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BEYOND $SU(N)$

$U(3) \times U(2)$ as the underlying symmetry of the strong and electroweak interactions

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HOW IT STARTED

It began with a mistake (mine)

My student, **Antonio Herrero-Brocal**, came to me and pointed out an arXiv paper that claimed to be based on a $U(2)$ **flavor symmetry** when, according to him, what it actually wrote down was $SU(2) \times U(1)$.

I told him, confidently, that these are *the same thing* because they share the **same algebra**. But equal algebras do not mean equal groups: they are *not* the same thing, and that error is exactly where this project began.

So we set out to take that difference seriously.

01

Preliminaries

Where the SM falls short and the group theory behind $U(N) \neq SU(N) \times U(1)$

THE STARTING POINT

The Standard Model: successful, yet ad hoc

A Yang–Mills theory on $SU(3) \times SU(2) \times U(1)$. Remarkably accurate, yet built on ad hoc empirical inputs rather than an organizing principle.

Hypercharge by hand

The Y assignments have no rationale inside the SM.

They are simply assumed.

Charge unquantized

Nothing in the SM makes $Q_p + Q_e = 0$ hold exactly.

A **coincidence** that follows from no deeper rule.

Missing representations

Some minimal irreps are absent from the SM (and not observed in Nature).

Example: an $SU(2)$ doublet with $Y = 0$.

AN ACCIDENTAL $U(1)$

Most $SU(N)$ models hide an extra $U(1)$

Fermions enter the renormalizable Lagrangian **in pairs**:

$$\bar{\psi}\, i\gamma^\mu D_\mu \psi, \quad y\,\bar{\psi}_L H \psi_R, \quad \dots$$

So $\psi \to e^{i\alpha}\psi$ is a symmetry **nobody imposed**: an **accidental** global $U(1)$.

QCD

quark number: $U(1)_B$

SM

$U(1)_B$ and $U(1)_L$

Not new: this $U(1)$ is the *common phase* of the $SU(N)$ multiplets themselves.

THE QUESTION

A symmetry we forget to gauge

Most $SU(N)$ models carry an accidental global $U(1)$, inherited from $SU(N)$ itself.

Why gauge only the $SU(N)$, and leave the $U(1)$ phase as a *global* symmetry?

A NEW PRINCIPLE

Taking the gauge principle to its limit

Promote the accidental $U(1)$ to a gauge symmetry, alongside $SU(N)$.

THE PRINCIPLE

$$SU(N) \to U(N)$$

A general rule for building gauge theories
[Herrero-Brocal, Perez-Soler, Vicente '25]

BACK TO MY MISTAKE

$U(N)$ is *not* $SU(N) \times U(1)$

$$\varphi : SU(N) \times U(1) \longrightarrow U(N), \qquad (g, e^{i\alpha}) \longmapsto e^{i\alpha} g$$

A GROUP HOMOMORPHISM

01 · ONTO

Every U is reached

Take any $U \in U(N)$. Its determinant is a phase, $\det U = e^{iN\alpha}$, so $g \equiv e^{-i\alpha} U$ has $\det g = 1$. Then

$$U = \varphi\left(g, e^{i\alpha}\right)$$

Every element has a preimage: φ is **surjective**.

02 · NOT ONE-TO-ONE

N pairs, one U

$$\left(\omega^{-k} g,\ \omega^{k} e^{i\alpha}\right) \longmapsto e^{i\alpha} g$$
$$k = 0, \ldots, N-1$$

The factors cancel: N different pairs give the same U.

Therefore, φ is **not injective**, and **the two groups are different.**

03 · HENCE

A quotient

$$U(N) \cong \frac{SU(N) \times U(1)}{\mathbb{Z}_N}$$

Remove the overcounting by identifying the pairs that give the same matrix.

$\mathfrak{u}(N) = \mathfrak{su}(N) \oplus \mathfrak{u}(1)$ Same algebra, same Feynman rules, different group, as $SU(2)$ vs $SO(3)$.

Let's apply these ideas to a new theory of
the strong and electroweak interactions

$$U(3) \times U(2)$$

02

The $U(3) \times U(2)$ model

2.1 The model

Charges, fermions, hypercharge, scalars

2.2 Consequences

The remnant $\mathbb{Z}_m$, neutrino masses, dark matter, and how to falsify all this

THE MODEL

The $U(3) \times U(2)$ model

$$G = U(3) \times U(2)$$

$$SU(3)_c \subset U(3), \quad SU(2)_L \subset U(2)$$

Algebra: $SU(3)_c \times SU(2)_L \times U(1)_3 \times U(1)_2$, one Abelian generator more than the SM.

