Perturbativity vs. Vacuum Stability: Impact of RG-running on a Parity Symmetric Radiative Neutrino Mass Model

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Introduction

Neutrinos:

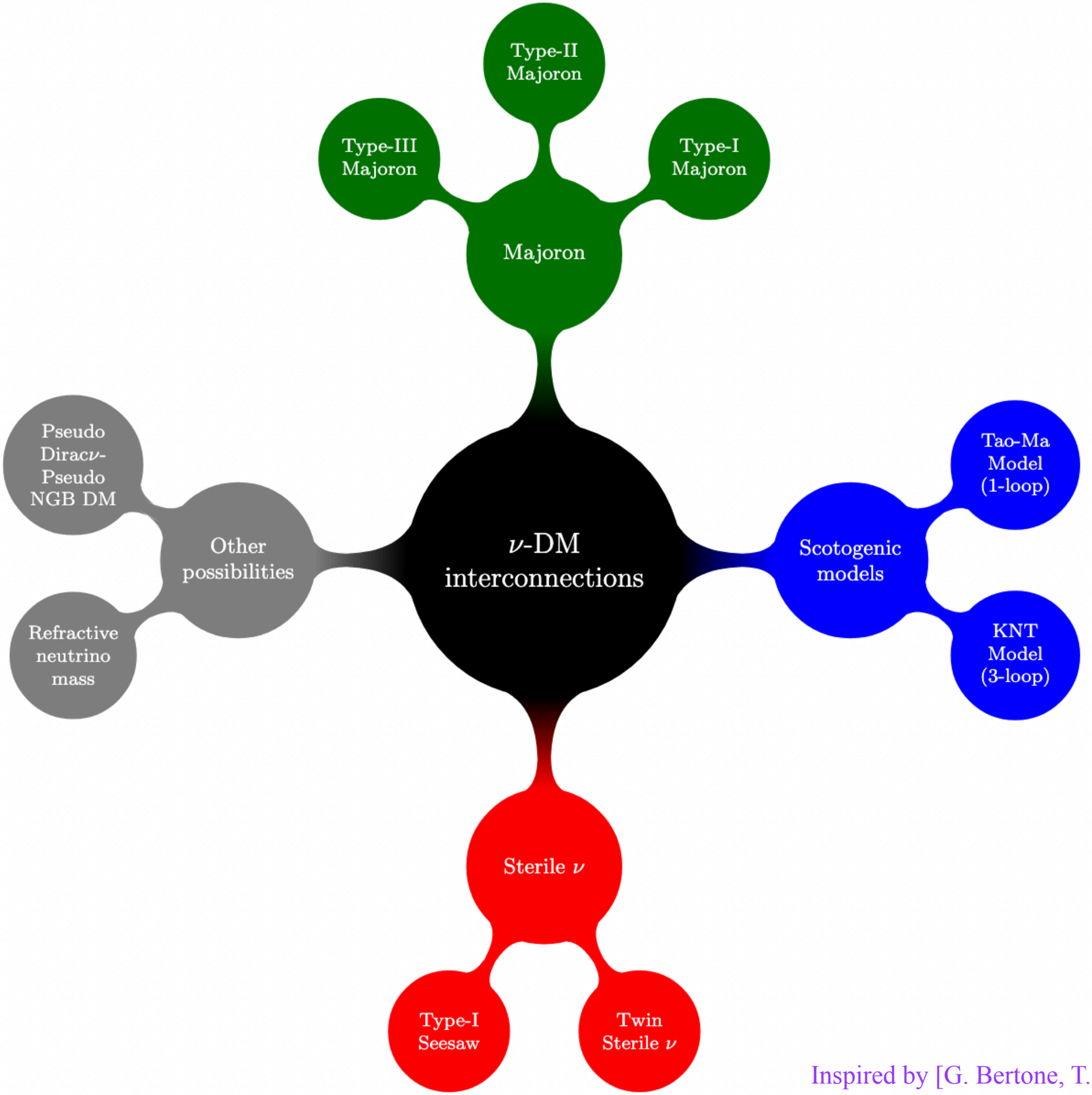
- Massive neutrinos? Theory (SM) $m_\nu = 0$
- Massive neutrinos? Neutrino oscillation data $m_\nu \neq 0$, $m_\nu < 0.45\,\text{eV}$

[See talk by M. C. Gonzalez-Garcia] [KATRIN, 2025]

- New physics: Tree-level seesaw $m_\nu \simeq \dfrac{v_{EW}^2}{\Lambda} \Rightarrow \Lambda \simeq 10^{10...15}\,\text{GeV}$
- New physics: Radiative mechanism $m_\nu \simeq \dfrac{v_{EW}^2}{(16\pi^2)^l \Lambda} \Rightarrow \Lambda \simeq 10^{2...5}\,\text{GeV}$

Dark Matter:

- Constitutes 27% of the energy budget of the Universe
- Neutrinos? Hot DM ✘
- Hexaquarks ($uuddss$)? Highly constrained? [G. R. Farrar, 2017], [M. Moore, T.R. Slatyer, 2024]
- BSM state? Direct detection, indirect detection, colliders
- Connection with neutrinos?



