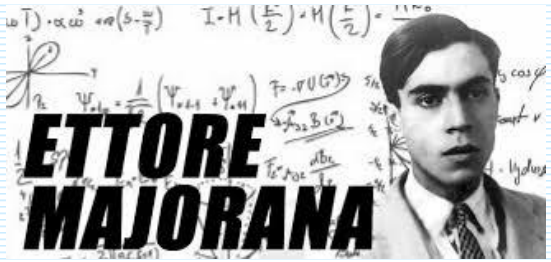
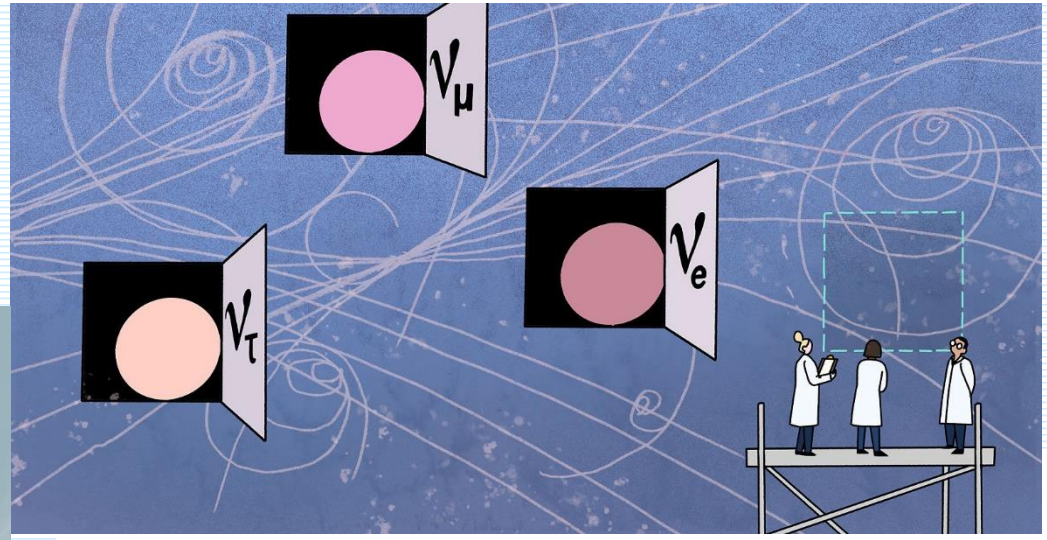
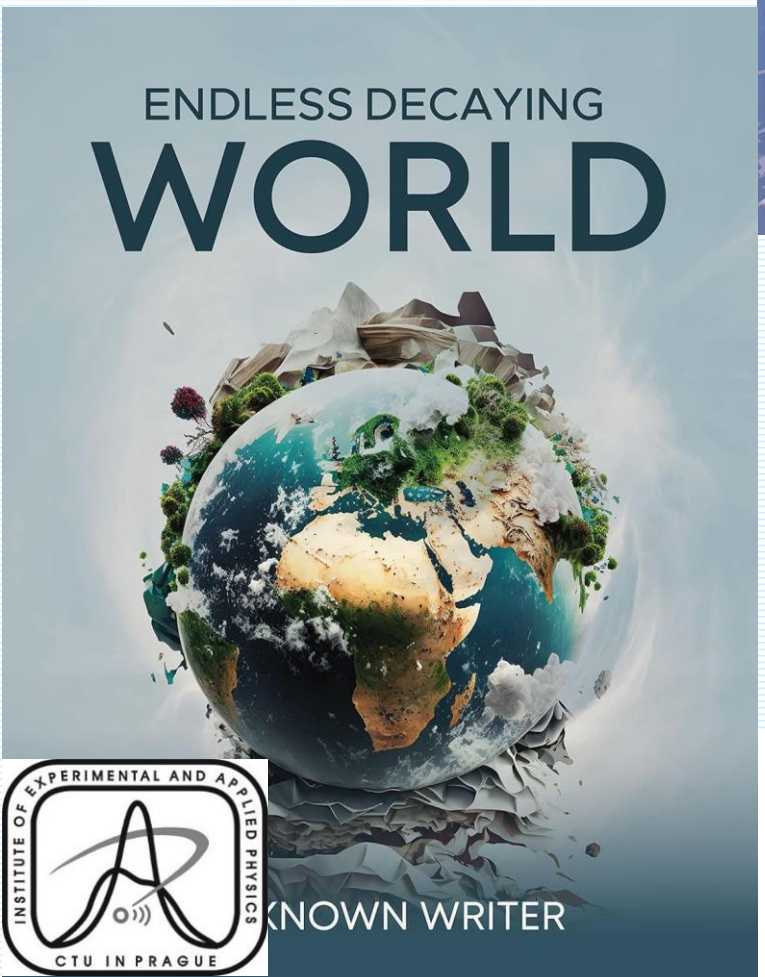
DISCRETE 2026