FACTOR	SUBGROUP	COUPLING	BOSON
$U(3)$	$SU(3)_c$	g_s	G_μ^a
	$U(1)_3$	g_3	A_μ^3
$U(2)$	$SU(2)_L$	g_L	W_μ^i
	$U(1)_2$	g_2	A_μ^2

$$D_\mu = \partial_\mu + i g_s G_\mu^a T^a + i g_L W_\mu^i T^i + i g_3 \, Q_3 A_\mu^3 + i g_2 \, Q_2 A_\mu^2$$

Every field carries **two Abelian charges**, Q_3 and Q_2.

TWO CONSEQUENCES

(i) Two couplings per $U(N)$

(ii) Constraints on Q_3, Q_2

CONSEQUENCE (I)

Two couplings, one group

$$D_\mu = \partial_\mu + i\, g_a A^a_\mu T^a$$

A single $U(N)$ admits **two independent gauge couplings**: one for the $SU(N)$ generators, one for the $U(1)$. Only the Lie algebra constrains them.

01 · DEMAND COVARIANCE

$$\delta\psi = i\alpha^b T^b \psi, \qquad \delta A^c_\mu = -\tfrac{1}{g_c}\partial_\mu \alpha^c + f^{abc}\alpha^b A^a_\mu$$

$D_\mu\psi \to U D_\mu \psi$ leaves the residue

$$(g_a - g_c)\, f^{abc}\, \alpha^b A^a_\mu\, T^c \psi = 0$$

Couplings must match only where $f^{abc} \neq 0$, i.e. within each simple factor. For $T^0 \propto \mathbb{1}$ all $f^{0bc} = 0$: its coupling is free.

02 · APPLIED TO $U(N)$

$$D_\mu = \partial_\mu + i\, g_N\, G^a_\mu T^a + i\, g_1\, Q\, A_\mu$$

The global structure fixes **which charges** Q are allowed (the charge constraints), **not** the ratio g_1/g_N.

$U(3) \times U(2)$ — Four couplings: $g_s,\ g_L$ for $SU(3),\ SU(2)$ and $g_3,\ g_2$ for $U(1)_3,\ U(1)_2$.

CONSEQUENCE (II)

$U(3) \times U(2)$: the charge constraints

The two $U(1)$ charges, Q_3 for $U(1)_3$ and Q_2 for $U(1)_2$, must satisfy **consistency conditions** set by their representation.

THE CHARGE CONSTRAINTS

$$Q_2 = 2j + 2\,n_2 \qquad Q_3 = p - q + 3\,n_3$$

$2j$ is the $SU(2)$ Dynkin label (j the isospin), (p, q) the $SU(3)$ labels, and $n_2, n_3 \in \mathbb{Z}$. With duality $d \equiv 2j \pmod 2$ and triality $t \equiv p - q \pmod 3$, this reads $Q_2 \equiv d \pmod 2$ and $Q_3 \equiv t \pmod 3$: the N-ality fixes the charge mod N, which is exactly what quantizes it.

$SU(2)$ doublet

Only **odd** Q_2 is allowed

→ $\mathbf{2}_1$ permitted, $\mathbf{2}_0$ **forbidden.**

$SU(3)$ triplet

Only $Q_3 \equiv 1 \pmod 3$ is allowed

→ $\mathbf{3}_1$ permitted, $\mathbf{3}_0$ **forbidden.**

KEY RESULT

Charge is quantized, and not every representation is allowed.

ANOMALY CANCELLATION

The fermion content is *derived*, not assumed

In the SM, the fermions are an input. In $U(3) \times U(2)$ they are an **output of anomaly cancellation**, thanks to its combination with the charge constraints.

TWO INGREDIENTS, BOTH FROM THE GROUP

01 **Charge constraints**

Turn the $SU(N)^2\,U(1)_N$ anomalies into **counting rules** for the number of fundamentals.

02 **Anomaly cancellation**

A separate, independent requirement on the spectrum as a whole.

ONLY TWO ASSUMPTIONS

A **Minimality**

Smallest representations (fundamentals and singlets), fewest fields.

