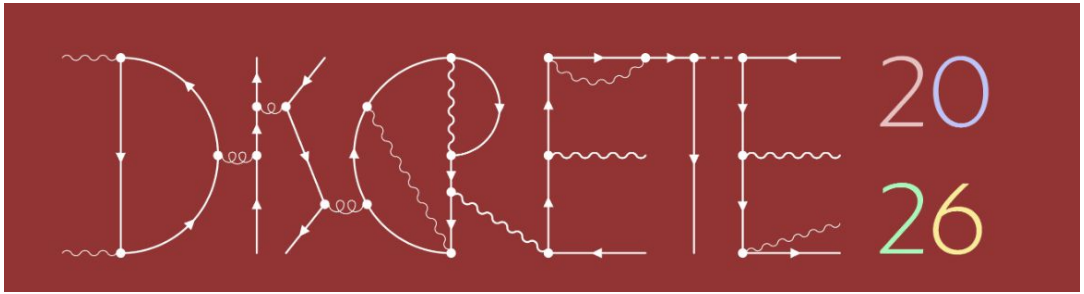
Parity-Violating Electron Scattering @ NNLO

- Physics Motivation
- Technical Aspects
- Results
- Outlook



FACULTY OF MATHEMATICS, PHYSICS AND INFORMATICS
Comenius University Bratislava

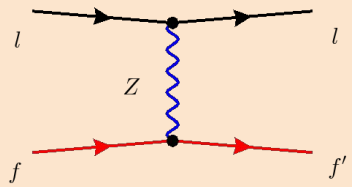
SLOVAK RESEARCH AND DEVELOPMENT AGENCY

Cluster of Excellence PRISMA++

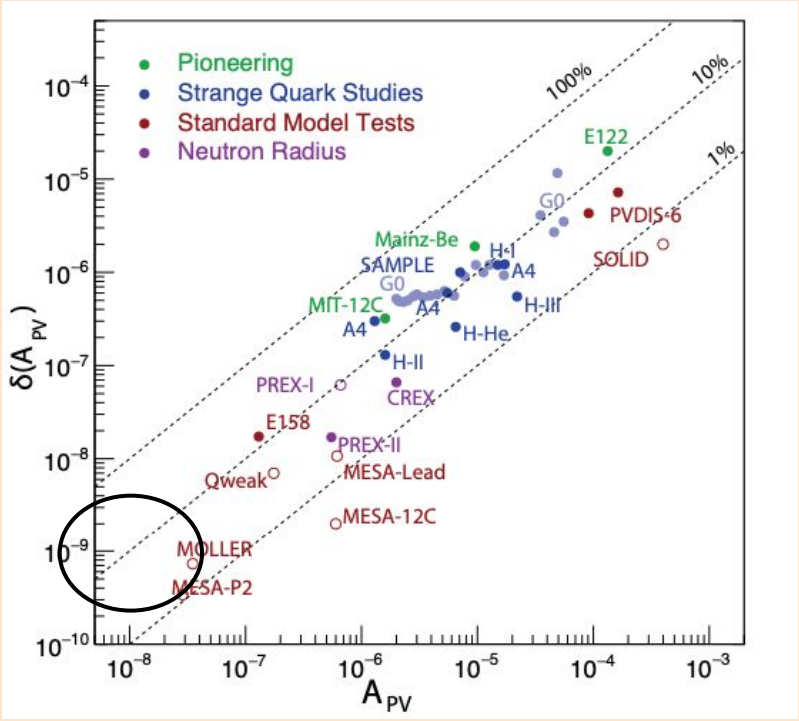
PITTPACC

In collaboration with J. Erler, A. Freitas, J. Kwak 2610.12287, 261x. xxxxx

A Brief History of Parity-Violating Electron Scattering *PVES*



$$A_{\mathrm{PV}} = \frac{\sigma_{\mathrm{L}} - \sigma_{\mathrm{R}}}{\sigma_{\mathrm{L}} + \sigma_{\mathrm{R}}}$$