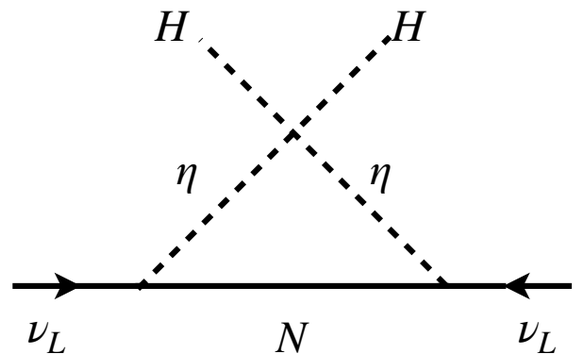
Inspired by [G. Bertone, T. Tait, 2018]

Scotogenic Models

- Dark matter assisted neutrino mass generation at loop-level
- Dark parity symmetry $\mathcal{Z}_2$*: Ensures DM stability and forbids tree-level m_ν

  * Scenario with no additional symmetry: neutrino mass generation with ***milli-charged DM***, see [M. Klasen, S. Jana, VPK, L. P. Wiggering, 2024]

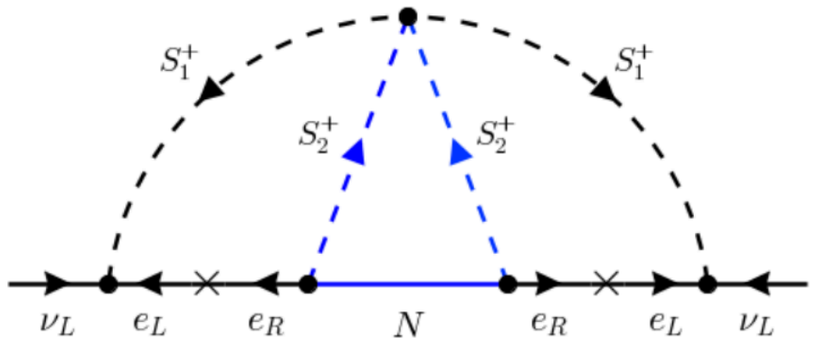
- Eg., Tao-Ma Model: [Z. Tao, 1996], [E. Ma, 2006]
  - Neutrino mass generated at one-loop level
  - Model content: Majorana fermion singlets N_i
    + a scalar doublet η
  - DM candidate: Scalar option $\{\mathrm{Re}(\eta^0), \mathrm{Im}(\eta^0)\}$
    or Fermionic option $\{N_i\}$
  - Smallness of neutrino mass: Small quartic coupling $\lambda_5 \simeq (10^{-12}, 10^{-6})$
    Small Yukawa coupling $Y \simeq (10^{-6}, 10^{-3})$



Scotogenic Models

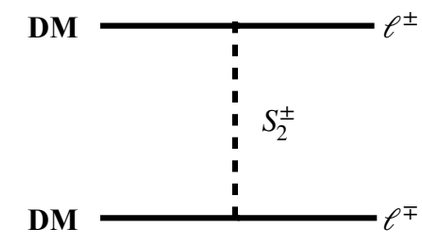
KNT Model [L.M. Krauss, S. Nasri, M. Trodden, 2002]

- Neutrino mass generated at three-loop level
- Model content: Majorona fermion singlets N_i and two singlet scalars $\{S_1^{\pm}, S_2^{\pm}\}$
- DM candidate: Fermionic DM, lightest of $\{N_i\}$
- Contrary to the Tao-Ma model, large values for Yukawas required



$$m_\nu = \frac{1}{(4\pi^2)^3}\frac{\lambda'_{S_1 S_2}}{m_{S_2}}\tilde{Y} M_e Y (M_{\text{eff}})^{-1} Y^T M_e \tilde{Y}^T$$

$$\sigma v \simeq \frac{Y^4}{48\pi}\frac{m_{\text{DM}}^2}{m_{S_2}^4} v^2$$



- Most of the parameter space of the model can be probed in cLFV experiments

[R. Cepedello, M. Hirsch, P. Rocha-Morán A. Vicente, 2020],
[O. Seto, T. Shindou, T. Tsuyuki, 2022]

New
- Induces significant running for scalar quartic couplings $\Longrightarrow$ Vacuum stability? Perturbativity?

KNT Model

- Particle content:
  - Three Majorana fermion singlets $\{N_i\}$
  - Two singly charged singlet scalars $\{S_1^{\pm}, S_2^{\pm}\}$
- Yukawa interactions:

$$\mathscr{L}_{\rm Yuk} \supset \tilde{Y} L_L^T C i\tau_2 L_L S_1^+ + Y \overline{N^c} S_2^+ l_R + \frac{M}{2} N^T C N + h\,.\,c\,.\,,$$

- Scalar potential:

$$\mathscr{V} \supset \mu_{S_1}^2 (S_1^+ S_1^-) + \mu_{S_2}^2 (S_2^+ S_2^-) + \frac{\lambda_{S_1}}{2} (S_1^+ S_1^-)^2 + \frac{\lambda_{S_2}}{2} (S_2^+ S_2^-)^2 + \lambda_{HS_1} (H^\dagger H)(S_1^+ S_1^-)$$
$$+ \lambda_{HS_2} (H^\dagger H)(S_2^+ S_2^-) + \lambda_{S_1 S_2} (S_1^+ S_1^-)(S_2^+ S_2^-) + \frac{1}{2} \left(\lambda'_{S_1 S_2} (S_1^+ S_2^-)^2 + h\,.\,c\,. \right).$$

- Masses of BSM scalars:

$$m_{S_1}^2 = \mu_{S_1}^2 + \frac{1}{2} \lambda_{HS_1} v^2,$$
$$m_{S_2}^2 = \mu_{S_2}^2 + \frac{1}{2} \lambda_{HS_2} v^2\,.$$

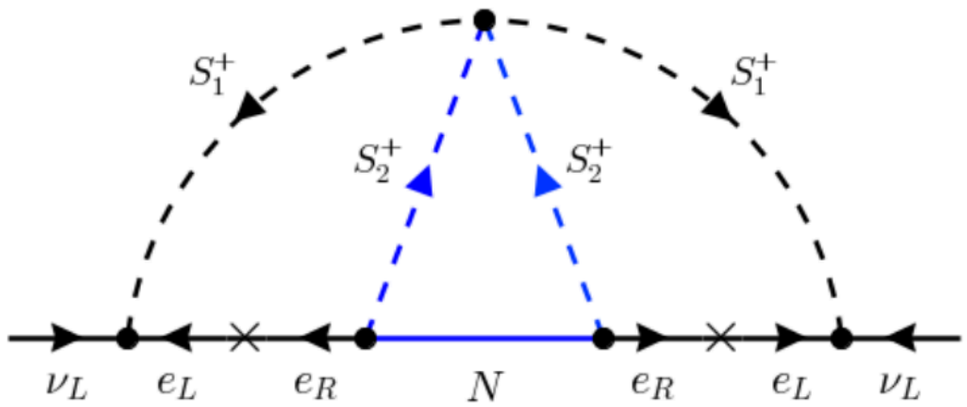
Experimental constraints

- Neutrino masses and mixings:

$$M_\nu = \frac{\lambda'_{S_1 S_2}}{(4\pi^2)^3 m_{S_2}} \tilde{Y} M_e Y (M_{\rm eff})^{-1} Y^T M_e \tilde{Y}^T \,.$$

$$(M_{\rm eff})^{-1} = {\rm diag}\left(F\left(\frac{M_1^2}{m_{S_2}^2}, \frac{m_{S_1}^2}{m_{S_2}^2}\right), F\left(\frac{M_2^2}{m_{S_2}^2}, \frac{m_{S_1}^2}{m_{S_2}^2}\right), F\left(\frac{M_3^2}{m_{S_2}^2}, \frac{m_{S_1}^2}{m_{S_2}^2}\right) \right),$$

$$F(x,y) = \frac{\sqrt{x}}{8 y^{\frac{3}{2}}} \int_0^\infty dr \frac{r}{r+x} \left(\int_0^1 dx \ln \frac{x(1-x)r + (1-x)y + x}{x(1-x)r + x} \right)^2 .$$



Yukawa parametrization:

Normal Hierarchy

$$\frac{\tilde{Y}_{13}}{\tilde{Y}_{23}} = \frac{c_{23}s_{12}}{c_{12}c_{13}} + e^{-i\delta_{CP}} \frac{s_{23}s_{13}}{c_{13}},$$

$$\frac{\tilde{Y}_{12}}{\tilde{Y}_{23}} = \frac{s_{23}s_{12}}{c_{12}c_{13}} - e^{-i\delta_{CP}} \frac{c_{23}s_{13}}{c_{13}}$$

Inverted Hierarchy

$$\frac{\tilde{Y}_{13}}{\tilde{Y}_{23}} = e^{i\delta_{CP}} \frac{c_{23}c_{13}}{s_{13}},$$

$$\frac{\tilde{Y}_{12}}{\tilde{Y}_{23}} = - e^{i\delta_{CP}} \frac{s_{23}c_{13}}{s_{13}}$$

$$Y = M_e^{-1} U_{\rm aux}^T (\hat{M}_{\rm aux})^{\frac{1}{2}} R (M_{\rm eff})^{\frac{1}{2}} \quad \text{with} \quad M_{\rm aux} = M_e Y (M_{\rm eff})^{-1} Y^T M_e \,.$$

Experimental constraints

- Dark matter phenomenology:
  - DM candidate: lightest of $\{N_i\}$
  - DM relic abundance $\Omega h^2 \simeq 0.12$ obtained via thermal freeze-out
  - Dominant annihilation channel: DM DM $\longrightarrow \bar{\ell}\ell$ induced by $Y_{ij}\overline{N_i^c}S_2^+ l_{R_j}$
  - Annihilation crosssections are p-wave and t-channel suppressed: large values for Y required
  - Constraints from DM direct and indirect detection expts. are relaxed
- Lepton flavor violating experiments:

$$\mathrm{BR}(\mu \to e\gamma) < 3.1 \times 10^{-13},$$
$$\mathrm{BR}(\tau \to \mu\gamma) < 4.2 \times 10^{-8},$$
$$\mathrm{BR}(\tau \to e\gamma) < 3.3 \times 10^{-8},$$
$$\mathrm{BR}(\mu \to 3e) < 1.0 \times 10^{-12},$$
$$\mathrm{CR}(\mu - e, Ti) < 4.3 \times 10^{-12}\,.$$

[See talk by A. M. Teixeira]

Theoretical constraints

- Vacuum stability conditions (co-positivity criteria):

$$\lambda_H > 0,$$
$$\lambda_{S_1} > 0,$$
$$\lambda_{S_2} > 0,$$
$$\lambda_{HS_1} + \sqrt{\lambda_H \lambda_{S_1}} > 0,$$
$$\lambda_{HS_2} + \sqrt{\lambda_H \lambda_{S_2}} > 0, \lambda_{S_1 S_2} - |\lambda'_{S_1 S_2}| + \sqrt{\lambda_{S_1} \lambda_{S_2}} > 0,$$
$$\left(\sqrt{\lambda_H \lambda_{S_1} \lambda_{S_2}} + \lambda_{HS_1} \sqrt{\lambda_{S_2}} + \lambda_{HS_2} \sqrt{\lambda_{S_1}} + (\lambda_{S_1 S_2} - |\lambda'_{S_1 S_2}|) \sqrt{\lambda_H} \right.$$
$$\left. + \sqrt{2(\lambda_{HS_1} + \sqrt{\lambda_H \lambda_{S_1}})(\lambda_{HS_2} + \sqrt{\lambda_H \lambda_{S_2}})(\lambda_{S_1 S_2} - |\lambda'_{S_1 S_2}| + \sqrt{\lambda_{S_1} \lambda_{S_2}})} \right) > 0.$$

- Perturbativity constraints:

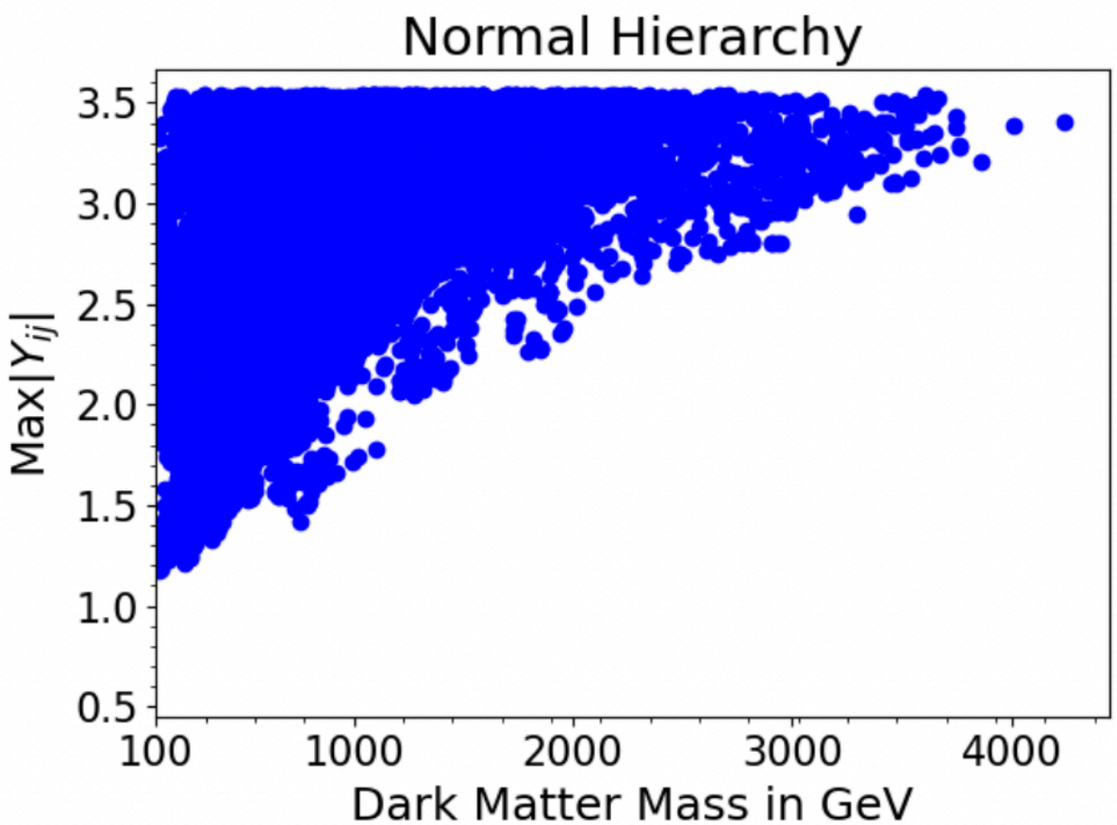
$$|\tilde{Y}_{ij}|^2, |Y_{ij}|^2, |\lambda_{S_1}|, |\lambda_{S_2}|, |\lambda_{HS_1}|, |\lambda_{HS_2}|, |\lambda_{S_1 S_2}|, |\lambda'_{S_1 S_2}| \lesssim 4\pi .$$

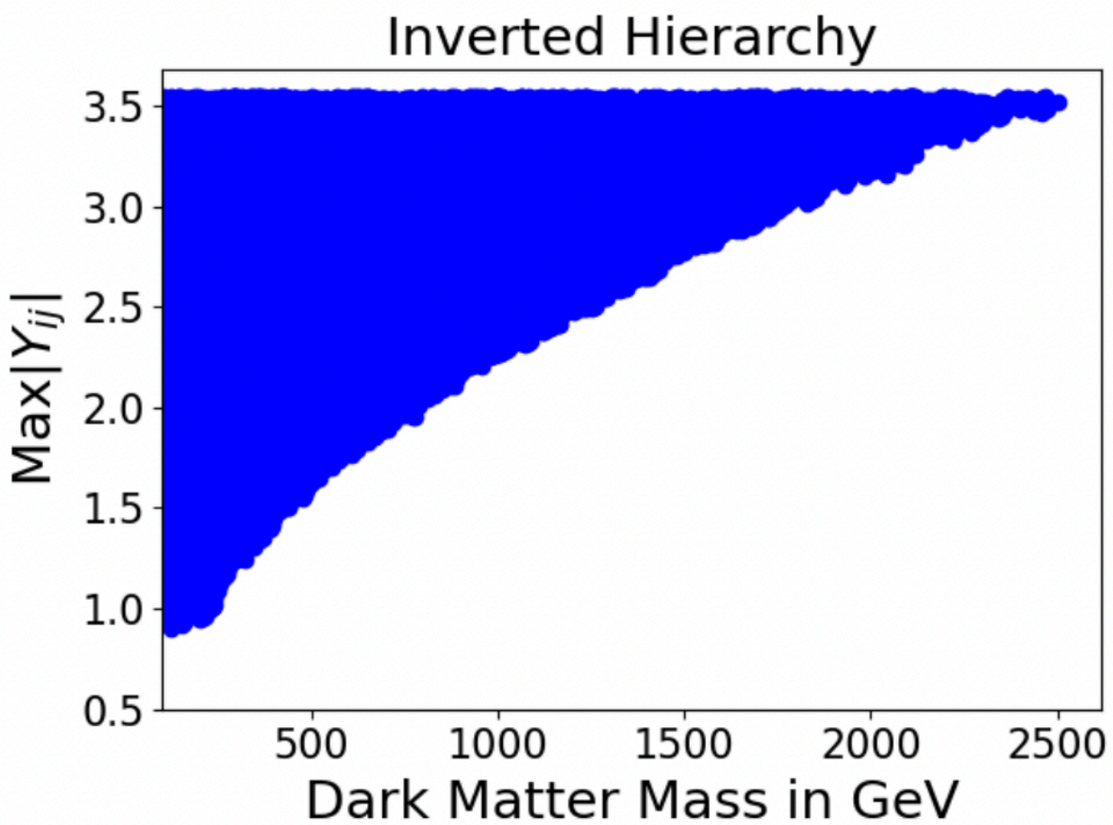
Viable parameter space at low energies

- Numerical scan performed over the free parameters

$$100\ \text{GeV} < M_{1,2,3} < 10\ \text{TeV},$$
$$100\ \text{GeV} < m_{S_{1,2}} < 10\ \text{TeV},$$
$$|\lambda_{S_1}, \lambda_{S_2}, \lambda_{HS_1}, \lambda_{HS_2}, \lambda_{S_1S_2}, \lambda'_{S_1S_2}|, |\tilde{Y}_{23}|^2 < 4\pi,$$
$$0 < \alpha_i < 2\pi, \text{(related to R)}$$
$$|(M_{\text{aux}})_{ij}| < m_i m_j \times (3 \times 1.05 \times 4\pi),$$