B **Irreducibility**

At least one bifundamental $(\mathbf{3}, \mathbf{2})$. Otherwise the two $U(N)$ factors decouple into two unrelated theories.

⇒ Exactly **one SM generation plus one additional singlet**

THE FERMIONS

$U(3) \times U(2)$: minimal fermion content

The minimal content is one SM generation plus one new singlet. Every charge follows from those of Q_L (q_i) and u_R (u_i).

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_3$	$U(1)_2$
Q_L	$\mathbf{3}$	$\mathbf{2}$	q_3	q_2
u_R	$\mathbf{3}$	$\mathbf{1}$	u_3	u_2
d_R	$\mathbf{3}$	$\mathbf{1}$	$2q_3 - u_3$	$2q_2 - u_2$
L_L	$\mathbf{1}$	$\mathbf{2}$	$-3q_3$	$-3q_2$
e_R	$\mathbf{1}$	$\mathbf{1}$	$-2q_3 - u_3$	$-2q_2 - u_2$
ν_R	$\mathbf{1}$	$\mathbf{1}$	$-4q_3 + u_3$	$-4q_2 + u_2$

The remaining charge constraints then hold automatically.

PREDICTION (I)

Quantized charge & hypercharge

PREDICTED HYPERCHARGE

$$Y = \alpha\, Q_3 + \beta\, Q_2$$

$$\alpha = \frac{4q_2 - u_2}{6\,\Delta}, \qquad \beta = -\,\frac{4q_3 - u_3}{6\,\Delta}$$

Such a combination always exists provided $\Delta \equiv q_2 u_3 - q_3 u_2 \neq 0$.
Otherwise the two $U(1)$s would coincide.

01 Charge is quantized

Anomaly cancellation alone fixes only the *relations* among charges, leaving q_i, u_i as continuous real parameters.

The compact $U(N)$ structure (the charge constraints) then **discretizes** them, so the SM's rational charge ratios emerge rather than being fitted.

02 Hypercharge is unique

The charge constraints rule out the other assignments the SM would tolerate:

Trivial $Y_i = 0 \;\; \forall i$

Vector-like $Y_Q = Y_L = Y_e = 0, \;\; Y_u = -Y_d$

Only the observed Y survives.

THE CORRECT SM HYPERCHARGES COME OUT ON THEIR OWN, INDEPENDENTLY OF q_i, u_i

Q_L $\frac{1}{6}$	u_R $\frac{2}{3}$	d_R $-\frac{1}{3}$
L_L $-\frac{1}{2}$	e_R -1	ν_R 0

PREDICTION (II)

Right-handed neutrinos & B–L

ν_R

Must exist

Internal consistency requires right-handed neutrinos as part of the minimal content.

It is the singlet **neutral under hypercharge**.

B − L

Now gauged

$B - L$ is predicted as a local gauge symmetry, no longer accidental and global.

$$B-L = \gamma\, Q_3 + \delta\, Q_2$$

$$\gamma = \frac{q_2 - u_2}{3\Delta}, \quad \delta = -\frac{q_3 - u_3}{3\Delta}$$

Z'

New heavy boson

Recovering the SM keeps Y at low energy, so the physical Z' aligns with the **orthogonal** combination, not necessarily $B - L$.

$Z' = B-L$ for any couplings only if:

$$u_3 = q_3, \qquad u_2 = 4q_2$$

$B-L$ is the phase of $U(3)$, Y that of $U(2)$. Minimal case: $(1, 1, 1, 4)$.

RECOVERING THE SM

The scalar sector & symmetry breaking

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_3$	$U(1)_2$
H	$\mathbf{1}$	$\mathbf{2}$	$u_3 - q_3$	$u_2 - q_2$
Φ	$\mathbf{1}$	$\mathbf{1}$	$s \times$ charges of ν_R	

$$\mathcal{L}_Y = y_d\,\bar{Q}Hd_R + y_u\,\bar{Q}\tilde{H}u_R + y_e\,\bar{L}He_R + y_\nu\,\bar{L}\tilde{H}\nu_R + \text{h.c.}$$

H compensates the charges of $\bar{Q}u_R$ and therefore also of $\bar{Q}d_R$, $\bar{L}e_R$ and $\bar{L}\nu_R$.

