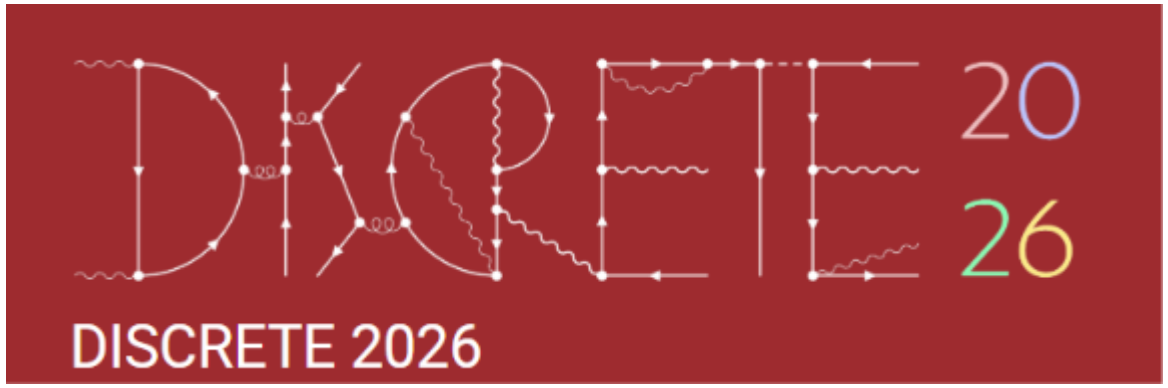
Exploring Majorana phases alongside effective Majorana masses



16.11. Bratislava

Miriam Chvostálová

Supervisor: Fedor Šimkovic

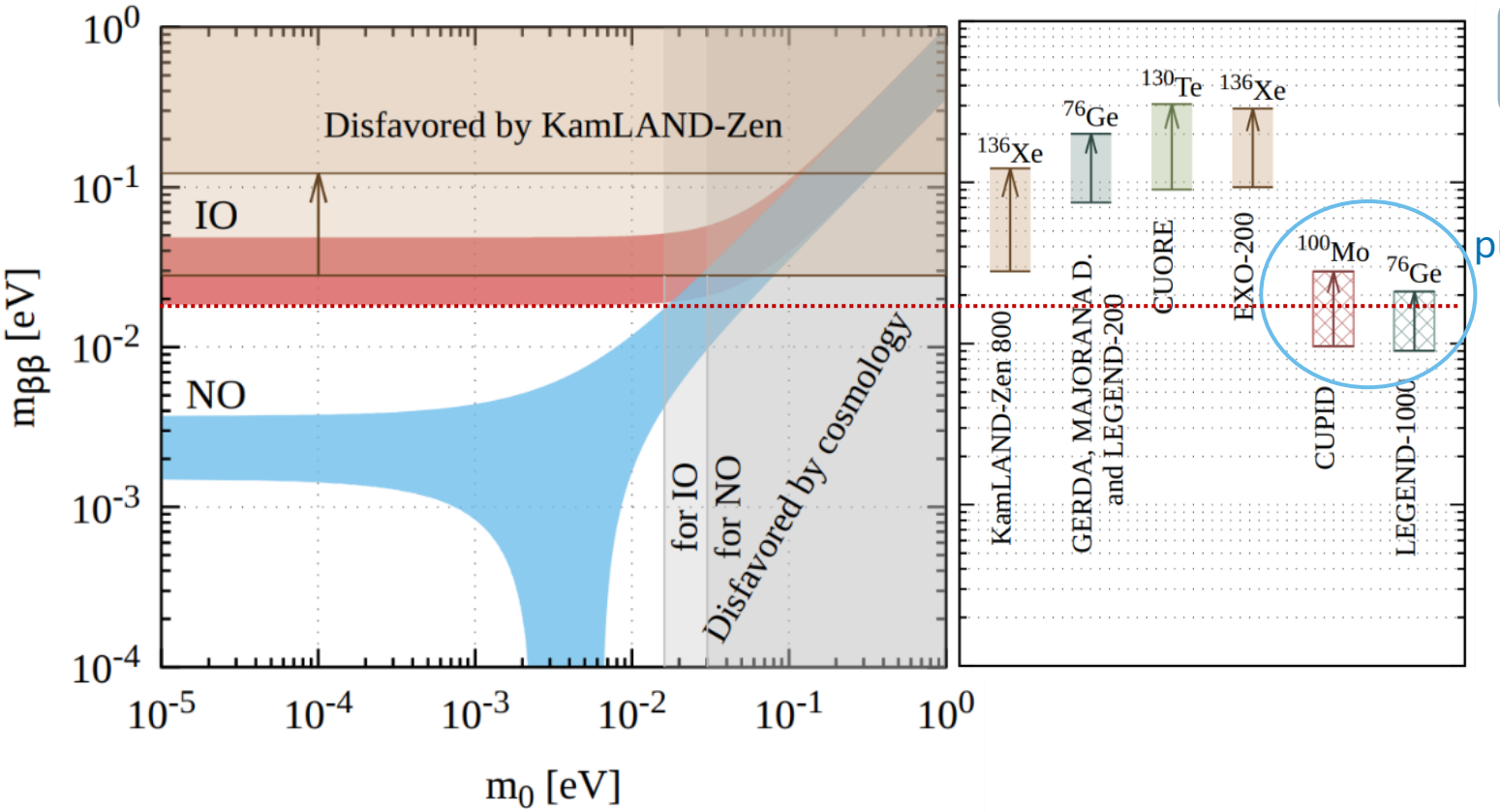
FACULTY OF MATHEMATICS, PHYSICS AND INFORMATICS
Comenius University
Bratislava

From $0\nu\beta\beta$ to effective Majorana mass $m_{\beta\beta}$

Observed $0\nu\beta\beta$ half-life limits: $\frac{1}{T^{0\nu}_{1/2}} = G^{0\nu}\, g_A^4\, \left|M^{0\nu}\right|^2 \frac{m^2_{\beta\beta}}{m_e^2}$

Effective Majorana mass

Nuclear matrix elment - significant uncertainities



planned experiments → Prospects for probing the entire IO region.