Neutrino oscillation parameters 3σ range



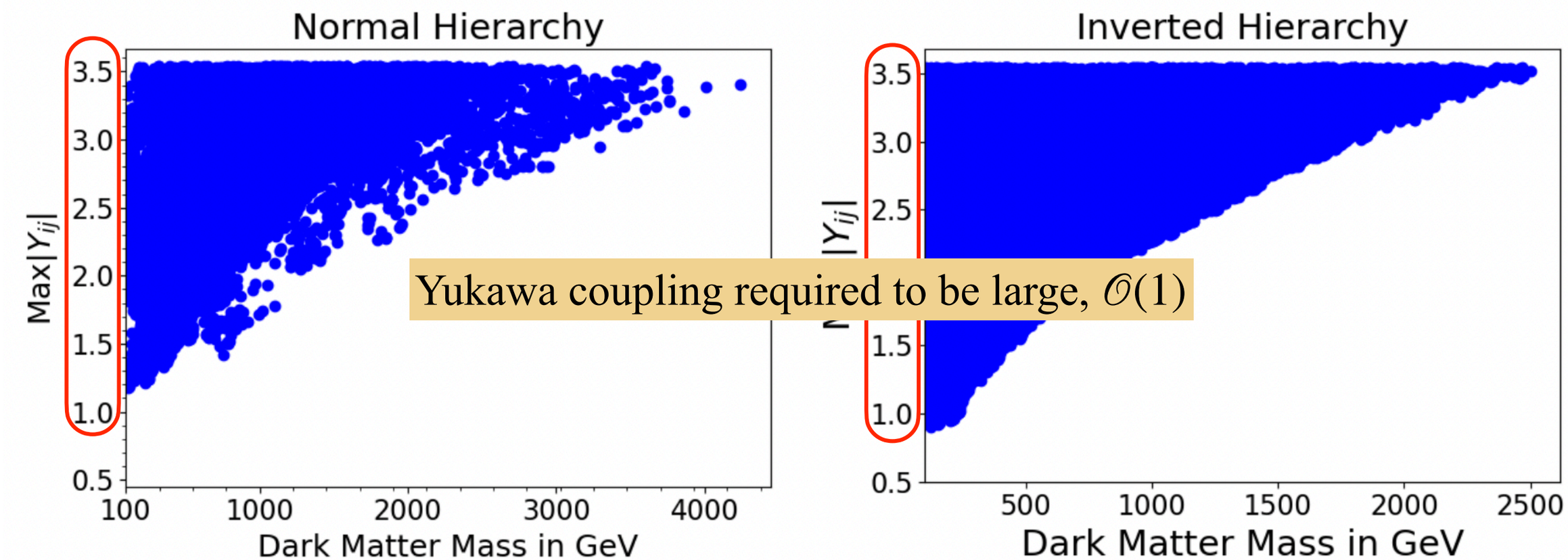


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Neutrino oscillation parameters 3σ range



Yukawa coupling required to be large, $\mathcal{O}(1)$

RG-equations

- One loop beta functions:

$$\beta_\kappa = 16\pi^2\mu\frac{d\kappa}{d\mu}$$

- Beta functions for Yukawa couplings:

$$\beta_{Y_u} = Y_u\left(\frac{3}{2}(Y_u^\dagger Y_u - Y_d^\dagger Y_d) + T - \frac{17}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right)$$

$$\beta_{Y_d} = Y_d\left(\frac{3}{2}(Y_d^\dagger Y_d - Y_u^\dagger Y_u) + T - \frac{5}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right)$$

$$\beta_{Y_e} = Y_e\left(\frac{3}{2}Y_e^\dagger Y_e - 2\tilde{Y}^\dagger\tilde{Y}\right) + \frac{1}{2}YY^\dagger Y_e + Y_e\left(T - \frac{15}{4}g_1^2 - \frac{9}{4}g_2^2\right)$$

$$\beta_{\tilde{Y}} = \tilde{Y}\left(4\tilde{Y}^\dagger\tilde{Y} + \frac{1}{2}Y_e^\dagger Y_e\right) + \frac{1}{2}Y_e^T Y_e^* \tilde{Y} + \tilde{Y}\left(4T_{\tilde{Y}} - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2\right)$$

$$\beta_Y = YY^\dagger Y + Y_e Y_e^\dagger Y + Y\,\mathrm{Tr}(YY^\dagger) - \frac{9}{5}Yg_1^2$$

RG-equations

- Beta functions for scalar quartic couplings:

$$\beta_{\lambda_H} = 12\lambda_H^2 + 2\lambda_{HS_1}^2 + 2\lambda_{HS_2}^2 + \frac{3}{4}\left(g_1^4 + 2g_1^2 g_2^2 + 3g_2^4\right) + \lambda_H\left(4T - 3(g_1^2 + 3g_2^2)\right) - 4T_4$$

$$\beta_{\lambda_{S_1}} = 10\lambda_{S_1}^2 + 4\lambda_{HS_1}^2 + 2\lambda_{S_1S_2}^2 + 2|\lambda'_{S_1S_2}|^2 + 12g_1^4 + \lambda_{S_1}\left(16T_{\tilde{Y}} - 12g_1^2\right) - 64T_{4\tilde{Y}}$$

$$\beta_{\lambda_{S_2}} = 10\lambda_{S_2}^2 + 4\lambda_{HS_2}^2 + 2\lambda_{S_1S_2}^2 + 2|\lambda'_{S_1S_2}|^2 + 12g_1^4 + \lambda_{S_2}\left(4T_Y - 12g_1^2\right) - 4\mathrm{Tr}(YY^\dagger YY^\dagger)$$

$$\beta_{\lambda_{HS_1}} = 4\lambda_{HS_1}^2 + 6\lambda_H\lambda_{HS_1} + 4\lambda_{S_1}\lambda_{HS_1} + 2\lambda_{S_1S_2}\lambda_{HS_2} + 3g_1^4 + \lambda_{HS_1}\left(2T + 8T_{\tilde{Y}} - \frac{3}{2}(5g_1^2 + 3g_2^2)\right) - 16T_{\tilde{Y}e}$$