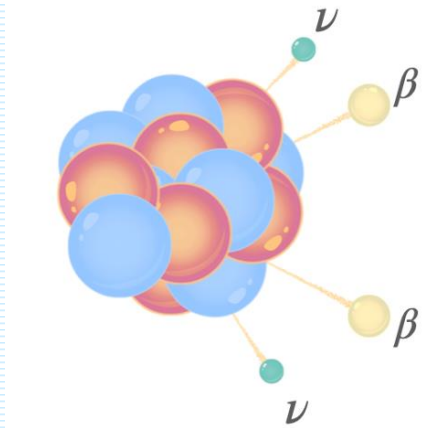
Comenius University, Bratislava, Slovakia, Nov. 13-17, 2026



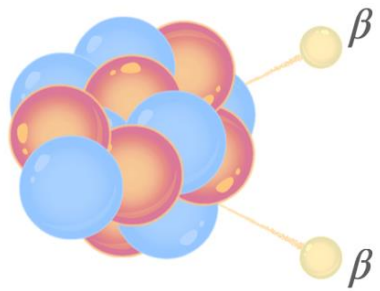




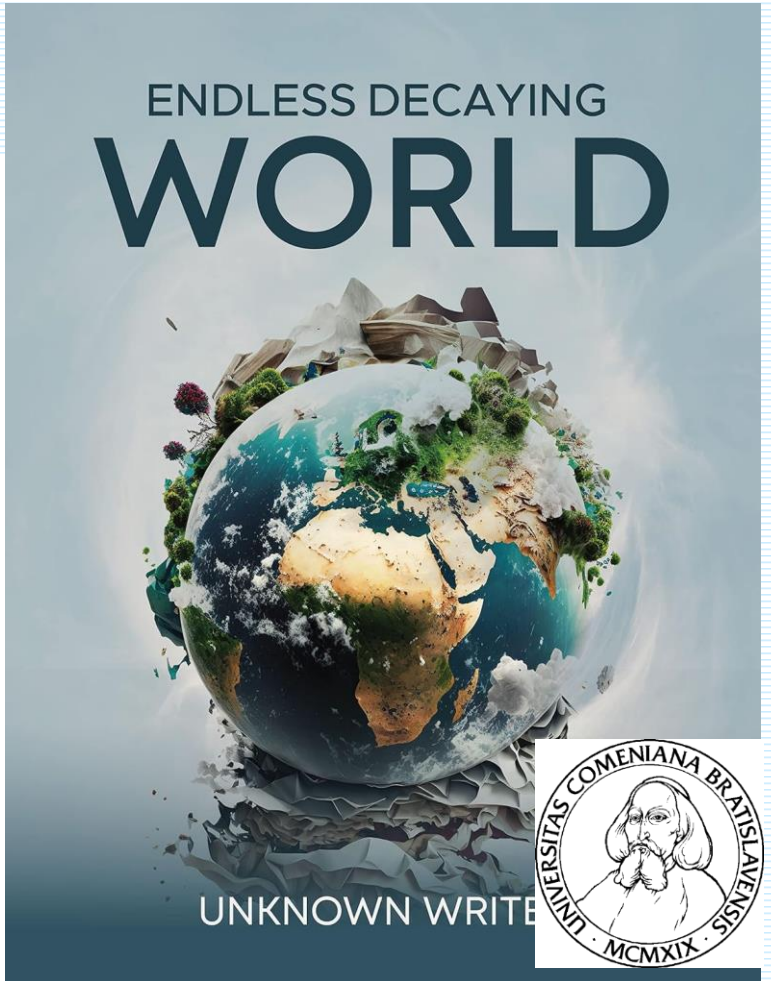




$(A,Z) \to (A, Z \pm 2) + 2\beta^{\mp} + 2\overset{(-)}{\nu}$



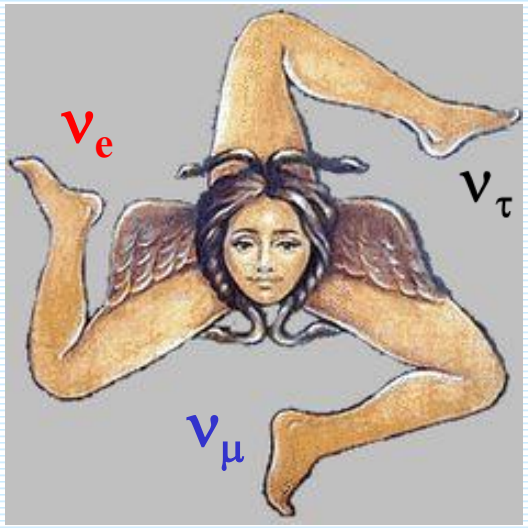
$(A,Z) \to (A, Z \pm 2) + 2\beta^{\mp}$



Neutrino masses, mixing, oscillations and 0νββ-decay

Fedor Šimkovic

OUTLINE



I. Introduction

II. Neutrino oscillations as a single Feynman diagram
(QFT formalism with plane waves, entanglement of production and detection of neutrinos)

III. Neutrino-antineutrino oscillations
(effective Majorana mass)

IV. Neutrino mass mechanisms of the $0\nu\beta\beta$-decay
(QCSS scenario of ν mass generation, LRS model Quasi-Dirac neutrinos)