Using $\sum m_i < 0.12$ eV (95 %, CMB + BAO), we obtain $m_1 < 0.030$ eV for NO, and $m_3 < 0.016$ eV for IO.

$T^{0\nu}_{1/2} > 3.8\times10^{26}$ yr for ^{136}Xe corresponds to $m_{\beta\beta} < (28 - 122)$ meV. (KamLAND-Zen 800)

Effective Majorana masses

$$m_{\beta\beta} = |M^M_{ee}| = \left||U_{11}|^2 e^{2i\phi_1} m_1 + |U_{12}|^2 e^{2i\phi_2} m_2 + |U_{13}|^2 m_3\right|$$

$$\mathcal{L}^{\mathcal{M}} = -\frac{1}{2}\,\overline{\nu_L}\,\boldsymbol{M^M}(\nu_L)^c + h.c.$$

Majorana neutrino mass matrix in the flavor basis

The effective Majorana mass is the magnitude of the corresponding Majorana mass matrix element

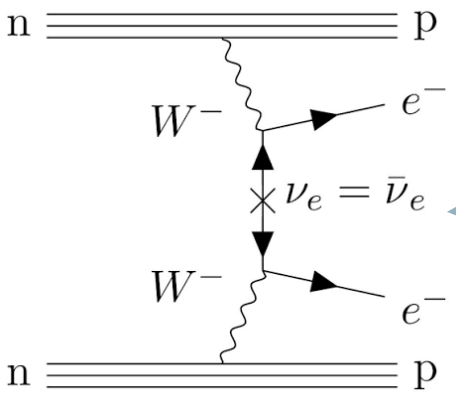
$$\boldsymbol{M^M} = \begin{pmatrix} M^M_{ee} & M^M_{e\mu} & M^M_{e\tau} \\ M^M_{\mu e} & M^M_{\mu\mu} & M^M_{\mu\tau} \\ M^M_{\tau e} & M^M_{\tau\mu} & M^M_{\tau\tau} \end{pmatrix} = \boldsymbol{U}\, diag(m_1, m_2, m_3)\boldsymbol{U^T}$$

symmetric: $(\boldsymbol{M^M})^{\mathrm{T}} = \boldsymbol{M^M}$

Parameters of mixing matrix **U** are probed by oscillation experiments

Majorana phases $\boldsymbol{\phi_1}, \boldsymbol{\phi_2}$ are inaccessible in standard neutrino oscillation experiments.

$0\nu\beta\beta$:



exchange of virtual Majorana neutrino

$\Delta L = 2$ processes to probe effective Majorana masses

$m_{\beta\beta} = \boldsymbol{m_{ee}}$

$(A, Z) \to (A, Z+2) + 2e^-$

$K^+ \to \pi^- e^+ e^+, \quad \mathcal{B} < 5.3\times10^{-11}$ (90% C. L.)

$\boldsymbol{m_{e\tau}}$

$D_s^+ \to \pi^- e^+ \tau^+$

$\tau^- \to e^+ \pi^- \pi^-, \quad \mathcal{B} < 2.0\times10^{-8}$ (90% C. L.)

$\boldsymbol{m_{\mu\tau}}$

$\tau^- \to \mu^+ \pi^- \pi^-, \quad \mathcal{B} < 3.9\times10^{-8}$ (90% C. L.)

$\boldsymbol{m_{e\mu}}$

$\mu^- + (A, Z) \to e^+ + (A, Z-2)$

$K^+ \to \pi^- \mu^+ e^+, \quad \mathcal{B} < 4.2\times10^{-11}$ (90% C. L.)

$\boldsymbol{m_{\mu\mu}}$

$K^+ \to \pi^- \mu^+ \mu^+, \quad \mathcal{B} < 4.2\times10^{-11}$ (90% C. L.)

$D^+ \to K^- \mu^+ \mu^+, \quad \mathcal{B} < 1\times10^{-5}$ (90% C. L.)

$\boldsymbol{m_{\tau\tau}}$

$B^+ \to K^- \tau^+ \tau^+$

Branching ratios: F. Takahashi et al. (Particle Data Group), *Int. J. Mod. Phys.* A 41 , 2630011 (2026)

Effective Majorana masses

$$\boldsymbol{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-\mathrm{i}\boldsymbol{\delta}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\mathrm{i}\boldsymbol{\delta}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{\mathrm{i}\boldsymbol{\delta}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{\mathrm{i}\boldsymbol{\delta}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\mathrm{i}\boldsymbol{\delta}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{\mathrm{i}\boldsymbol{\phi_1}} & 0 & 0 \\ 0 & e^{\mathrm{i}\boldsymbol{\phi_2}} & 0 \\ 0 & 0 & e^{\mathrm{i}\boldsymbol{\delta}} \end{pmatrix}$$

For **U** parameters, we used best-fits from global analysis [1].