Φ breaks the extra $U(1)$ but must leave Y unbroken. Like ν_R, it is neutral under Y, so its charges are an integer multiple of those of ν_R (once common factors are removed): only that integer multiple, s, is free.

SYMMETRY BREAKING

$$U(3) \times U(2)$$

$\downarrow \langle\Phi\rangle$

$$\left(SU(3)_c \times SU(2)_L \times U(1)_Y\right)/\Gamma$$

$\downarrow \langle H\rangle$

$$\left(SU(3)_c \times U(1)_{\text{em}}\right)/\Gamma'$$

$\Gamma \in \{\mathbb{1}, \mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_6\}$ and Γ' are **outputs** of the charge assignment, not inputs.

The fermions fix every scalar charge, up to one integer.

NOT ONE MODEL

$U(3) \times U(2)$ is a family of models

ONE MODEL = FIVE INTEGERS

$$(q_3,\ q_2,\ u_3,\ u_2;\ s)$$

$$q_3, u_3 \equiv 1\ (3),\quad q_2 \text{ odd},\quad u_2 \text{ even}$$

Up to relabeling

Changes of basis $M \in GL(2,\mathbb{Z})$ that preserve triality and duality are not physical.

The fifth integer

s fixes the charges of Φ. After $\langle\Phi\rangle$, a discrete $\mathbb{Z}_m$ with $m = |s|$ survives the breaking.

All members of the family reproduce the SM at the renormalizable level. They differ in **what they forbid** (Γ) and **what they leave behind** (a residual discrete symmetry $\mathbb{Z}_m$).

(q_3, q_2, u_3, u_2)	$6Y$	Γ	**Forbids**
$(1, -1, -8, 14)$	$3Q_3 + 2Q_2$	$\mathbb{1}$	forbids neither
$(1, 1, 1, 4)$	Q_2	$\mathbb{Z}_2$	forbids the $Y = 0$ doublet only
$(1, 3, 4, 2)$	Q_3	$\mathbb{Z}_3$	forbids the charge-$\frac{1}{3}$ singlet only
$(1, 1, -2, 0)$	$-2Q_3 + 3Q_2$	$\mathbb{Z}_6$	forbids both

02

The $U(3) \times U(2)$ model

2.1 The model

Charges, fermions, hypercharge, scalars

2.2 Consequences

The remnant $\mathbb{Z}_m$, neutrino masses, dark matter, and how to falsify all this

THE REMNANT SYMMETRY

What $\langle\Phi\rangle$ leaves behind: $\mathbb{Z}_m$

$\langle\Phi\rangle$ breaks the extra $U(1)$, but not always completely. The rotation by $2\pi/m$ along the direction of the charges of Φ gives it a trivial phase:

$$\langle\Phi\rangle \;\to\; e^{2\pi i s/m}\,\langle\Phi\rangle = \langle\Phi\rangle, \qquad m = |s|$$

the vacuum is invariant: a $\mathbb{Z}_m$ **survives** (nontrivial only for $m > 1$)

Gauge, not global

A remnant of $U(3) \times U(2)$, so it is exact: quantum gravity cannot break it [Krauss–Wilczek '89].

Order set by Φ

Unlike Γ, fixed by the fermions, $m = |s|$ is the one free integer of the scalar sector.

A NATURAL CONSEQUENCE

Neutrino masses

ν_R is required and is **neutral under hypercharge**, carrying only the extra-$U(1)$ charge. So both the nature of neutrino masses and the origin of their smallness are decided by *how that* $U(1)$ *is broken*, i.e. by the integer s that fixes the charges of Φ.

(i) Φ charges = 2 × ν_R charges

Type-I seesaw

A Majorana mass $M_R = \lambda\langle\Phi\rangle$ is induced.

$$\frac{\lambda}{2}\,\overline{\nu_R^c}\,\Phi^*\,\nu_R$$

(ii) Φ charges = ν_R charges

Pseudo-Dirac

No renormalizable Majorana term, but a **suppressed** one, $M_R \sim \frac{v_\Phi^2}{\Lambda}$.

$$\frac{1}{\Lambda}\,\overline{\nu_R^c}\,\Phi^{*2}\,\nu_R$$

An extra singlet ψ allows the inverse seesaw.

(iii) most other choices

Exactly Dirac

No Majorana term at any order, protected by the **gauge** remnant $\mathbb{Z}_m$: safe against Planck-suppressed violation.