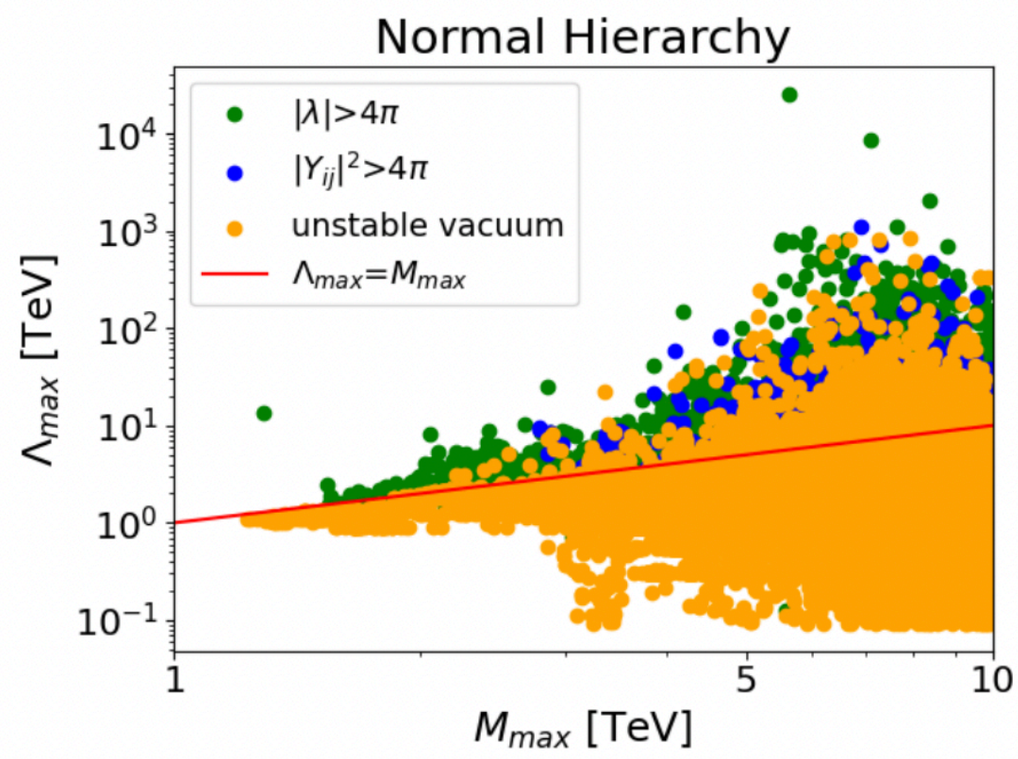
$$\beta_{\lambda_{HS_2}} = 4\lambda_{HS_2}^2 + 6\lambda_H\lambda_{HS_2} + 4\lambda_{S_2}\lambda_{HS_2} + 2\lambda_{S_1S_2}\lambda_{HS_1} + 3g_1^4 + \lambda_{HS_2}\left(2T + 2T_Y - \frac{3}{2}(5g_1^2 + 3g_2^2)\right) - 16T_{Ye}$$

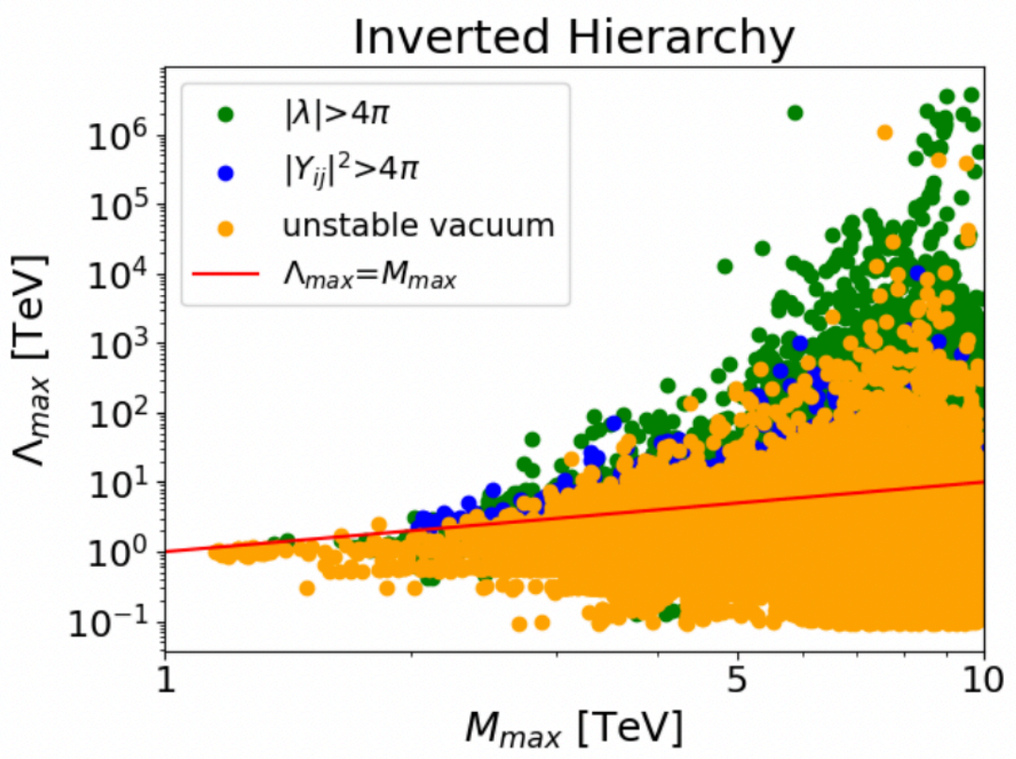
$$\beta_{\lambda_{S_1S_2}} = 4\lambda_{S_1S_2}^2 + 4\lambda_{S_1}\lambda_{S_1S_2} + 4\lambda_{S_2}\lambda_{S_1S_2} + 4\lambda_{HS_1}\lambda_{HS_2} + 8|\lambda'_{S_1S_2}|^2 + 12g_1^4 + \lambda_{S_1S_2}\left(8T_{\tilde{Y}} + 2T_Y - 12g_1^2\right)$$