Without loss of generality:
$\boldsymbol{\phi_1}, \boldsymbol{\phi_2} \in \langle -\pi/2, \pi/2 \rangle$

$$\boldsymbol{m_{ee}} = \left| m_1 e^{2i\boldsymbol{\phi_1}} c_{12}^2 c_{13}^2 + m_2 e^{2i\boldsymbol{\phi_2}} s_{12}^2 c_{13}^2 + m_3 s_{13}^2 \right|$$

$$\boldsymbol{m_{e\mu}} = \left| m_1 e^{2i\boldsymbol{\phi_1}} c_{12}c_{13}\left(-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\boldsymbol{\delta}}\right) + m_2 e^{2i\boldsymbol{\phi_2}} s_{12}c_{13}\left(c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\boldsymbol{\delta}}\right) + m_3 s_{13}s_{23}\, c_{13} e^{i\boldsymbol{\delta}} \right|$$

$$\boldsymbol{m_{e\tau}} = \left| m_1 e^{2i\boldsymbol{\phi_1}} c_{12}c_{13}\left(s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\boldsymbol{\delta}}\right) + m_2 e^{2i\boldsymbol{\phi_2}} s_{12}c_{13}\left(-c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\boldsymbol{\delta}}\right) + m_3 s_{13}c_{23}c_{13}\, e^{i\boldsymbol{\delta}} \right|$$

$$\boldsymbol{m_{\mu\mu}} = \left| m_1 e^{2i\boldsymbol{\phi_1}} \left(s_{12}c_{23} + c_{12}s_{23}s_{13}e^{i\boldsymbol{\delta}}\right)^2 + m_2 e^{2i\boldsymbol{\phi_2}} \left(c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\boldsymbol{\delta}}\right)^2 + m_3 s_{23}^2 c_{13}^2\, e^{2i\boldsymbol{\delta}} \right|$$

$$\boldsymbol{m_{\mu\tau}} = \left| m_1 e^{2i\boldsymbol{\phi_1}} \left(-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\boldsymbol{\delta}}\right)\left(s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\boldsymbol{\delta}}\right) + m_2 e^{2i\boldsymbol{\phi_2}} \left(c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\boldsymbol{\delta}}\right)\left(-c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\boldsymbol{\delta}}\right) + m_3 s_{23}c_{23}c_{13}^2\, e^{2i\boldsymbol{\delta}} \right|$$

$$\boldsymbol{m_{\tau\tau}} = \left| m_1 e^{2i\boldsymbol{\phi_1}} \left(s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\boldsymbol{\delta}}\right)^2 + m_2 e^{2i\boldsymbol{\phi_2}} \left(-c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\boldsymbol{\delta}}\right)^2 + m_3 c_{23}^2\, c_{13}^2 e^{2i\boldsymbol{\delta}} \right|$$

[1] I. Esteban, M.C. Gonzalez-Garcia et al., Nufit-6.0: updated global analysis of three-flavor neutrino oscillations, *Journal of High Energy Physics* (2024)