1970s–1980s

Establish weak neutral currents

Validate the electroweak SM and measure $\sin^2\theta_W$

SLAC E122, Mainz-Be, MIT-12C

1990s–2010s

Hadron & nuclear structure

Probe strangeness, neutron skins, and nucleon structure

SAMPLE, HAPPEX, G0, A4, PREX/CREX

2000s

Clean leptonic test

Measure the electron weak charge and test the running of $\sin^2\theta_W$

SLAC E158

2010s–2020s

Precision weak charges

$Q_W{}^p$ and low-Q^2 observables become precision SM benchmarks

Qweak, APV, PVDIS

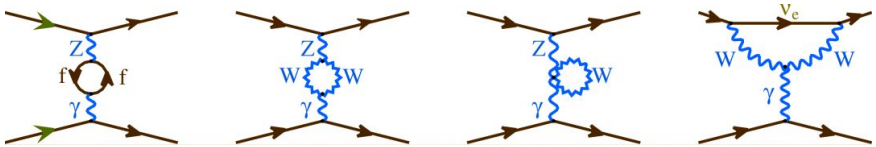
2020s+

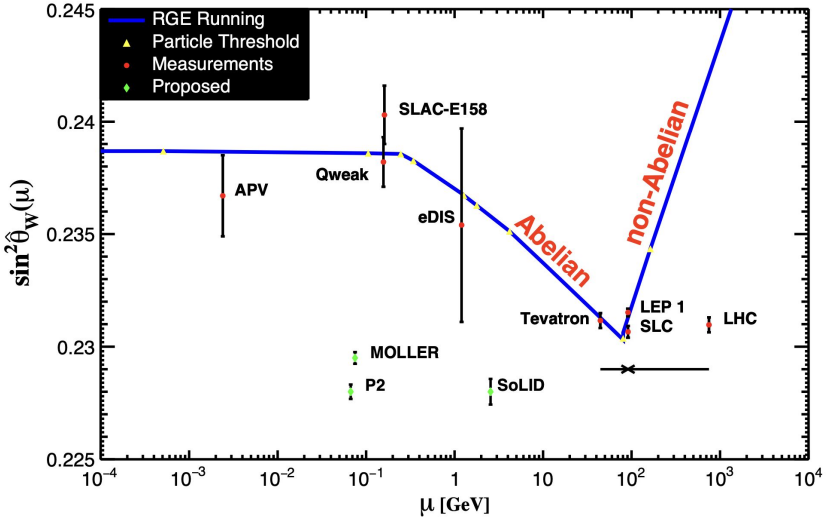
Precision frontier for BSM

MOLLER, P2, and SoLID probe new neutral-current physics complementary to the LHC

MOLLER, P2, SoLID , Chiral Belle II see Steven. R's talk

The Running of the Weak-mixing Angle vs. PVES



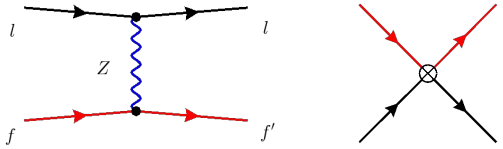


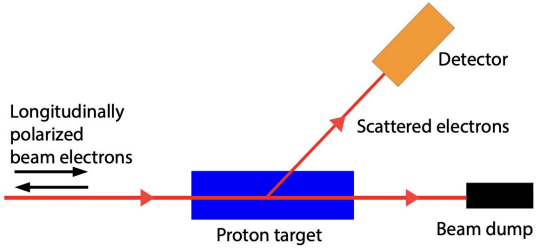
$$\hat{s}^2_W(\mu) \equiv \sin^2\hat{\theta}_W(\mu) = \frac{\hat{g}'^2(\mu)}{\hat{g}^2(\mu)+\hat{g}'^2(\mu)}$$

PVES is powerful in probing the structure of the SM crossing the energy scales from Thompson Limit to the Z-pole.

Ferro-Hernández & JE, arXiv:1712.09146

PVES as an *Amp*lifier





$$g_{AV}^{ef}[\bar{e}\gamma^\mu\gamma_5 e][\bar{f}\gamma_\mu f]$$
$$g_{VA}^{ef}[\bar{e}\gamma^\mu e][\bar{f}\gamma_\mu\gamma_5 f]$$

$$g_{AV}^{ef} = \frac{1}{2} - 2|Q_f|\sin^2\bar{\theta}(\mu)$$
$$g_{VA}^{ef} = \frac{1}{2} - 2\sin^2\bar{\theta}(\mu)$$

Averaged out from unpolarized target!

$$A_{\rm PV} = \frac{\sigma_{\rm L} - \sigma_{\rm R}}{\sigma_{\rm L} + \sigma_{\rm R}}$$

$$Q_{\rm W}^p \sim -2(2g_{AV}^{eu} + g_{AV}^{ed}) \approx 1 - 4\sin^2\bar{\theta}(\mu)$$
$$Q_{\rm W}^e \sim -2g_{AV}^{ee} \approx 1 - 4\sin^2\bar{\theta}(\mu)$$

} ~ ***0.07***

$$A_{\rm PV} \sim \frac{2\,\mathcal{M}_\gamma\mathcal{M}_Z}{|\mathcal{M}_\gamma|^2} \sim G_F Q^2\, Q_W$$

@ MOLLER $\quad Q_{\rm W}^e \sim 2.3\% \rightarrow \delta s_{\rm W}^2 \sim 0.1\%$

@ P2 $\quad Q_{\rm W}^p \sim 1.5\% \rightarrow \delta s_{\rm W}^2 \sim 0.14\%$

A per-cent measurement of the small weak charge is per-mille search for new NC amplitude!

Phenomenology Indication

1. SMEFT dim-6 operator

New heavy physics at scale Λ induces 4-fermion operators

$$\mathcal{L}_{\rm SMEFT} \supset \sum_i \frac{C_i}{\Lambda^2}\,\mathcal{O}_i^{(6)}$$

ee channel (MOLLER):
$\mathcal{O}_{ee} \sim (\bar{e}\gamma^\mu\gamma_5 e)(\bar{e}\gamma_\mu e)$

eq channel (P2):
$\mathcal{O}_{eq} \sim (\bar{e}\gamma^\mu\gamma_5 e)(\bar{q}\gamma_\mu q)$

2. Matching to low-energy PVES

After EWSB, operators shift weak neutral-current couplings

$$\mathcal{L}_{\rm PV}^{\rm eff} = \frac{G_{\rm F}}{\sqrt{2}}\, g_{AV}^{ff}\,(\bar{e}\gamma^\mu\gamma_5 e)(\bar{f}\gamma_\mu f) + \frac{C_i}{\Lambda^2}\,(\bar{e}\gamma^\mu\gamma_5 e)(\bar{f}\gamma_\mu f)$$

Shift in weak charge

$$\Delta Q_W^f \sim C_i\,\frac{v^2}{\Lambda^2}$$

3. Sensitivity at low Q^2

PV asymmetry is proportional to the weak charge

$$A_{\rm PV} \simeq \frac{G_{\rm F}\,Q^2}{4\pi\alpha\sqrt{2}}\,Q_W^f$$

$$Q_W^f = Q_W^{f,\rm SM} + \Delta Q_W^f$$

MOLLER & P2 aim at $\delta Q_W^f \sim 10^{-3}$

$$\Rightarrow \Lambda \sim \frac{v}{\sqrt{\delta Q_W^f}} \sim 7\text{–}10\ \text{TeV}\ (C\sim 1)$$

4. Conventional CI normalization

Often quoted in collider convention with strong coupling $g^2 = 4\pi$

$$\mathcal{L}_{\rm CI} = \frac{4\pi}{\Lambda^2}\,(\bar{e}\Gamma e)(\bar{f}\Gamma f)$$