Exact criterion: a Majorana mass appears at some order **iff** twice the charges of ν_R are an integer multiple of those of Φ, so that $\overline{\nu_R^c}\,\nu_R$ can be compensated by a power of Φ.

WHERE IT LEADS

Beyond the minimal model

Dark matter

The remnant $\mathbb{Z}_m$ could **stabilize a dark matter particle** added to the minimal model.

Unification

Promoting $SU(5) \to U(5)$ gives

$$U(5) \cong \frac{SU(5) \times U(1)}{\mathbb{Z}_5}$$

Which coincides with **flipped** $SU(5)$.
And $U(3) \times U(2) \subset U(5)$.

The message generalizes: any GUT on $SU(N)$ should be promoted to $U(N)$, unifying all interactions in a single group with two distinct couplings.

A CURIOSITY

The naive product $SU(3)_c \times SU(2)_L \times U(1)_Y$ does *not* fit inside $SU(5)$, only $\big(SU(3) \times SU(2) \times U(1)\big)/\mathbb{Z}_6$ fits. **Global structure, once again.**

Plenty of novel model-building directions still to explore.

HOW WE WOULD TELL THEM APART

Perturbatively identical, physically not

$$U(3) \times U(2) \quad \underset{\simeq}{\text{PERTURBATIVELY}} \quad SU(3)_c \times SU(2)_L \times U(1)_a \times U(1)_b$$

Same algebra, same Feynman rules.

The familiar Z' searches and neutral gauge-boson mixing carry over unchanged.

But non-perturbative effects know about the **global structure** of the group, not just its algebra: that is where the two part ways.

01

Axions & topology

Add an axion for strong CP: the quantized axion–gauge-boson couplings and the formation of domain walls are fixed by the global structure.

02

Forbidden representations

Set by Γ: a $Y = 0$ doublet is forbidden iff $\Gamma \supseteq \mathbb{Z}_2$, a free charge-$\frac{1}{3}$ singlet iff $\Gamma \supseteq \mathbb{Z}_3$. Observe one and that model is dead.

03

Beyond phenomenology

A symmetry-based reason for $U(1)$-extension features that would otherwise be entirely ad hoc.

03

Conclusions

What to take home

TAKEAWAYS

Same algebra, different group: $SU(N) \to U(N)$

Gauging the phase of every $SU(N)$ turns the SM into $U(3) \times U(2)$. The global structure of the group does the work.

EXPLAINS

Charge quantization and the observed hypercharge, uniquely, and why some harmless representations never show up.

PREDICTS

One SM generation plus ν_R, forced by anomaly cancellation, a gauged $B - L$ and a heavy Z'.

A FAMILY OF MODELS

All members reproduce the SM. They differ in what they forbid, Γ, and in what they leave behind, $\mathbb{Z}_m$.

CONSEQUENCES

$\mathbb{Z}_m$ decides whether neutrinos are seesaw, pseudo-Dirac or exactly Dirac, and may stabilize dark matter.
$U(3) \times U(2) \subset U(5)$: flipped $SU(5)$.

Take the gauge principle to its limit.

Questions welcome.

Backup

The SM gauge group · group theory basics · anomaly cancellation · Z' phenomenology

THE SM GAUGE GROUP

Pop quiz: what is the gauge group of the SM?

$$G_{SM} = \frac{SU(3) \times SU(2) \times U(1)}{\Gamma}$$

NOT FIXED!

A $\Gamma = \mathbb{1}$	B $\Gamma = \mathbb{Z}_2$	C $\Gamma = \mathbb{Z}_3$	D $\Gamma = \mathbb{Z}_6$

Same Feynman rules, same LHC. They differ in the **allowed representations**, monopoles and line operators [O'Raifeartaigh '88, Hucks '91, Tong '17].

Correct answer: nobody knows.
Perturbation theory is sworn to secrecy.

Actually, even this assumes hypercharge is a compact $U(1)$: it could just as well be $\mathbb{R}$, and then none of these finite quotients Γ would be available, simply because any real value of hypercharge would then be allowed, with no quantization at all.

THE SM GAUGE GROUP

Glashow fixed the algebra, not the group

Glashow's 1961 paper is credited with the electroweak *group* $SU(2)_L \times U(1)_Y$ [Glashow '61].

What he wrote down are infinitesimal transformations and their commutators: the **algebra** $\mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$.