IV. Towards fixing parameters of ν mass matrix
(Processes described with Frobenius covariants instead of elements of the neutrino mixing matrix)

Acknowledgments: S. Kovalenko, M. Krivoruchenko, A. Babíč, A. Khatun and other colleagues and friends.



Lukáš, thank you for a brilliant introductory talk on the DBD subject

Majorana fermions



Ettore Majorana

Teoria simmetrica dell'elettrone e del positrone
(A symmetric theory of electrons and positrons).
Il Nuovo Cimento, 14: 171–184, 1937.) 171

ν is its own antiparticle

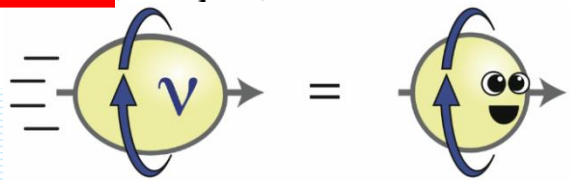


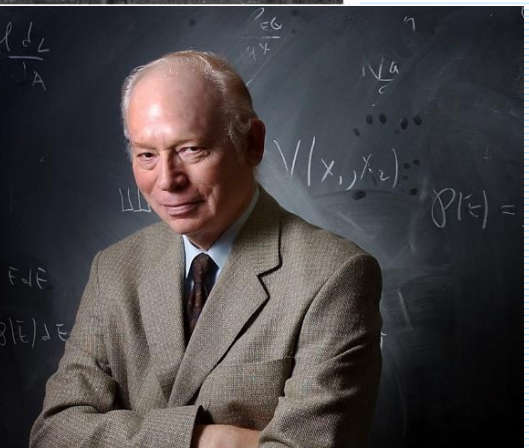
Bruno Pontecorvo

Inverse beta processes and nonconservation of lepton charge
Zhur. Eksptl'. i Teoret. Fiz. 34, 247 (1958)