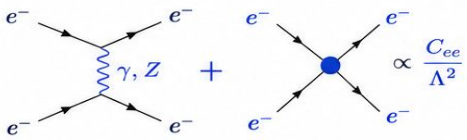
Thus,

$$\Lambda_{\rm CI} = \sqrt{4\pi}\,\Lambda$$

$\delta Q_W^f \sim 10^{-3} \Rightarrow \Lambda_{\rm CI} \sim 30\text{–}50$ TeV

MOLLER (ee)

Parity-violating Møller scattering isolates purely leptonic neutral current



Measures $Q_W^e = 1 - 4\sin^2\theta_W + \Delta Q_W^e$

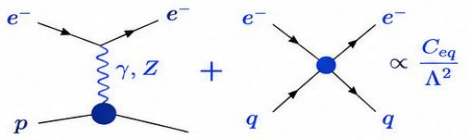
Goal: $\delta(Q_W^e) \simeq 1.0\times 10^{-3}$

$\Rightarrow \Lambda \sim 7\text{–}10$ TeV $(C\sim 1)$

or $\Lambda_{\rm CI} \sim 30\text{–}50$ TeV

P2 (ep)

Parity-violating e–p elastic scattering probes electron–quark neutral current



Measures $Q_W^p = 1 - 4\sin^2\theta_W + \Delta Q_W^p$

Goal: $\delta(Q_W^p) \simeq 1.3\times 10^{-3}$

$\Rightarrow \Lambda \sim 7\text{–}10$ TeV $(C\sim 1)$

or $\Lambda_{\rm CI} \sim 30\text{–}50$ TeV

Why this powerful

- PVES picks out weak–EM interference ⇒ extremely clean theoretically
- Small SM weak charges $(1 - 4\sin^2\theta_W \sim 0.05)$ ⇒ per-mille sensitivity to ΔQ_W
- Complementary to LEP/LHC: different chiral structures and kinematic regime $(Q^2 \ll M_Z^2)$

MOLLER targets $\mathcal{O}_{ee}$ (4e operators)
P2 targets $\mathcal{O}_{eq}$ (eeqq operators)
Together: a precision microscope for TeV-scale neutral currents

Take-away

$\Delta Q_W \sim Cv^2/\Lambda^2$ together with : $\delta Q_W \sim 10^{-3}$



probe purely leptonic and semileptonic contact interactions at scales comparable to, and chiral-complementary with, LEP/LHC constraints



Precise theoretical prediction of the SM becomes important.!

$\delta_{1-\text{loop}} A_{LR} \sim 40\%, \qquad \delta_{1-\text{loop}} \sin^2\theta_W \sim 3\%. \quad >> \quad \delta^{\text{exp}} s_w^2$

Framework Set-up

PV asymmetry

$$A_{\rm LR}(s,y) \equiv \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R} = \frac{G_\mu Q^2}{\sqrt{2}\,\pi\alpha(0)} \frac{1-y}{1+y^4+(1-y)^4} Q_W^e(s,y), \qquad y = \frac{Q^2}{s} = -\frac{t}{s}$$

Asymptotically, we organize the effective weak charge in such a way

$$Q_W^e(s,y) = 1 - 4\hat{s}_W^2(0) + \sum_{n\geq 1} \left(\frac{\alpha(0)}{\pi}\right)^n \sum_{k,\ell\geq 0} b_{n,k,\ell}(s,y) \left[1-4\hat{s}_W^2(0)\right]^k L_Q^\ell, \qquad L_Q \equiv \ln\frac{M_Z^2}{Q^2}, \quad Q^2 = ys$$

n **loop number,** k **order of Born suppression,** l **order of** L_Q

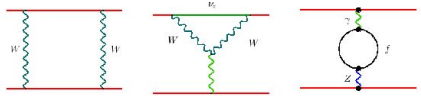
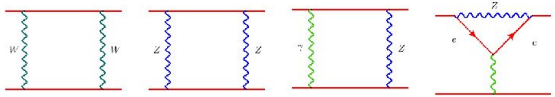
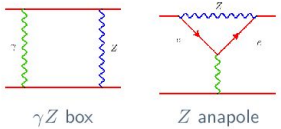
Systematic way of resumming the large-log ?

NLO EW correction to MOLLER

$$Q^e_{W,\mathrm{el}}(s,y) = Q^e_{W,\mathrm{Born}} + \frac{\alpha(0)}{\pi}\left[b_{1,0,0} + b_{1,0,1}L + Q^e_{W,\mathrm{Born}}\left(b_{1,1,0} + b_{1,1,1}L\right)\right] + \mathcal{O}\left(\alpha^2, \alpha(Q^e_{W,\mathrm{Born}})^2\right)$$