$$\beta_{\lambda'_{S_1S_2}} = \lambda'_{S_1S_2}\left(2\lambda_{S_1} + 2\lambda_{S_2} + 8\lambda_{S_1S_2} + 8T_{\tilde{Y}} + 2T_Y - 12g_1^2\right)$$

Effects of RG evolution

Each of the low energy viable solutions evolved from weak scale to a scale where atleast one of the theoretical constraints is violated





$\Lambda_{\max}$ = Scale of inconsistency: scale at which one of the theoretical condition is violated

$M_{\max} = \mathrm{Max}\{M_1, M_2, M_3, m_{S_1}, m_{S_2}\}$: highest mass scale of the model

- $\Lambda_{\max}$ is not high for low energy viable solutions
- Significant portion of the paremeter space lies below $\Lambda_{\max} = M_{\max}$ line

Effects of RG evolution

- Fraction of low energy viable solutions that are consistent up to a certain energy scale

Consistency scale	Fraction of parameter space	
	Normal Hierarchy	Inverted Hierarchy
$M_{\max}$	4.46%	9.23%
$2M_{\max}$	1.15%	2.96%
$3M_{\max}$	0.57%	1.66%
$4M_{\max}$	0.30%	1.22%
$5M_{\max}$	0.22%	0.99%

- Majority of the paremeter space ($> 90\,\%$) lies below $\Lambda_{\max} = M_{\max}$ line
- These parameter points, which are otherwise phenomenologically compatible, are inconsistent from a theoretical standpoint, since additional new physics (beyond the KNT model) requires below $\Lambda_{\max} \lesssim M_{\max}$

Vacuum Stability vs. Perturbativity

- Fraction of the low energy viable points that are excluded by various theoretical constraints for the case where $\Lambda_{\max} \lesssim M_{\max}$

Inconsistency scenario	Fraction of parameter space	
	Normal Hierarchy	Inverted Hierarchy
Unstable vacuum	85.46%	72.44%
$\lvert\lambda\rvert > 4\pi$	12.31%	26.56%
$\lvert Y_{ij}\rvert^2 > 4\pi$	2.22%	1.00%

- Vacuum stability conditions and perturbatity conditions of λ's impose the most stringent constraint on the model parameter space
- Among the various vacuum stability conditions, $\lambda_{S_2} > 0$ accounts for more than 99 % of the parameter points are excluded by the vacuum stability conditions

Understanding the effect

- Effect of Yukawa couplings:

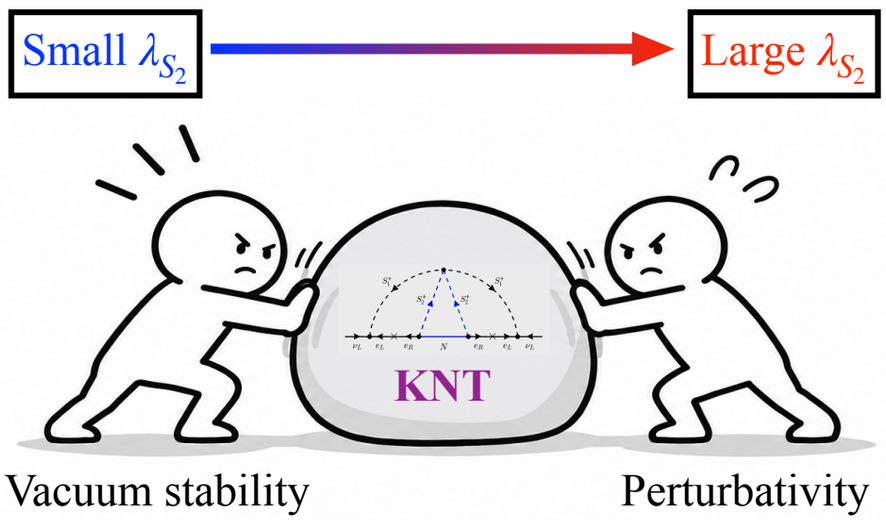
$$\beta_{\lambda_{S_2}} = 10\lambda_{S_2}^2 + 4\lambda_{HS_2}^2 + 2\lambda_{S_1S_2}^2 + 2|\lambda'_{S_1S_2}|^2 + 12g_1^4 + \lambda_{S_2}\left(4T_Y - 12g_1^2\right) - 4\mathrm{Tr}(YY^\dagger YY^\dagger)$$

  - Drives λ_{S_2} to negative values $\Longrightarrow$ Destabilize the vacuum

- However, λ_{S_2} does not play any role in m_ν or Ωh^2 $\Rightarrow$ allowed to take large values

$$\beta_{\lambda_{S_2}} = 10\lambda_{S_2}^2 + 4\lambda_{HS_2}^2 + 2\lambda_{S_1S_2}^2 + 2|\lambda'_{S_1S_2}|^2 + 12g_1^4 + \lambda_{S_2}\left(4T_Y - 12g_1^2\right) - 4\mathrm{Tr}(YY^\dagger YY^\dagger)$$

  - Lead to large values for λ_{S_2} $\Longrightarrow$ Becomes nonperturbative



[AI (ChatGPT)-generated image]