It follows from the above assumptions that in vacuum a neutrino can be transformed into an antineutrino and vice versa. This means that the neutrino and antineutrino are "mixed" particles, i.e., a symmetric and antisymmetric combination of two truly neutral Majorana particles ν_1 and ν_2 of different combined parity.[5]

ν ↔ anti-ν oscillation





INTERNATIONAL JOURNAL OF HIGH-ENERGY PHYSICS
CERNCOURIER
VOLUME 57 NUMBER 9 NOVEMBER 2017

Steve Weinberg
ν-mass generation via d=5 eff. oper. related to unknown high energy scale (GUT?)

thought massless back in 1979. Weinberg does not take credit for predicting neutrino masses, but he thinks it's the right interpretation. What's more, he says, the non-renormalisable interaction that produces the neutrino masses is probably also accompanied with non-renormalisable interactions that produce proton decay and other things that haven't been observed, such as violation of baryon-number conservations. "We don't know anything about the details of those terms, but I'll swear they are there."

Neutrino Mass Term

Dirac (3x3)

$$L^D_{\rm mass} = -\sum_{\alpha\beta} \overline{\nu}_{\alpha R}\ M^D_{\alpha\beta}\ \nu_{\beta_L} + H.c.$$

$$= -\sum_{k=1}^{3} m_k \overline{\nu}_k \nu_k$$

$$\alpha, \beta = e, \mu, \tau, \qquad V^\dagger\ M^D\ U = M^D_{\rm diag}$$

Majorana (3x3)

$$L^M_{\rm mass} = \frac{1}{2}\sum_{\alpha\beta} \nu^T_{\alpha L} C^\dagger\ M^L_{\alpha\beta}\ \nu_{\beta_L} + H.c.$$

$$= \frac{1}{2}\sum_{k=1}^{3} m_k \nu^T_k C^\dagger \nu_k$$

$$\alpha, \beta = e, \mu, \tau$$

$$M^L_{\alpha\beta} = M^L_{\beta\alpha} \qquad U^T\ M^M\ U = M^M_{\rm diag}$$

Dirac-Majorana (6x6: see-saw, Quasi-Dirac ν's)

$$L^{D+M}_{\rm mass} = -\sum_{\alpha\beta} \overline{N}_{\alpha R}\ M^{D+M}_{\alpha\beta}\ N_{\beta_L} + H.c.$$

$$= -\sum_{k=1}^{6} m_k \overline{\nu}_k \nu_k$$

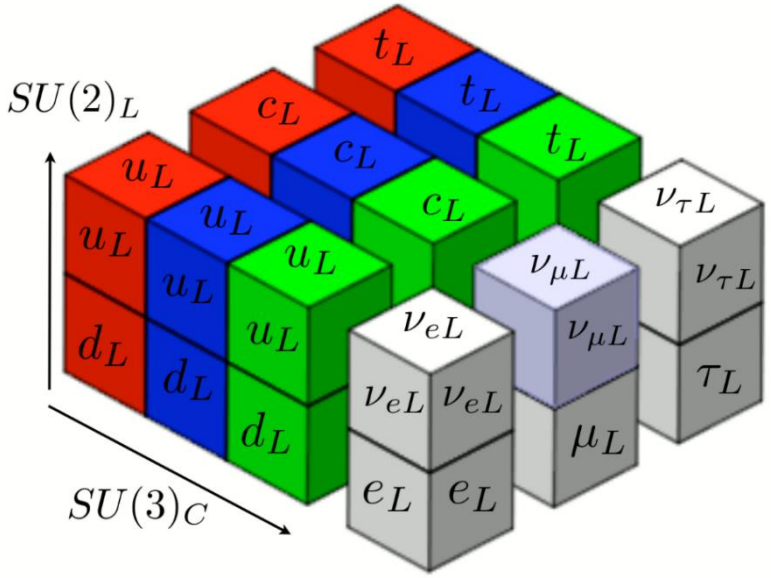
$$N = \begin{pmatrix} \nu_L \\ \nu^C_R \end{pmatrix}, \qquad \alpha, \beta = e, \mu, \tau, s_1, s_2, s_3$$