$$Q^e_{W,\mathrm{Born}} \equiv 1 - 4\hat{s}^2_W(0)$$

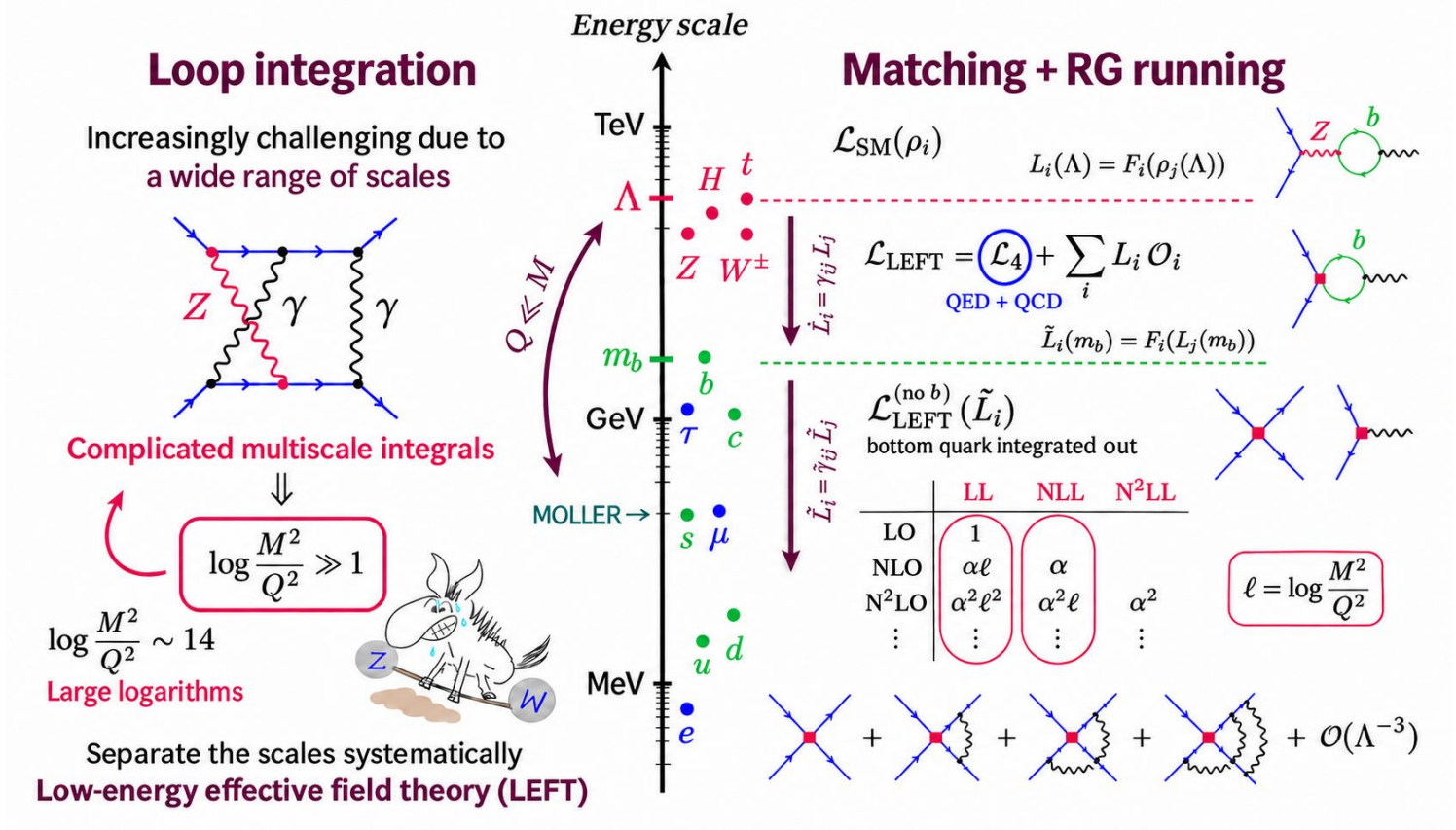
$$L = \ln(M_Z^2/s)$$

Coefficient	Diagrams	Contributions
Angle input $\hat{s}^2_W(0)$	 Fermionic γZ Bosonic γZ	Zero-transfer γZ mixing enters $\hat{s}^2_W(0)$ through RG evolution and threshold matching.
$b_{1,0,0}$	 WW box W anapole Finite transfer	$+1$ from the WW box. $+8/9$ from the W anapole and angle-matching remainder. Born-unsuppressed part of finite-transfer γZ and $\gamma\gamma$ vacuum polarization.
$b_{1,0,1}$	—	Zero. The explicit electroweak log starts at $k = 1$.
$b_{1,1,0}$	 WW box ZZ box γZ box Z anapole	$+1$ from the WW box and $-1/2$ from the ZZ box. Finite terms from the γZ boxes, Z anapole and virtual QED, including two-photon exchange. Neutral-current normalization to muon-decay G_μ. Born-suppressed part of finite-transfer vacuum polarization.
$b_{1,1,1}$	 γZ box Z anapole	$-3/2$ from the γZ boxes. $-1/3$ from the Z-loop anapole.

Each $b_{1,k,\ell}$ multiplies $\frac{\alpha(0)}{\pi}\left[1 - 4\hat{s}^2_W(0)\right]^k L^\ell$, where $L = \ln(M_Z^2/s)$. Diagrams are representative.

Marciano, Sirlin 80'; Czarnecki and Marciano, 96'; Denner and Pozzorini, 98; Petriello, 03; J.Erler, A.Kurylov, M.J.Ramsey-Musolf 03'; Kolomensky et al, 05'; Zykunov et al, 05'; Zykunov, 09'; Aleksejevs et al 10',11',12', *S. Kollatzsch, D.Moreno, D. Radic, A.Signer '25 (**LEFT + LL Res**)*

Framework Set-up (*LEFT*) see S.Kollatzsch, D. Radic, MITP workshop



Framework Set-up (*LEFT*)

PV asymmetry

$$A_{\mathrm{LR}}(s,y) \equiv \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R}$$

EFT Lagrangian

$$\mathcal{L}_{\mathrm{LEFT}} = \mathcal{L}_{\mathrm{QED}} + \mathcal{L}_{\mathrm{QCD}} + \sum_i C_i\, \mathcal{O}_i(\ell, q)$$

Observable

$$A_{LR} = -\frac{s\, y(1-y)}{(1-y+y^2)^2} \frac{C^{V,LL}_{ee,1111} - C^{V,RR}_{ee,1111}}{2\pi\bar{\alpha}} + \dots$$

Resummation

$$\begin{pmatrix} C_{\ell\ell} \\ C_{\ell q} \\ \vdots \end{pmatrix}_{\mu_s} = \underbrace{U^{(5q)}_{\mathrm{LL}}(\mu_s, M_Z)}_{\text{full coupled LEFT evolution}} \begin{pmatrix} C_{\ell\ell} \\ C_{\ell q} \\ \vdots \end{pmatrix}_{M_Z}$$

SM fixed-order input

Five-flavour hadronic prescription $\Longrightarrow$ $\boxed{\mu_s \geq m_b \text{ in this implementation}}$