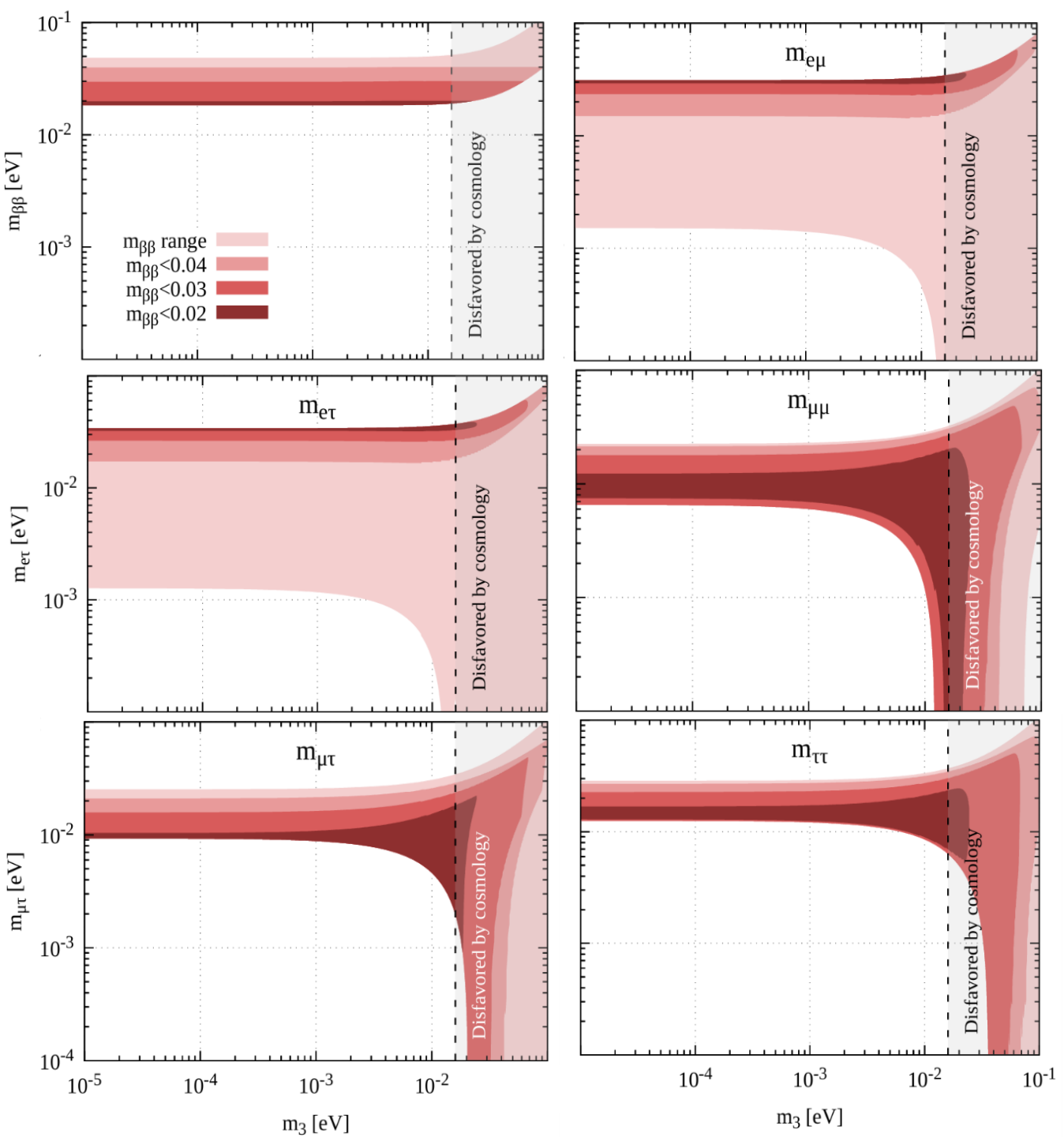
Effective Majorana masses for IO

Restricting the parameter space (m_3, ϕ_1, ϕ_1)

full range
$m_{\beta\beta}$< 40 meV
$m_{\beta\beta}$< 30 meV
$m_{\beta\beta}$< 20 meV

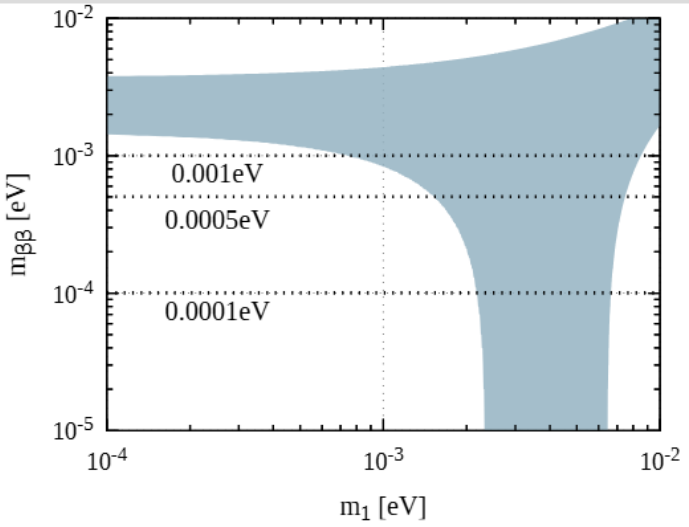
Range of effective masses for IO in meV

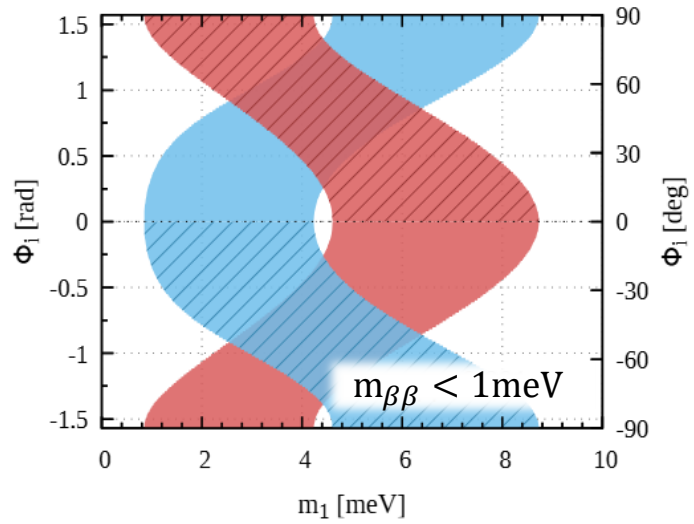
$m_{\beta\beta}$	max	51.26	40.00	30.00	20.00
	min	18.31	18.31	18.31	18.31
$m_{e\mu}$	max	34.37	34.37	34.37	34.34
	min	0	14.31	22.85	28.83
$m_{e\tau}$	max	37.19	37.19	37.19	37.18
	min	0	16.63	25.78	31.97
$m_{\mu\mu}$	max	32.28	30.59	26.87	20.15
	min	0	0	0	0.02
$m_{\mu\tau}$	max	34.19	28.78	23.53	18.15
	min	2.013	2.013	2.01	2.01
$m_{\tau\tau}$	max	36.77	34.48	30.17	23.48
	min	6.36	6.36	6.36	7.00