$$U^T\ M^{D+M}\ U = M^{D+M}_{\rm diag}$$



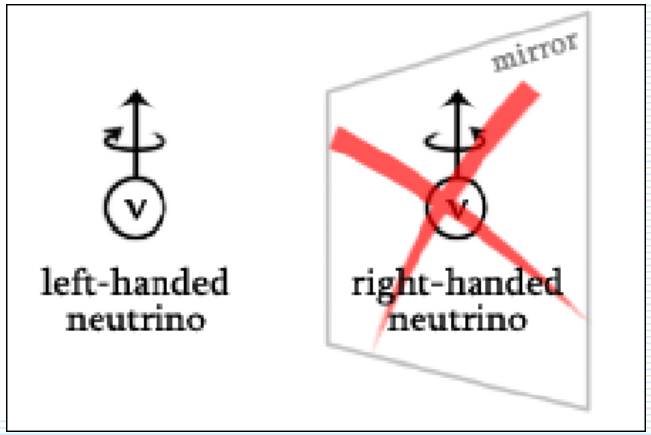


Standard Model
(an astonishing successful theory, based on few principles)



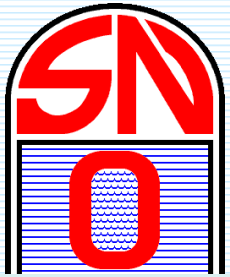
ν is a special particle in SM:

- **It is the only fermion that does not carry electric charge (like γ, g, H^0)**
- **There are only left-handed ν's (ν_{eL}, $\nu_{\mu L}$, $\nu_{\tau L}$)**
- **ν-mass can not be generated with any renormalizable coupling with the Higgs fields through SSB**

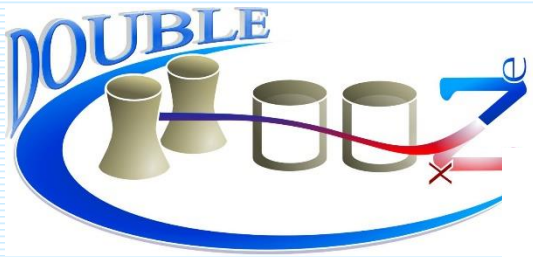


Neutrino Masses and Mixing imply ν's oscillations













ν's oscillation experiments ⇒ tiny neutrino masses (!)

⇒ Beyond SM physics (!)





, etc

$$R_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{12} & s_{12} \\ 0 & -s_{12} & c_{12} \end{pmatrix}$$

$$\tilde{R}_{13} = \begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{pmatrix}$$