Effective Weak charge vs. LEFT

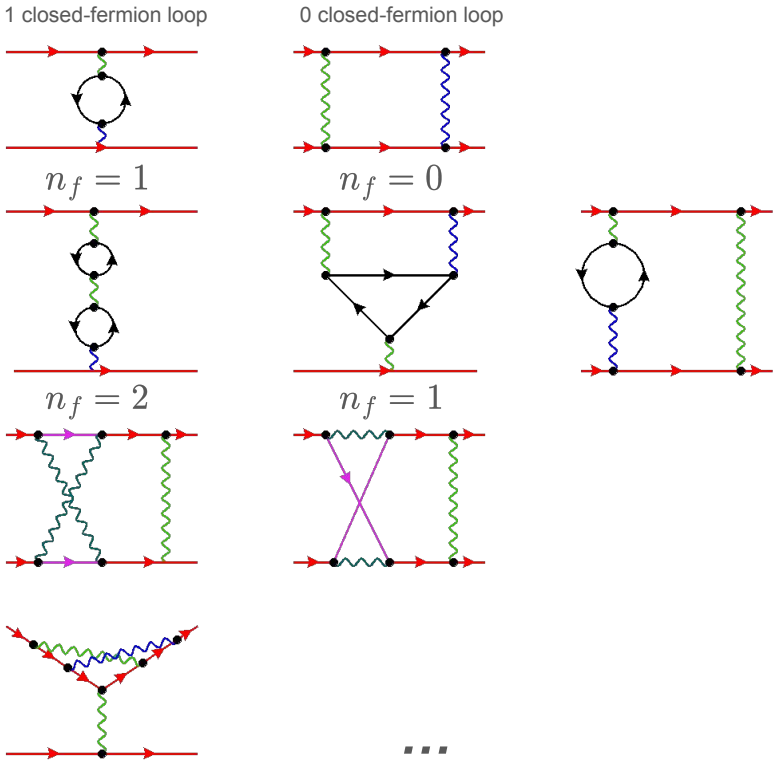
- $Q_W(s,y)$ separates the β-function of the $\gamma - Z$ mixing effects from the EW large log L_Q. Easy to incorporate hadronic effects when $Q^2 < m_b^2$.
- Resumming the manifestly gauge-invariant L_Q^n at each order is challenging.

- LEFT organize the L_Q, $\log(m_f^2/M_Z^2)$ systematically.
- How to reach $\mu_s < m_b$?
- Physical meaning of the $\sin\theta_W$?

EW Radiative Correction Categorization for MOLLER

1-loop

2-loop



$$\Delta Q_W^{e,(2)} = \Delta Q_{W,\mathrm{ferm}}^{e,(2)} + \Delta Q_{W,\mathrm{bos}}^{e,(2)} \qquad L = \ln(M_Z^2/s)$$

$$\Delta Q_{W,\mathrm{ferm}}^{e,(2)} = \left(\frac{\alpha(0)}{\pi}\right)^2 \sum_{n_f=1}^{2} \sum_{k=0}^{1} \sum_{\ell=0}^{2} b_{2,k,\ell}^{(n_f)} \left[1 - 4\hat{s}_W^2(0)\right]^k L^\ell$$

Du, Freitas, et al

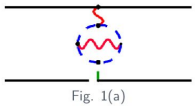
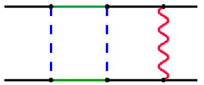
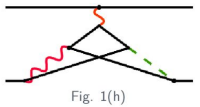
$$\Delta Q_{W,\mathrm{bos}}^{e,(2)} = \left(\frac{\alpha(0)}{\pi}\right)^2 \sum_{k=0}^{1} \sum_{\ell=0}^{2} b_{2,k,\ell}^{(0)} \left[1 - 4\hat{s}_W^2(0)\right]^k L^\ell$$

Chen, Freitas, Kwak, Erler 26

Czarnecki 26

Bosonic NNLO corrections to the weak charge

$$\Delta Q_{W,\mathrm{bos}}^{e,(2)}(s,y)\Big|_{\text{retained},\,k\le 1} = \left(\frac{\alpha(0)}{\pi}\right)^2 \left[b_{2,0,0}^{(0)}(y) + b_{2,0,1}^{(0)} L_Q + \left[1 - 4\hat{s}_W^2(0)\right] b_{2,1,2}^{(0)} L_Q^2\right], \qquad L_Q = \ln\frac{M_Z^2}{Q^2};$$

Coefficient	Representative diagram	Contribution
$b_{2,0,0}^{(0)}(y)$ Finite	 Fig. 1(a)	**Two-loop γ–Z mixing** Hard self-energy contribution
$b_{2,0,1}^{(0)}$ Single log L_Q	 Fig. 1(i)	**$WW\gamma$ double box** Photonic correction to the WW amplitude
$b_{2,1,2}^{(0)}$ Double log L_Q^2	 Fig. 1(h)	**$\gamma\gamma Z$ vertex topology** Carries the Born factor $1 - 4\hat{s}_W^2(0)$

Diagrams from Chen et al., arXiv:2610.12287, Fig. 1. Coefficients are obtained from the renormalized sum.

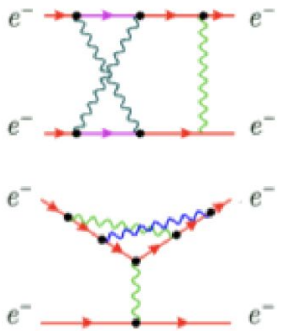
- **Expansion by regions.** Use $m_e^2 \ll Q^2 \ll M_{W,Z}^2$ to evaluate the integrals analytically.
- **IR cancellation.** Keep m_e (collinear) and m_γ (soft). The regulator logs cancel in the virtual asymmetry.
- **γ_5 prescription.** Use NDR with diagram-dependent projections and subtractions.
- **Cross-checks.** Compare independent calculations and verify UV and IR finiteness.