Perturbation theory never needed more. Weinberg, Salam and the textbooks kept the habit [Weinberg '67, Salam '68].

The interaction Lagrangian,

$$g\mathbf{Z}_\mu \cdot \mathbf{J}_\mu = g\mathbf{Z}_\mu \cdot [(\mathbf{Z}_{\mu\nu} \times \mathbf{Z}_\nu) + i\psi\beta\gamma_\mu \mathbf{O}\psi], \tag{3.1}$$

includes both a symmetrical self-interaction of the Z-triplet and the Yukawa interaction of the bosons to the Fermions. The common coupling strength is in accord with the universality both of the electric charge and of the weak interaction coupling constant. For partial-symmetry, the three imaginary anti-symmetric matrices $\mathbf{O}$ must satisfy the commutation relations of an angular momentum,

$$\mathbf{O} \times \mathbf{O} = i\mathbf{O}. \tag{3.2}$$

In that case three partially-conserved currents

$$\mathbf{J}_\mu = \mathbf{Z}_{\mu\nu} \times \mathbf{Z}_\nu + \psi\beta\gamma_\mu \mathbf{O}\psi \tag{3.3}$$

result from the three infinitesimal transformations (i.e., partial-symmetries)

$$\psi \to (1 + i\mathbf{a} \cdot \mathbf{O})\psi, \quad \mathbf{Z} \to (1 + i\mathbf{a} \cdot \mathbf{t})\mathbf{Z}. \tag{3.4}$$

GLASHOW, NUCL. PHYS. 22 (1961) 579, P. 582

"The gauge group" usually means *the gauge algebra*.

THE SM GAUGE GROUP

Each group comes with its own menu

$(SU(3), SU(2), Y)$	$\Gamma = \mathbb{1}$	$\mathbb{Z}_2$	$\mathbb{Z}_3$	$\mathbb{Z}_6$
STANDARD				
$Q_L \sim (\mathbf{3}, \mathbf{2}, \frac{1}{6})$	✓	✓	✓	✓
$e_R \sim (\mathbf{1}, \mathbf{1}, -1)$	✓	✓	✓	✓
$H \sim (\mathbf{1}, \mathbf{2}, \frac{1}{2})$	✓	✓	✓	✓
EXOTIC				
$(\mathbf{1}, \mathbf{2}, 0)$	✓	✗	✓	✗
$(\mathbf{1}, \mathbf{1}, \frac{1}{2})$	✓	✗	✓	✗
$(\mathbf{3}, \mathbf{1}, \frac{1}{6})$	✓	✗	✓	✗
$(\mathbf{1}, \mathbf{1}, \frac{1}{3})$	✓	✓	✗	✗
$(\mathbf{3}, \mathbf{1}, 0)$	✓	✓	✗	✗
$(\mathbf{1}, \mathbf{2}, \frac{1}{6})$	✓	✓	✗	✗
$(\mathbf{1}, \mathbf{1}, \frac{1}{6})$	✓	✗	✗	✗
$(\mathbf{3}, \mathbf{2}, \frac{7}{6})$	✓	✓	✓	✓
$(\mathbf{8}, \mathbf{1}, 0)$	✓	✓	✓	✓

Which representations are allowed is decided by the **group**, not the algebra.

A representation is allowed iff Γ acts trivially on it [O'Raifeartaigh '88].

The SM fields fit **all four** groups.

The bigger Γ, the **shorter the menu**.

ANOMALY CANCELLATION IN THE SM

Anomalies don't fix the hypercharges

A common belief, but anomaly cancellation in one generation has three solutions, not one.

ANOMALY CONDITIONS

$SU(3)^2U(1)$	$2Y_Q - Y_u - Y_d = 0$
$SU(2)^2U(1)$	$3Y_Q + Y_L = 0$
$\mathrm{grav}^2U(1)$	$6Y_Q - 3Y_u - 3Y_d + 2Y_L - Y_e = 0$
$U(1)^3$	$6Y_Q^3 - 3Y_u^3 - 3Y_d^3 + 2Y_L^3 - Y_e^3 = 0$

	Y_Q	Y_u	Y_d	Y_L	Y_e
SM	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1
TRIVIAL	0	0	0	0	0
VECTOR-LIKE	0	a	$-a$	0	0

Each solution up to an overall normalization. In the vector-like one, u_R and d_R carry opposite charges and form a vector-like pair.

