quantities

The non-anomalous basis gives the answers without computing

- Which operators give a mass
- When the mass vanishes ($c_L \to 0$, $m_q \to 0$)
- Whether there are domain walls

→ **Identifying the anomaly-free symmetry turns non-perturbative questions into symmetry arguments**

Backup: proof of B – L nature

2 Second argument: the completely anomaly-free limit

- Suppose B − L is completely anomaly free
- ⇒ it can be consistently gauged: $U(1)_{B-L}$
- J is absorbed by Z'_{B-L} as its longitudinal polarisation
- **Being absorbed requires an exact shift symmetry ⇒ J cannot be the pNGB of the anomalous L**

Gauged B – L

$$\phi \to e^{2i\alpha(x)}\phi \quad \Longrightarrow \quad J \to J + 2v_\phi\,\alpha(x)$$

J can be gauged away only if its shift symmetry is exact

A massive pNGB of L could not be eaten

→ **Absorption by Z′ requires an exact shift symmetry: J belongs to B − L, not to L**

Backup: proof of B − L nature

3 Third argument: break everything explicitly (1/2)

Start from $\mathcal{L}_{inv}$ (B and L conserved, ⟨ϕ⟩ breaks L) and add every explicit breaking:

$$\mathcal{L} = \mathcal{L}_{\text{inv}} + c_{B-L}\,\mathcal{O}_{B-L} + c_{B+L}\,\mathcal{O}_{B+L} + c_B\,\mathcal{O}_B + c_L\,\mathcal{O}_L$$

Polar parametrisation

$$\phi = \frac{1}{\sqrt{2}}\,(v_\phi + \rho)\,e^{iq_\phi \pi(x)}, \qquad \pi \equiv \frac{J}{q_\phi v_\phi}$$

Local B − L rotation of every field

$$f \;\to\; e^{-iq_f \pi(x)}\, f \qquad \Longrightarrow \qquad \phi \to \frac{1}{\sqrt{2}}\,(v_\phi + \rho)$$

q_f = B − L charge of f. B − L is free of gauge anomalies, so no π FF term is generated

→ **π is removed from ϕ: it now appears only through $\partial_\mu J$ or through B − L-violating operators**

Backup: proof of B − L nature

3 Third argument: break everything explicitly (2/2)

✓ Rotate with B − L

no anomalous term: B − L is free of gauge anomalies

$$\mathcal{L} \to \widetilde{\mathcal{L}}_{\text{inv}}(\partial_\mu J) + c_{B-L}\,\mathcal{O}_{B-L} + c_{B+L}\,e^{iQ_{B+L}\pi}\,\mathcal{O}_{B+L} + c_B\,e^{iQ_B\pi}\,\mathcal{O}_B + c_L\,e^{iQ_L\pi}\,\mathcal{O}_L$$

L Rotate with L

L is anomalous: an extra π WW, π $B\!B$ term appears

$$\mathcal{L} \to \widetilde{\mathcal{L}}_{\text{inv}}(\partial_\mu J) + c_{B-L}\,e^{iQ^L_{B-L}\pi}\,\mathcal{O}_{B-L} + c_{B+L}\,e^{iQ^L_{B+L}\pi}\,\mathcal{O}_{B+L} + c_B\,e^{iQ^L_B\pi}\,\mathcal{O}_B + c_L\,\mathcal{O}_L + \pi(x)\,\mathcal{A}_L\,\frac{g^2\,W\widetilde{W} - g'^2\,B\widetilde{B}}{32\pi^2}$$

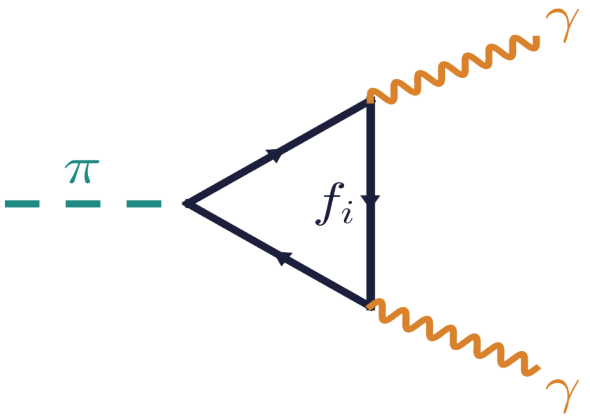
→ **Same Lagrangian: the anomalous term can be rotated away, the phases of B − L-violating operators cannot**

Backup: toy model, an exact NGB that couples to γγ

field	SM rep.	B − L
ψ_L	(1, 1, 2)	+1
ψ_R	(1, 1, 2)	−1
χ_L	(1, 1, 1)	−4
χ_R	(1, 1, 1)	+4
ϕ	(1, 1, 0)	**+2**

- χ charges make B − L free of gauge anomalies
- Add $\mathcal{O}$ breaking B and L but not B − L, so θ_{EW} is physical
- π is massless and is the NGB of B − L

$$\mathcal{L}_Y^{\rm BSM} = y\,\phi\,\bar{\psi}_L\psi_R + \text{h.c.}\,, \qquad \phi = \frac{1}{\sqrt{2}}\left(v_\phi + \phi^0 + i\pi\right)$$



$$\mathcal{M} = -i\,\frac{q_\psi^2 e^2}{4\pi^2 v_\phi}\,\epsilon^{\alpha\beta\rho\sigma}\epsilon^*_\alpha(p_1)\epsilon^*_\beta(p_2)\,p_{1\rho}\,p_{2\sigma} \;\neq\; 0$$

→ **No electromagnetic anomaly, exact NGB, yet πγγ ≠ 0: formally identical to the ALP coupling**

Backup: why the toy model works

π only sees ψ

χ is massless and decoupled. In the linear parametrisation π sees only the ψ-number U(1), which is anomalous. The coupling matches its anomaly.

EFT view

Integrate out ψ: the B − L anomaly from χ is compensated by the shift of the π FF operator, which is reabsorbable, hence unphysical.

Shift symmetry intact (Adler zero): $\mathcal{M} \propto \frac{Y_{ii}}{(\hat{M}_f)_{ii}} \epsilon^{\alpha\beta\rho\sigma} \epsilon^*_\alpha \epsilon^*_\beta \, p_{1\rho} \, p_{\pi\sigma} \xrightarrow{p_\pi \to 0} 0$

→ **SU(2) version: the anomaly picture would predict an instanton mass and walls. Neither occurs, the coupling does**