Workflow of the Calculation

01

Diagram generation

Two-loop bosonic Møller diagrams



FeynArts

02

Algebraic simplification

$$\gamma^\mu\gamma^\alpha\gamma_\mu = (2-d)\gamma^\alpha$$

$$\begin{aligned}\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] \\ = 4\big(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} \\ + g^{\mu\sigma}g^{\nu\rho}\big)\end{aligned}$$

FeynCalc

03

Expansion by regions

Scale hierarchy

$$m_e^2, s, |t|, |u| \ll M_V^2$$

Hard–hard

$$q_1, q_2 \sim M_V$$

Hard–soft

$$q_1 \sim M_V,\ q_2 \sim \sqrt{|t|}$$

$$V = W, Z$$

Beneke and Smirnov
In-house code

04

Integral reduction

Integration by parts

$$\int d^dk\, d^d\ell\, \frac{\partial}{\partial k^\mu}\frac{v^\mu}{\Pi_i D_i^{a_i}} = 0$$

$$D_i = \ell_i^2 - m_i^2$$

$$\sum_i c_i(a_j, d) I_i = 0$$

Reduce to a smaller integral basis

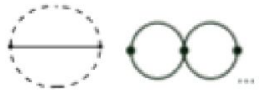
FIRE, etc.

05

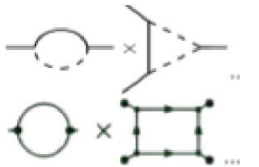
Master integrals

$$\mathcal{A} = \sum_i c_i(\epsilon) I_i$$

Two-loop vacuum bubbles



Products of one-loop topologies



Renormalization scheme

Mixed on-shell and $\overline{\text{MS}}$ prescriptions

On shell M_Z, M_H, m_f and e fixed by $\alpha(0)$.

$\overline{\text{MS}}$ The weak mixing angle $\hat{s}^2_W(\mu)$.

W mass

The bare electroweak mass terms give

$$\frac{M^2_{W,\text{B}}}{M^2_{Z,\text{B}}} = \frac{g^2_\text{B}}{g^2_\text{B} + g'^2_\text{B}} = 1 - s^2_{W,\text{B}}.$$

With on-shell M_Z and the $\overline{\text{MS}}$ angle,

$$M^2_W = M^2_Z\big[1 - \hat{s}^2_W(\mu)\big].$$

Expanding the bare relation to one loop gives

$$\delta M^2_W = \big(1 - \hat{s}^2_W\big)\delta M^2_{Z,\text{OS}} - M^2_Z\,\delta\hat{s}^2_W.$$

The low-energy prediction uses $\hat{s}^2_W(0)$ in the mass relation.

From the Z pole to the Thomson limit

Start with $\hat{s}^2_W(M_Z)$ in the full Standard Model.

$$\hat{s}^2_W(M_Z) \xrightarrow{\text{RG evolution and threshold matching}} \hat{s}^2_W(0)$$

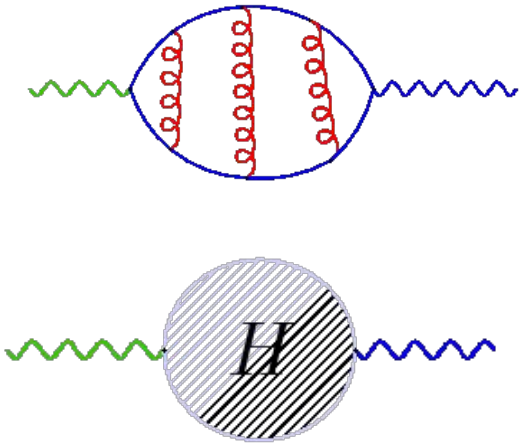
Use $\hat{s}^2_W(0)$ as the input for the fixed-order prediction.

$$\hat{s}^2_W(M_Z) \longrightarrow \hat{s}^2_W(0) + \big[\hat{s}^2_W(M_Z) - \hat{s}^2_W(0)\big].$$

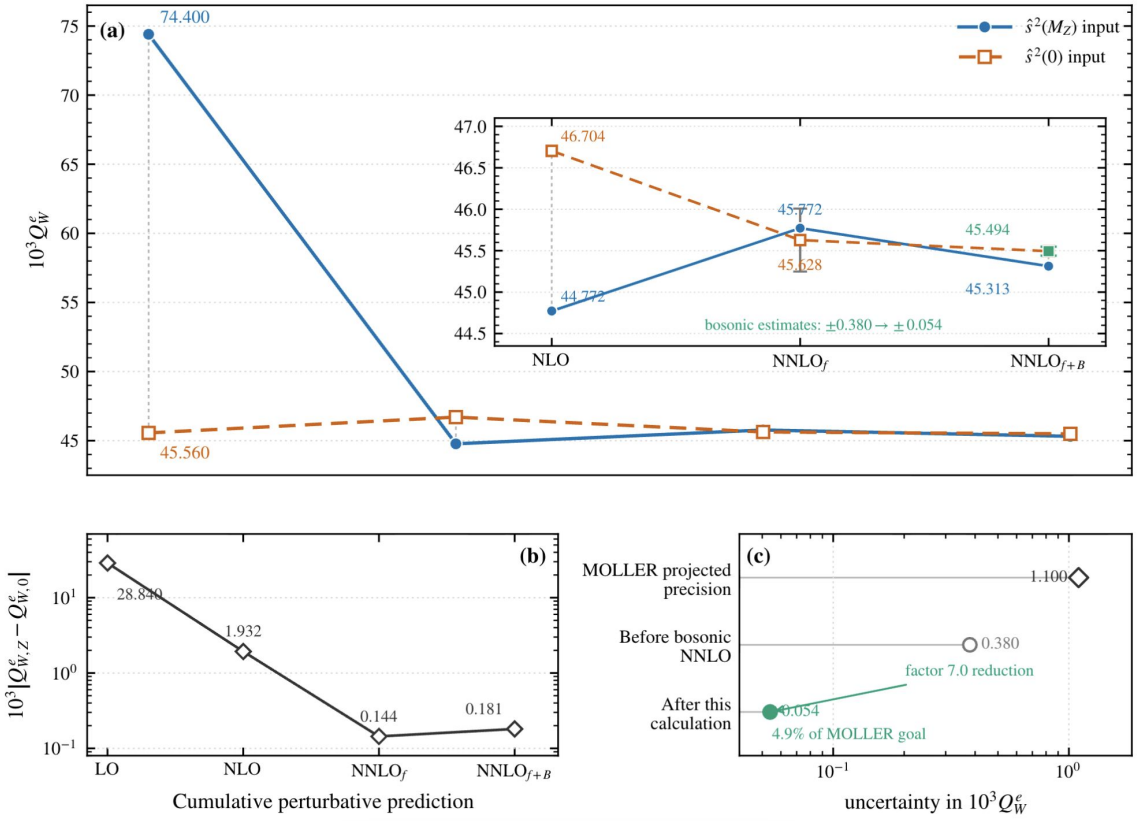
Compute the bracket order by order from the running and threshold matching. Substitute this expression into A_LR and truncate at the same order.