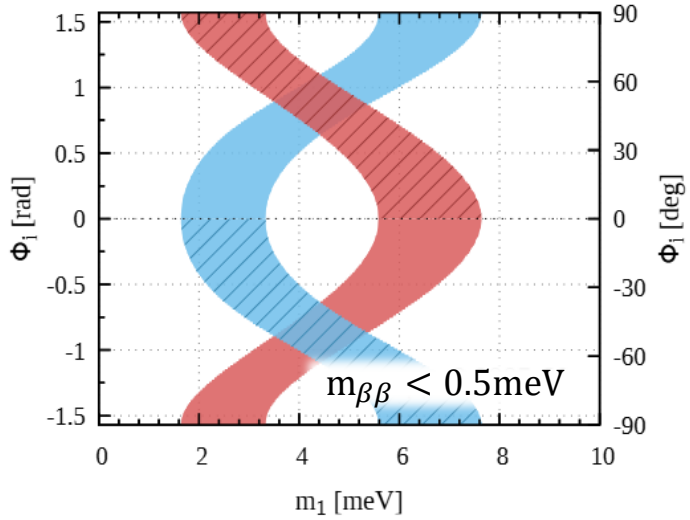


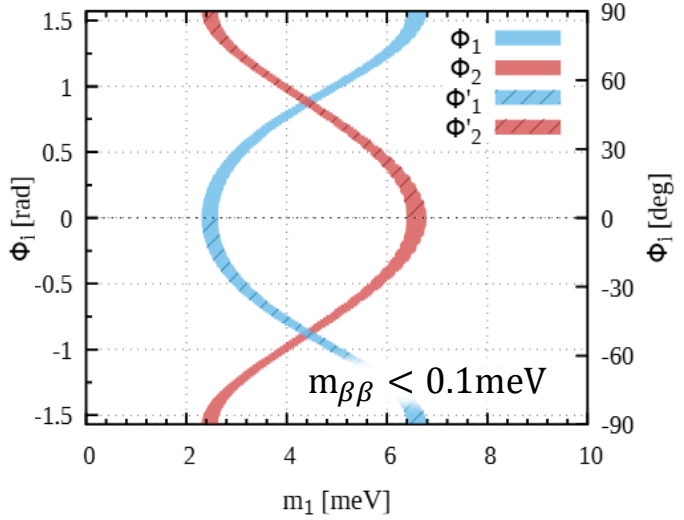
Majorana phases for supressed $m_{\beta\beta}$

- Significance of ϕ_1, ϕ_2 in determining the observability of $0\nu\beta\beta$-decay in the case of NO, where $m_1 \sim 10^{-3}$eV