$$U = R_{23}\tilde{R}_{13}R_{12}$$

3 mixing angles
CP-phase

$$|\nu_\alpha\rangle = \sum_{j=1}^{3} U^*_{\alpha j}|\nu_j\rangle$$

$$(\alpha = e,\ \mu, \tau)$$

3 neutrino masses, 2 mass squared differences

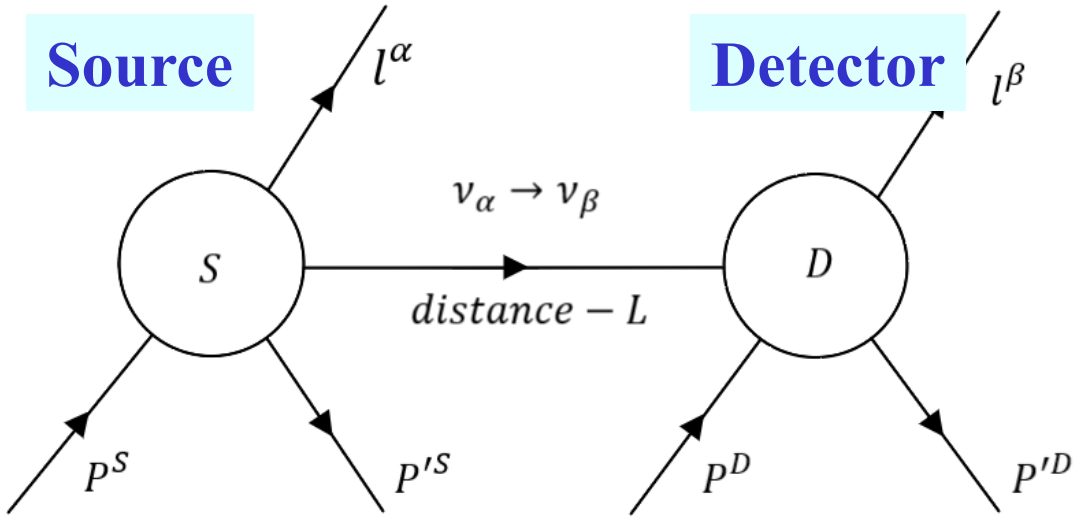
$$\delta m^2 = m_2^2 - m_1^2, \quad \Delta m^2 = m_3^2 - (m_1^2 + m_2^2)/2$$

best − fit	1σ	2σ	3σ
	Normal hierarchy (NH)		
$\delta m^2/10^{-5}$ eV^2	7.34	7.20-7.51	7.05-7.69	6.92-7.90
$\Delta m^2/10^{-3}$ eV^2	2.485	2.453-2.514	2.419-2.547	2.2389-2.578
$\sin^2\theta_{12}/10^{-1}$	3.05	2.92-3.19	2.78-3.32	2.65-3.47
$\sin^2\theta_{13}/10^{-2}$	2.22	2.14-2.28	2.07-2.34	2.01-2.41
$\sin^2\theta_{23}/10^{-1}$	5.45	4.98-5.65	4.54-5.81	4.36-5.95
δ/π	1.28	1.10-1.66	0.95-1.90	0-0.07 $\oplus$ 0.81-2.00
	Inverted hierarchy (IH)		
$\delta m^2/10^{-5}$ eV^2	7.34	7.20-7.51	7.05-7.69	6.92-7.91
$-\Delta m^2/10^{-3}$ eV^2	2.465	2.434-2.495	2.404-2.526	2.374-2.556
$\sin^2\theta_{12}/10^{-1}$	3.03	2.90-3.17	2.77-3.31	2.64-3.45
$\sin^2\theta_{13}/10^{-2}$	2.23	2.17-2.30	2.10-2.37	2.03-2.43
$\sin^2\theta_{23}/10^{-1}$	5.51	5.17-5.67 $\oplus$ 5.31-6.10	4.60-5.82	4.39-5.96
δ/π	1.52	1.37-1.65	1.23-1.78	1.09-1.90

Global neutrino oscillations analysis
(PRD 101, 116013 (2020))

Neutrino oscillations
(Quantum Mechanics Approach)

Rev. Mod. Phys. 59, 671 (1987)



Massive neutrinos and neutrino oscillations

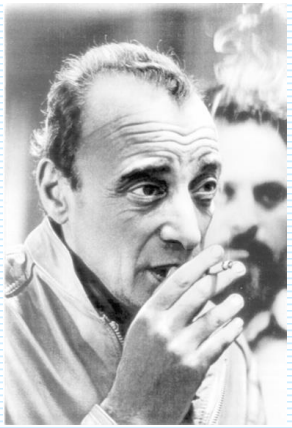
S. M. Bilenky

Joint Institute of Nuclear Research, Dubna, Union of Soviet Socialist Republics

S. T. Petcov

Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, 1784 Sofia, People's Republic of Bulgaria