Why $\hat{s}_W(0)$ @ Lower Scale

- Resum $\ln(m_f^2/M_Z^2)$ systematically. Including higher order perturbative QCD (via RGE and matching) Erler, Ferro-Herrández
- Incorporate the non-perturbative light-quark effects and their uncertainties (via flavor separation) Erler, Ramsey-Musolf
- Finite-Q^2contribution is very small (~10^-6).



Preliminary Results



- *sw(0)*-scheme converges much faster.
- Two-loop bosonic correction minimize the uncertainty down to 4.9% to the precision goal (~0.1% to the measurement)

Comparison Between Two Results

$$\Delta Q_{W,\mathrm{bos}}^{e,(2)}(s,y)\Big|_{\text{retained},\,k\le 1} = \left(\frac{\alpha(0)}{\pi}\right)^2 \left[b_{2,0,0}^{(0)}(y) + b_{2,0,1}^{(0)} L_Q + \left[1 - 4\hat{s}_W^2(0)\right] b_{2,1,2}^{(0)} L_Q^2\right], \qquad L_Q = \ln\frac{M_Z^2}{Q^2}$$

Bosonic NNLO contributions to $10^3 \Delta Q_W^e$

$y = 0.4$ ▪ Common $\hat{s}_W^2(0)$ scheme and L_Q basis ▪ Through first order in $1 - 4\hat{s}_W^2(0)$

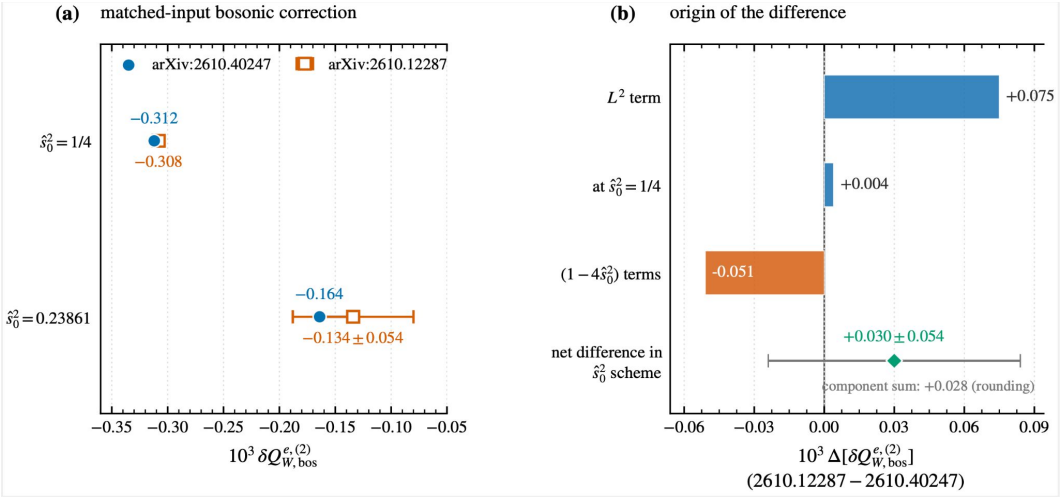
Term	Chen et al.	Czarnecki	Comparison
b_{200} · Finite	-0.038	-0.042	Small offset. M_H dependence matches a structure in the one-loop G_μ normalization.
b_{201} · Single log	-0.270	-0.270	Exact agreement
b_{212} · Suppressed double log	**$+0.172$**	**$+0.086$**	**Main discrepancy (factor of two)**
b_{211} · Suppressed single log	Omitted, est. 0.048^*	$+0.059$	
b_{210} · Suppressed finite	Omitted, est. 0.012^*	$+0.003$	

* Unsigned estimates from grouping the amplitude-class uncertainties in Chen et al., Table 1.

- **Suppressed double-log are off by factor of 2, (missing diagrams? RG exponential expansion of the two-loop coefficients?)**