Surviving parameter regions

- Surviving fraction of low energy viable parameter points can be probed in various charged lepton flavor violating experiments

Process	Future sensitivity	Fraction of parameter space	
		Normal Hierarchy	Inverted Hierarchy
$\mu \to e\gamma$	BR $< 6.0 \times 10^{-14}$ [34]	88.20%	91.45%
$\tau \to e\gamma$	BR $< 3.0 \times 10^{-9}$ [44]	4.42%	0.00%
$\tau \to \mu\gamma$	BR $< 10^{-9}$ [44]	7.72%	3.49%
$\mu \to eee$	BR $< 10^{-16}$ [45, 46]	99.55%	100%
$\mu - e, Al$	CR $< 6.0 \times 10^{-17}$ [47, 48]	99.73%	99.93%

- (More or less) The entire viable parameter space of the model is within the reach of cLFV experiments

Summary and Outlook

- KNT model:
  - ☑ Unified (and minimal) framework for smallness of m_ν and the dark matter abundance
  - ☑ Unlike other scotogenic models, Yukawa couplings are required to be $\mathcal{O}(1)$, consequently the model is testable at variety of experiments
  - ☑ Such large values of Yukawa couplings induce significant running for scalar quartic couplings
  - ☑ Morethan 90 % of low energy viable solutions of the model found to be theoretcally inconsistent from vacuum stability conditions and perturbativity conditions once the RG-effects are taken to account
  - ☑ Surviving (small) fraction of the parameter space can be probed in cLFV experiments
- RG-effects are important, particularly in scenarios where Yukawa couplings are large, see for e.g., SM + heavy neutrinos [J.Elian-Miro, et al., 2011], [W. Rodejohann, H. Zhang, 2012], etc.

  Fermionic WIMPs from Tao-Ma model [M. Lindner, M. Platscher, C.E. Yaguna, A. Merle, 2016]

  Leptophilic DM model [O. Seto, T. Shindou, T. Tsuyuki, 2025]
- Scrutinize other scotogenic models of fermionic WIMPs, e.g., AKS model

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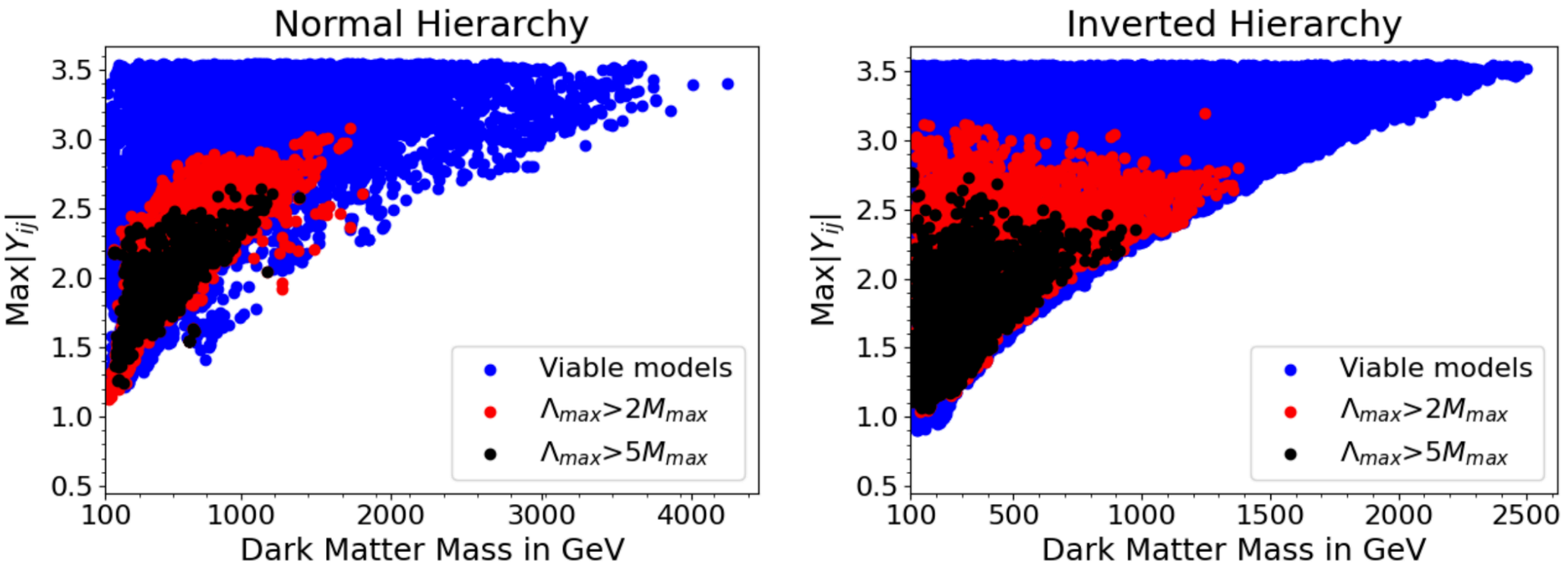
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Thank you for your attention!

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$\Lambda_{\max}$ = Scale of inconsistency: scale at which one of the theoretical condition is violated

$M_{\max} = \mathrm{Max}\{M_1, M_2, M_3, m_{S_1}, m_{S_2}\}$: highest mass scale of the model



- Restrictions induced by RG-effects are more severe for large values of Yukawa couplings: effects are driven by largest entry of Y

Analytical estimation of Λ_{max}

- The problematic term can be identified with

$$\beta_{\lambda_{S_2}} \sim -4\mathrm{Tr}(YY^\dagger YY^\dagger)$$

- By considering only this term, we estimate Λ_{max} (denoted as Λ_{est})

$$\log\left(\frac{\Lambda_{\mathrm{est}}}{\mu_0}\right) = \left.\frac{4\pi^2\lambda_{S_2}}{\mathrm{Tr}(YY^\dagger YY^\dagger)}\right|_{\mu=\mu_0}$$

assuming no running for Y