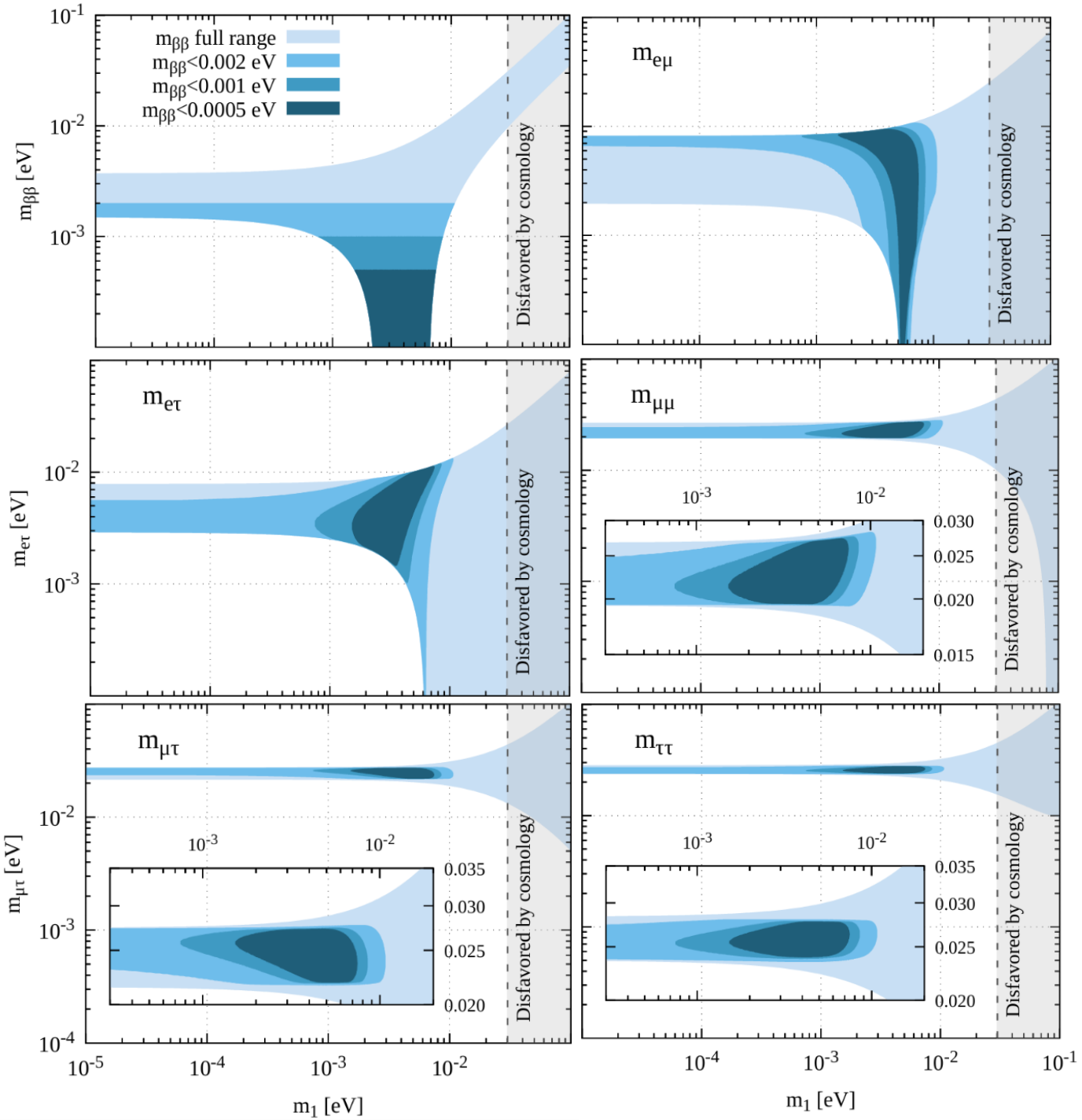


Effective Majorana masses for NO

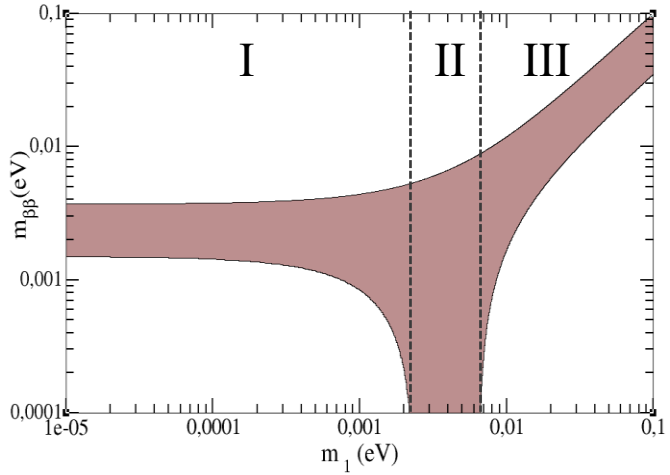
full range
$m_{\beta\beta}$< 2 meV
$m_{\beta\beta}$< 1 meV
$m_{\beta\beta}$< 0.5 meV

Range of effective masses for NO

$m_{\beta\beta}$ [meV]	max	30.90	2.00	1.00	0.50
	min	0	0	0	0
$m_{e\mu}$ [meV]	max	24.69	10.83	9.88	9.48
	min	0	0	0	0
$m_{e\tau}$ [meV]	max	25.81	13.34	11.99	11.38
	min	0	0	1.02	1.45
$m_{\mu\mu}$ [meV]	max	42.84	28.29	27.63	27.37
	min	10.43	19.26	19.42	19.48
$m_{\mu\tau}$ [meV]	max	43.03	27.75	27.43	27.31
	min	13.50	21.65	21.82	21.88
$m_{\tau\tau}$ [meV]	max	44.32	28.03	27.83	27.77
	min	15.67	23.46	23.77	23.92

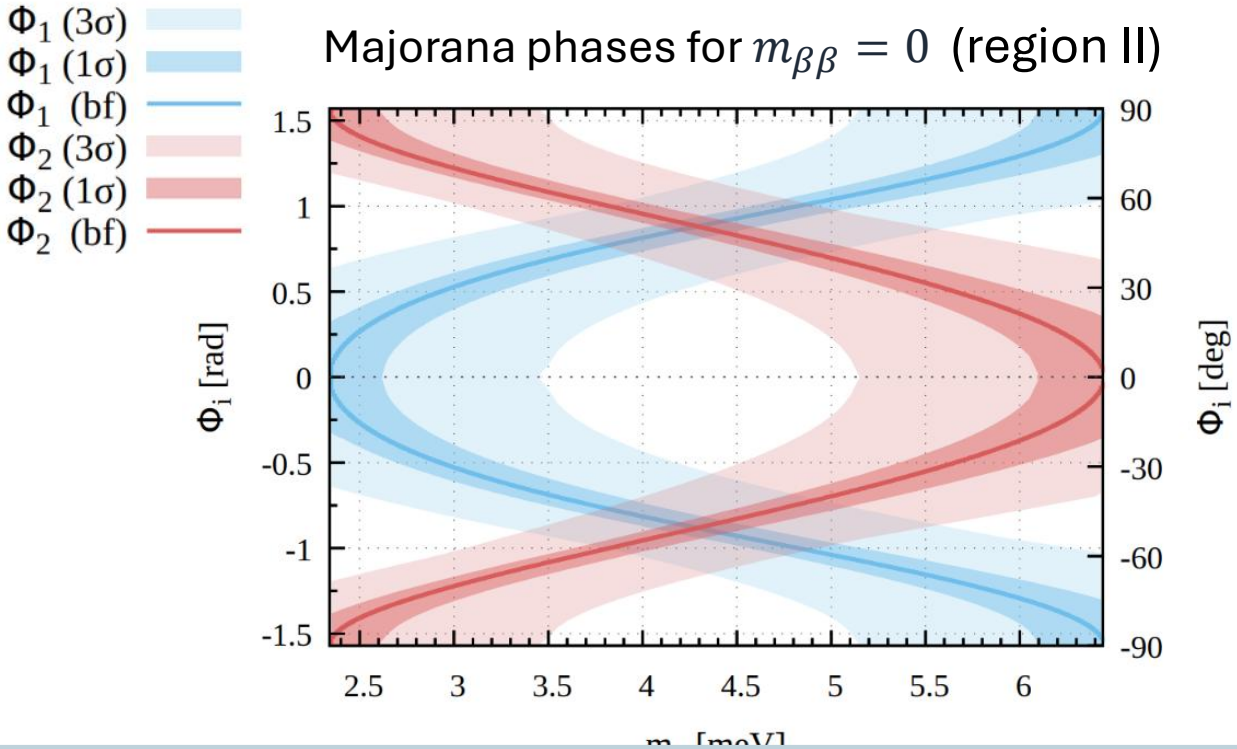


Majorana phases for $m_{\beta\beta} = 0$



$$m_{\beta\beta} = \left|\rho_1 e^{2i\phi_1} + \rho_2 e^{2i\phi_2} + \rho_3\right|$$

$$\rho_i \equiv |U_{1i}|^2 m_i$$



$$\min_{\phi_1,\phi_2} m_{\beta\beta} = \begin{cases} \text{region I}: & |\rho_2 - \rho_3| - \rho_1 \\ \text{region II}: & 0 \\ \text{region III}: & \rho_1 - (\rho_2 + \rho_3) \end{cases} \quad \begin{array}{ll} \text{if } \rho_1 < |\rho_2 - \rho_3| & \phi_1 = 0, \phi_2 = \pm\pi/2 \\ \text{if } |\rho_2 - \rho_3| \le \rho_1 \le \rho_2 + \rho_3 & \text{continuous change of } \phi_1, \phi_2 \\ \text{if } \rho_2 + \rho_3 < \rho_1 & \phi_1 = \pm\pi/2, \phi_2 = 0 \end{array}$$