$$e^{-aL} = 1 - aL + \frac{1}{2}a^2L^2 + \cdots$$

Outlook

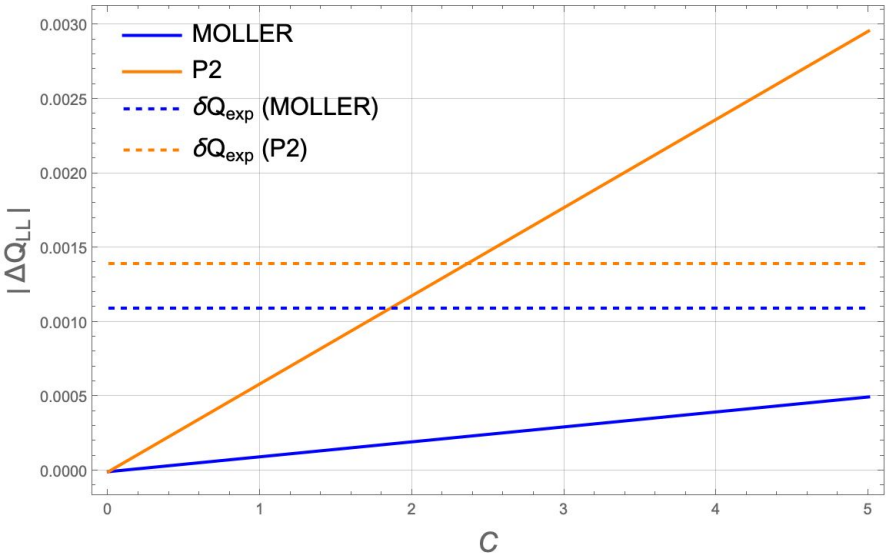


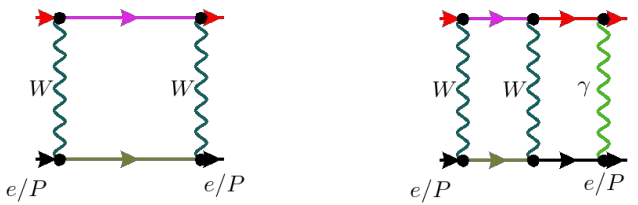
- We have two independent NNLO calculation to the Møller scattering at low-energy, which suffice the theoretical input for MOLLER experiment.
- A small discrepancy (*factor of 2*) is under investigation. Overall agreement has been achieved.
- A thorough error estimation, with data-driven(*e+e-/Lattice*) method to the light-quark contributions is expected in our new paper.
- Working towards NNLO *e-p PVES.*

Back-ups

the Large Logs (LL) @ PVES @ *NNLO*

A naive estimate





$$\Delta Q^{e,WW\,(1)} = \frac{\alpha}{4\pi \hat{s}^2}$$

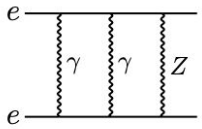
$$\Delta Q_W^{p,WW} = \frac{\alpha}{4\pi s^2}\left[2+5\left(1-\frac{\alpha_s(M_W^2)}{\pi}\right)\right]$$

$$\Delta Q_W^{WW\gamma} \sim C\frac{\alpha}{\pi}\log\frac{M_W^2}{Q^2}\Delta Q_W^{e,WW}$$

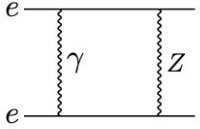
The photonic-EW corrections is more severe in e-p scattering

Uncertainties from Missing EW Calculations

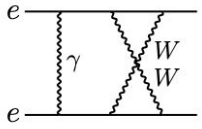
2610.12287's error estimation scheme ($L = \ln\left(Q^2/m_Z^2\right)$):



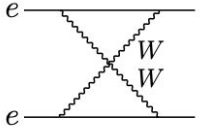
$$\sim -\left(\frac{3}{2}+\frac{1}{3}\right)\frac{\alpha}{\pi}L$$



$$\approx 0.151\times 10^{-3} \qquad [0.174\times 10^{-3}?]$$

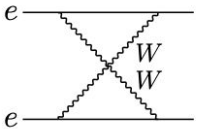


$$\sim -\left(\frac{3}{2}+\frac{1}{3}\right)\frac{\alpha}{\pi}L$$



$$\approx -0.328\times 10^{-3} \qquad [-0.270\times 10^{-3}]$$

two-loop remainder $\sim -6.9\frac{\alpha}{\pi}$



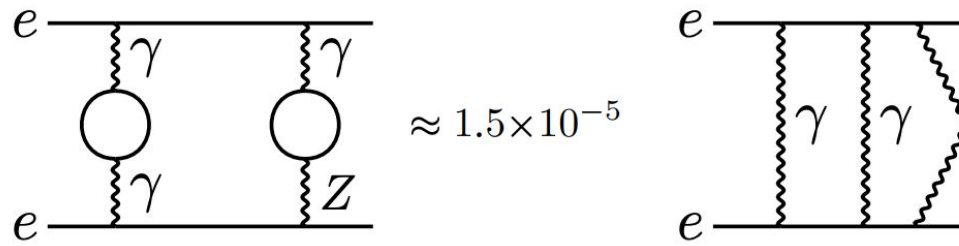
$$\approx -0.092\times 10^{-3} \qquad [-0.038\times 10^{-3}]$$

This is a demonstration using two-loop: the numbers following ≈ is the estimate; [the numbers in brackets] are actual calculations. In other words, these are not parts of the remaining uncertainty.

Kwak.26

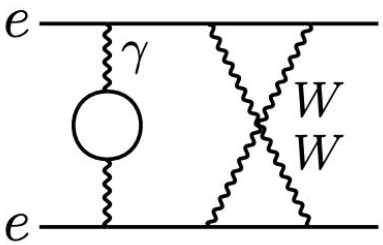
Uncertainty from Missing EW Calculations

Estimates for the largest remaining contributions (**PRELIMINARY**):

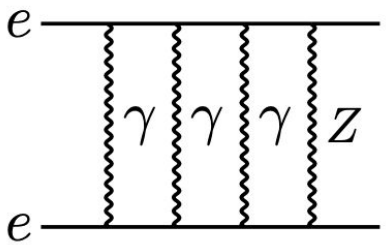


$\approx 1.5\times10^{-5}$

$\approx 2\times10^{-5}$



$\approx 1.5\times10^{-4}$



$\approx 10^{-5}$

Kwak,26

Adding all uncertainties in quadrature gives $\sim 1.9 \times 10^{-4}$.
This is probably conservative; refinement coming soon.



Master Prescription: $\mathcal{M}_{\rm ren} = \sum_i \int [d^D\ell_1 d^D\ell_2]\, C^{\rm sub}_{i,\rm ren}(\ell_1,\ell_2,\{p\};D)\,\left[T_i^{\rm out}(\ell_2,\{p\};\gamma_5)\right]_{D\to 4}$

$$C^{\rm sub}_{i,\rm ren} = C^{\rm sub}_{i,\rm reg} + C^{\rm CT}_{i,\rm reg}$$

Subtraction Prescription:

$\mathcal{M}_{\rm ren} =$ 