Anomalies allow all three. | *The SM one is chosen by hand.*

And all three *assume* the fermion content, which is itself put in by hand.

THE CENTER

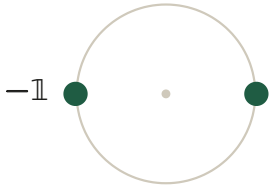
The center of $SU(N)$

The **center** of a group: the elements that **commute with everything**.

In $SU(N)$, Schur's lemma makes them multiples of the identity, $z = \lambda\,\mathbb{1}$, and $\det z = \lambda^N = 1$. So

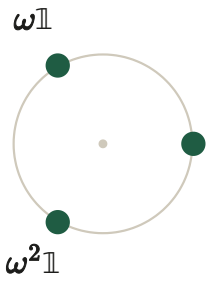
$$Z\big(SU(N)\big) = \{\,\omega^k\,\mathbb{1}\,\}_{k=0}^{N-1} \cong \mathbb{Z}_N$$
$$\omega = e^{2\pi i/N}$$

A **discrete** group sitting inside a continuous one.



$SU(2)$: $\mathbb{Z}_2$

$\pm\mathbb{1}$: two square roots of unity.



$SU(3)$: $\mathbb{Z}_3$

$\omega = e^{2\pi i/3}$: three cube roots of unity.

N-ALITY

How a representation feels the center

On an irrep R, a central element acts as **a single phase**:

$$R(\omega \mathbb{1}) = \omega^{\,n}\,\mathbb{1}, \qquad n \in \mathbb{Z}_N$$

n is the N**-ality** of R.

E.g. $-\mathbb{1} \in SU(2)$ acts on the doublet $\mathbf{2}$ as -1, so $n = 1$.

The cases $N = 2$ and $N = 3$ have their own names: **duality** d and **triality** t.

- It **adds** under $\otimes$: a conserved $\mathbb{Z}_N$ label.
- The adjoint has $n = 0$: gauge bosons are blind to the center.

$SU(2)$: duality $d = 2j \bmod 2$

IRREP	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{4}$	$\mathbf{5}$
d	0	**1**	0	**1**	0

$SU(3)$: triality $t = p - q \bmod 3$

IRREP	$\mathbf{1}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\mathbf{6}$	$\mathbf{8}$
t	0	**1**	**2**	**2**	0

NOT SO ACCIDENTAL AFTER ALL

The center of $SU(N)$ can grow into a $U(1)$

01

The center of $SU(N)$ is always a symmetry:

$$\psi \to \omega^n \psi$$

A discrete $\mathbb{Z}_N$.

02

Extend it to a **continuous** phase:

$$\psi \to e^{i\alpha Q}\psi$$

with $Q \equiv n$. At $\alpha = 2\pi/N$ it **is** the center.

03

It survives if every term pairs indices, $\bar{\psi}\psi$. Only $\varepsilon_{i_1 \ldots i_N}\psi^{i_1}\cdots\psi^{i_N}$ terms may break it: their charge may be $N \neq 0$, but $N \equiv 0 \pmod N$, so $\mathbb{Z}_N$ survives.

QCD: every renormalizable term is $\bar{q}q$, so the center of $SU(3)_c$ grows into $U(1)_B$, with $3B = Q$. The first $\varepsilon_{ijk}\, qqq$ term is dimension 6.

$$e^{2\pi i B} = \omega^t \quad \Longleftrightarrow \quad 3B \equiv t \pmod 3$$

The accidental $U(1)$ is the center of $SU(N)$, made continuous: its charge is the N-ality

[Herrero-Brocal '26]

BACK TO MY MISTAKE · A FAMILIAR CASE

You already know one: $SO(3)$ is *not* $SU(2)$

$$\rho : \; SU(2) \longrightarrow SO(3), \qquad \rho(g)_{ij} = \tfrac{1}{2}\mathrm{Tr}\left(\sigma_i \, g \, \sigma_j \, g^\dagger\right)$$

TWO ELEMENTS, ONE ROTATION

$$\rho(-g) = \rho(g)$$

The sign appears twice, in g and $g^\dagger$, and cancels.