Neutrino to antineutrino oscillations

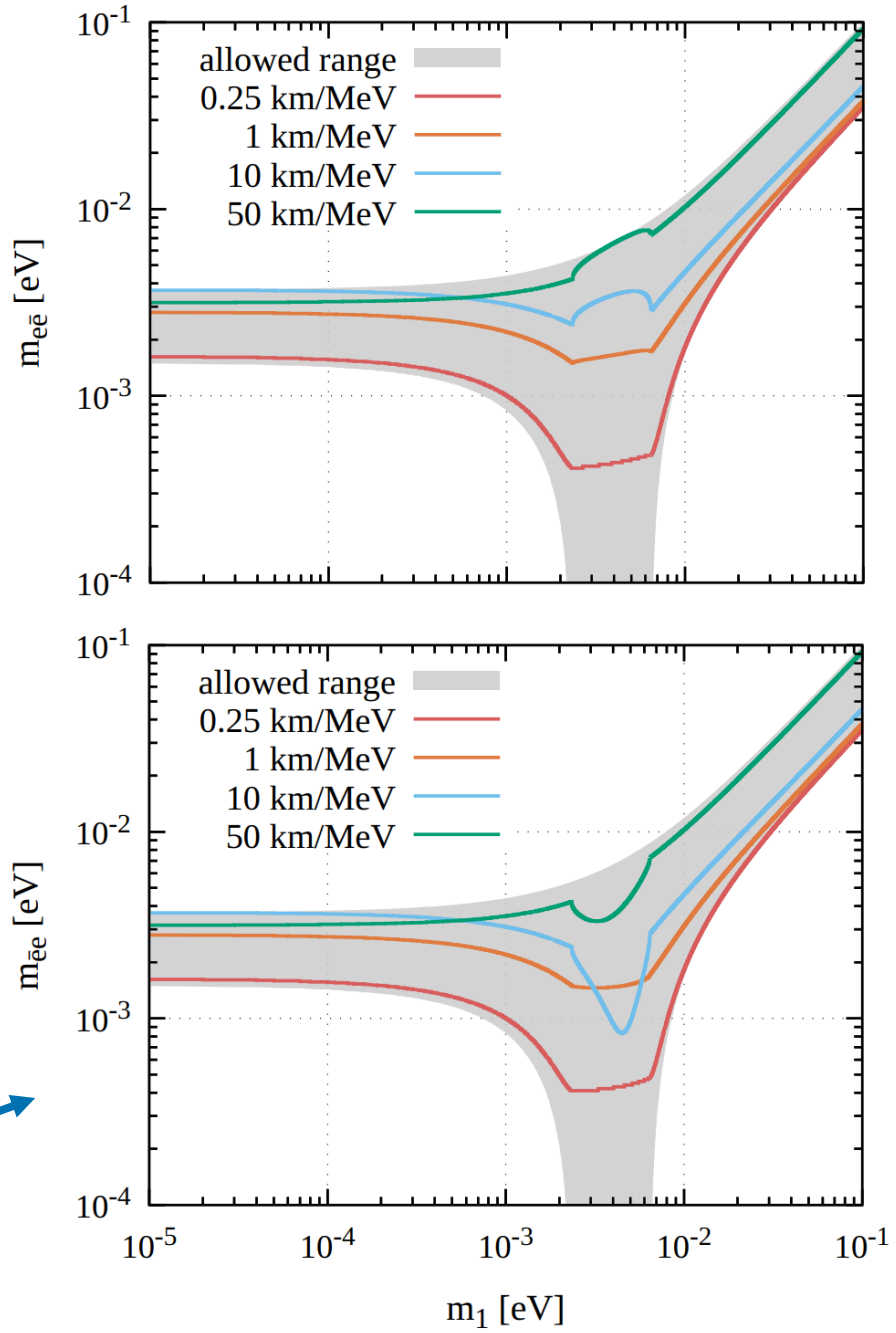
$$\Delta L = 2$$

$$P_{\nu_\alpha \to \overline{\nu}_\beta} = \frac{\left(m^{\mathrm{L}}_{\alpha\overline{\beta}}\right)^2}{E^2} \qquad P_{\overline{\nu}_\alpha \to \nu_\beta} = \frac{\left(m^{\mathrm{L}}_{\overline{\alpha}\beta}\right)^2}{E^2}$$

$$m^{\mathrm{L}}_{\alpha\overline{\beta}} = \left|\sum_i U^*_{\alpha i}\, U^*_{\beta i}\, m_i e^{-i\frac{m_i^2}{2E}L}\right| \qquad m^{\mathrm{L}}_{\overline{\alpha}\beta} = \left|\sum_i U_{\alpha i}\, U_{\beta i}\, m_i e^{-i\frac{m_i^2}{2E}L}\right| \quad [3]$$