The theory of neutrino mixing and neutrino oscillations, as well as the properties of massive neutrinos (Dirac and Majorana), are reviewed. More specifically, the following topics are discussed in detail: (i) the possible types of neutrino mass terms; (ii) oscillations of neutrinos (iii) the implications of *CP* invariance for the mixing and oscillations of neutrinos in vacuum; (iv) possible varieties of massive neutrinos (Dirac, Majorana, pseudo-Dirac); (v) the physical differences between massive Dirac and massive Majorana neutrinos and the possibilities of distinguishing experimentally between them; (vi) the electromagnetic properties of massive neutrinos. Some of the proposed mechanisms of neutrino mass generation in gauge theories of the electroweak interaction and in grand unified theories are also discussed. The lepton number nonconserving processes $\mu \to e\gamma$ and $\mu \to 3e$ in theories with massive neutrinos are considered. The basic elements of the theory of neutrinoless double-β decay are discussed as well. Finally, the existing data on neutrino masses, oscillations of neutrinos, and neutrinoless double-β decay are briefly reviewed. The main emphasis in the review is on the general model-independent results of the theory. Detailed derivations of these are presented.





$$\Gamma_{osc} = \int \frac{d\Phi_\nu(E_\nu)}{dE_\nu} \frac{\mathcal{P}_{\alpha\beta}(E_\nu, L)}{4\pi L^2} \sigma(E_\nu)\, dE_\nu$$

$$S \to S' + \ell_\alpha^+ + \nu_\alpha$$

$$\nu_\alpha \to \nu_\beta$$

$$\nu_\beta + D \to D' + \ell_\beta^-$$

$$S + D \to \ell_\alpha^+ + \ell_\beta^- + S' + D'$$

Process is governed by the oscillation probability

$$\mathcal{P}_{\alpha\beta}(E_\nu, L) = \left| \sum_{j=1}^{3} U_{\alpha j}^* U_{\beta j} e^{-i\, m_j^2\, L/(2E_\nu)} \right|^2$$

Fedor Simkovic

$$\langle f|S^{(2)}|i\rangle = -i \int d^4x_1 J_S^{\mu}(P'_S, P_S) e^{i(P_\alpha + P'_S - P_S)\cdot x_1} \times$$

$$\int d^4x_2 J_D^{\mu}(P'_D, P_D) e^{i(P_\beta + P'_D - P_D)\cdot x_2} \sum_{k=1}^{3} U^*_{\alpha k} U_{\beta k} \times$$

$$\overline{v}(P_\alpha;\lambda_\alpha)\gamma_\mu(1-\gamma_5)\; D(x_2 - x_1, m_k)\; (1-\gamma_5)\gamma_\nu u(P_\beta;\lambda_\beta)$$

Neutrino oscillations as a single Feynman diagram (within QFT, Walter Grimus approach revisited)]

J. Phys. G 51, 035202 (2024)

- **Choose a reference frame.**
- **The physical representation is an outgoing spherical wave of ν from the source.**

The neutrino emission and detection are identified with the charged-current vertices of a single second-order Feynman diagram for the underlying process, enclosing neutrino propagation between these two points.

$$D(x;m) = \theta(x_0) D^-(x;m) + \theta(-x_0) D^+(x;m),$$

$$D^{\pm}(x;m) = \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{\mp(-\mathbf{q}\cdot\vec{\gamma} + \omega\gamma^0) + m}{2\omega} e^{\pm i(-\mathbf{q}\cdot\mathbf{x} + \omega x_0)}$$

Integration over time variables results in energy conservation and energy denominator

$$2\pi i \; \frac{\delta(E_\beta + E'_D - E_D + E_\alpha + E'_S - E_S)}{\omega + E_\alpha + E'_S - E_S + i\varepsilon}.$$