EXAMPLE: ROTATE BY 2π

$$-\mathbb{1} \longmapsto \mathbb{1}$$

In $SO(3)$ you are back to the start; in $SU(2)$, not yet.

$$SO(3) \cong SU(2)/\mathbb{Z}_2$$

WHAT THE QUOTIENT MEANS

In $U(N)$, the $U(1)$ charge is *no longer free*

Given a $U(N)$ element, there are **exactly N ways** of splitting it into an $SU(N)$ matrix times a $U(1)$ phase,

$$(g,\, e^{i\alpha}) \;\to\; (\omega^{-k} g,\, \omega^{k} e^{i\alpha}), \quad k = 0, \ldots, N-1$$

and **all N must give the same result** for the action of $U(N)$ on a field to be well defined. On a field with N-ality n and charge Q, they differ by $\omega^{k(Q-n)}$, so

$$Q \equiv n \pmod{N}$$

The $SU(N)$ rep fixes the $U(1)$ charge, mod N.

EXAMPLE · $-\mathbb{1}_2 \in U(2)$ ON A DOUBLET

Splitting	$Q = 0$	$Q = 1$
$(-\mathbb{1}_2,\, +1)$	$(-1)(+1)^0 = -1$	$(-1)(+1)^1 = -1$
$(\mathbb{1}_2,\, -1)$	$(+1)(-1)^0 = +1$	$(+1)(-1)^1 = -1$

$\mathbf{2}_0$ ✗ **not a rep of $U(2)$.** The two splittings disagree.

$\mathbf{2}_1$ ✓ **well defined.** The sign just migrates from one factor to the other.

Charge quantization and **forbidden representations**, from group theory alone.

ANOMALY CANCELLATION · THE CONDITIONS

How the conditions fix the matter

HOW THE ANOMALIES SPLIT

$$U(N)^3 \longrightarrow SU(N)^3,\ SU(N)^2\,U(1),\ U(1)^3$$
$$\mathrm{grav}^2\,U(N) \longrightarrow \mathrm{grav}^2\,U(1)$$

Before charge quantization: the familiar conditions of $SU(3) \times SU(2) \times U(1)^2$.

Combined with the **charge constraints**, they become:

$SU(3)^2\,U(1)_3$

$$N_{3L} = N_{3R} + 3\,k_3$$

Counts $SU(3)$ fundamentals, with $k_3 \in \mathbb{Z}$.

$SU(2)^2\,U(1)_2$

$$N_{2L} = 2\,k_2$$

An even number of doublets, $k_2 \in \mathbb{Z}$: the global Witten condition. [Witten '82]

$U(1)_i\,\mathrm{grav}^2$

$$2\ SU(N)\ \text{singlets}$$

Gravity alone allows one complete singlet; the cubic and mixed anomalies force two.

Minimal and irreducible: $k_3 = 0$, $k_2 = 2$ ⇒ **one SM generation plus a singlet.**

ANOMALY CANCELLATION · CHARGE QUANTIZATION?

Anomalies: consistency, plus extra assumptions

No new particles, no new groups. Just demand that gauge and gravitational anomalies cancel within one generation [Geng–Marshak '89, Babu–Mohapatra '89, Minahan et al. '90].

WITHOUT ν_R

Hypercharge fixed ✓

Up to normalization, but **only if we assume**:

+ the fermion content, put in by hand
+ Yukawa couplings (or discarding the trivial and vector-like solutions)

WITH ν_R

$Y \to Y + \epsilon\,(B - L)$

$B - L$ becomes anomaly free, so charge is dequantized. A Dirac mass leaves ϵ free. A Majorana mass needs $Y(\nu_R) = -\epsilon = 0$, an **extra assumption**.

Anomalies alone do not quantize charge: *extra assumptions are always needed.*

Z' PHENOMENOLOGY

A Z' that cannot hide

Its couplings are **not free**: anomaly cancellation and hypercharge fix them, so each member of the family predicts its own coupling pattern. Three statements hold for all of them:

Always couples to charged leptons

LHC dilepton resonance searches, $pp \to Z' \to \ell^+\ell^-$, are the universal probe.

Always couples to ν_R

There is always an invisible decay channel, $Z' \to \nu_R\nu_R$.

$B - L$ is special

H carries no $B - L$, so there is no tree-level Z–Z' mixing: the least constrained case.

Different members of the family leave distinguishable Z' signatures.

[Herrero-Brocal, Martín Lozano, Perez-Soler, Terrones, Vicente, work in progress]