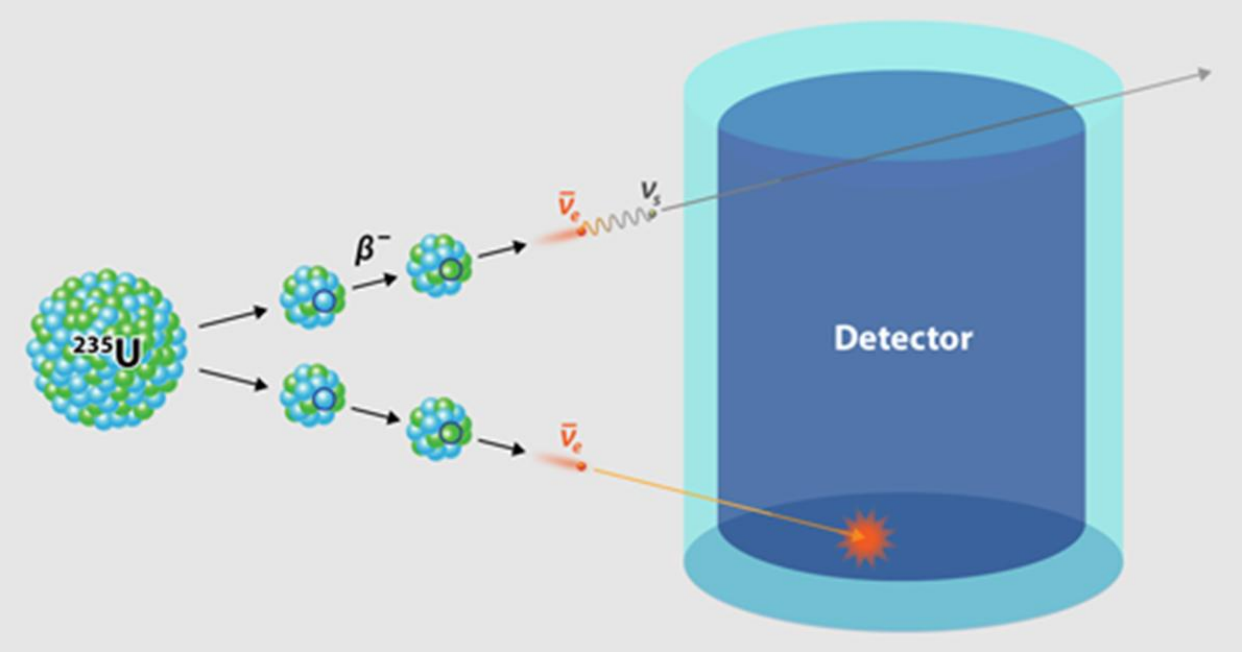
$$m^{L=0}_{\mathrm{e}\overline{\mathrm{e}}} = m^{L=0}_{\overline{\mathrm{e}}\mathrm{e}} = m_{\beta\beta}$$

- Allowed range for $m^{\mathrm{L}}_{\alpha\overline{\beta}}$, $m^{\mathrm{L}}_{\overline{\alpha}\beta}$ and $m_{\beta\beta}$ is identical
- We calculated $\boldsymbol{m^{\mathrm{L}}_{e\overline{e}}}$, $\boldsymbol{m^{\mathrm{L}}_{\overline{e}e}}$ for different $\mathbf{L/E}$ values, using Majorana phases that give $\boldsymbol{m^{\min}_{\beta\beta}}$



[3] S. Kovalenko and F. Šimkovic. Neutrino oscillations as a single feynman diagram. *Journal of Physics G: Nuclear and Particle Physics*, 51(3):035202, 2024.

Neutrino to antineutrino oscillations



$$R_L^{\bar{\nu}\nu} = 5.33 \times 10^{-13} \left(\frac{M_T}{1\text{kg}}\right) \left(\frac{1\text{g}\cdot\text{mol}^{-1}}{\mu}\right) \times \left(\frac{m_{\beta\beta}}{1\text{eV}}\right)^2 \left(\frac{\text{W}}{1\text{GWatt}}\right) \left(\frac{1\text{m}}{\text{L}}\right)^2 F(Z,E_e) B_{GT} \left(\frac{\text{events}}{\text{s}}\right)$$

$M_T = 1$ ton

$W = 1$ Gwatt

$L = 10$ m

$F(Z,E_e)\, B_{GT} = 5 \times 0.33$

$\mu \approx 100$ g mol^{-1}

$$R_L^{\bar{\nu}\nu} \approx 10^{-6} \left(\frac{m_{\beta\beta}}{1\text{eV}}\right)^2 \left(\frac{\text{events}}{\text{year}}\right)$$

Conclusion

Neutrinoless double beta decay currently provides the most sensitive experimental approach to testing the Majorana nature of neutrinos. The non-observation of $0\nu\beta\beta$-decay in experiments with finite sensitivity in the NO scenario cannot exclude the Majorana nature of neutrinos.

We investigated how the constraint on $m_{\beta\beta}$ restricts the allowed values of the other effective masses.

We considered the $\nu_e \rightarrow \overline{\nu}_e$ and $\overline{\nu}_e \rightarrow \nu_e$ oscillations in vacuum. Even in regions where $m_{\beta\beta}$ is strongly suppressed, the corresponding effective masses for $\nu_e \rightarrow \overline{\nu}_e$ and $\overline{\nu}_e \rightarrow \nu_e$ oscillations can remain significantly above zero for suitable $\mathrm{L/E}$.
Although these processes are strongly suppressed, they provide an alternative probe of Majorana neutrino properties.

Thank you for your attention!