Neutrino propagation

$$\int \frac{d\mathbf{q}}{(2\pi)^3} \frac{\not{p} + m_k}{2\omega(\omega + E_\alpha + E'_S - E_S + i\varepsilon)} e^{i\mathbf{q}\cdot(\mathbf{x}_2 - \mathbf{x}_1)}$$

$$\simeq \frac{1}{4\pi} \frac{e^{ip_k|\mathbf{x}_2 - \mathbf{x}_1|}}{|\mathbf{x}_2 - \mathbf{x}_1|} \; (\not{Q}_k + m_k) \simeq e^{i\mathbf{p}_k\cdot\mathbf{x}_D} \; e^{-i\mathbf{p}_k\cdot\mathbf{x}_S} \; \frac{e^{ip_k L}}{L} \; (\not{Q}_k + m_k)$$

$$Q_k \equiv (E_\nu, \mathbf{p}_k), \quad \mathbf{p}_k = p_k \, (\mathbf{x}_2 - \mathbf{x}_1) / |\mathbf{x}_2 - \mathbf{x}_1|, \quad p_k = \sqrt{E_\nu^2 - m_k^2}$$

$$E_\nu = E_S - E'_S - E_\alpha \; (source) = E_\beta + E'_D - E_D \; (detector)$$

Energy conservation

Amplitude (there is no factorization of source and detector!)

Energy conservation

Momentum conservation at source

Momentum conservation at detector

$$\langle f|S^{(2)}|i\rangle = (2\pi)^7 \delta(E_f - E_i \sum_k U_{\alpha k} U^*_{\beta k} \frac{e^{ip_k L}}{4\pi L} \times$$

$$T_k^{\alpha\beta} \; \delta^3_{V_S}(\mathbf{p}_k + \mathbf{p}_\alpha + \mathbf{p}'_S - \mathbf{p}_S) \; \delta^3_{V_D}(\mathbf{p}_\beta + \mathbf{p}'_D - \mathbf{p}_D - \mathbf{p}_k)$$

$$with$$

$$E_f - E_i = E_\beta + E'_D - E_D + E_\alpha + E'_S - E_S$$

$$T_k^{\alpha\beta} = J^\mu_S(P'_S, P_S) J^\nu_D(P'_D, P_D) \; \overline{v}(P_\alpha; \lambda_\alpha) \gamma_\mu (1 - \gamma_5) \; \not{Q}_k \; \gamma_\nu u(P_\beta; \lambda_\beta)$$

Lepton current

Master formula

J. Phys. G 51, 035202 (2024)

$$d\Gamma^{\alpha\beta}(L) = \sum_{km} U_{\alpha k} U^*_{\beta k} U_{\alpha m} U^*_{\beta m} \frac{e^{i(p_k - p_m)L}}{4\pi L^2} \times \mathcal{F}^{\alpha\beta}_{km}$$

$$\delta(\mathbf{p}_k + \mathbf{p}_\alpha + \mathbf{p}'_S - \mathbf{p}_S)\delta(\mathbf{p}_\beta + \mathbf{p}'_D - \mathbf{p}_D - \mathbf{p}_m)$$

$$\frac{(2\pi)^7}{4E_S E_D}\, \delta(E_\beta + E'_D - E_D + E_\alpha + E'_S - E_S)\times$$

$$\frac{1}{\hat{J}_S \hat{J}_D} \frac{d\mathbf{p}_\alpha}{2E_\alpha\ (2\pi)^3} \frac{d\mathbf{p}_\beta}{2E_\beta\ (2\pi)^3} \frac{d\mathbf{p}'_S}{2E'_S\ (2\pi)^3} \frac{d\mathbf{p}'_D}{2E'_D\ (2\pi)^3}$$

with

$$\mathcal{F}^{\alpha\beta}_{km} = 4\pi \sum_{\text{spin}} \frac{1}{2} \left(T^{\alpha\beta}_k \left(T^{\alpha\beta}_m\right)^* + T^{\alpha\beta}_m \left(T^{\alpha\beta}_k\right)^* \right)$$

$$\langle \Phi^{S,D}(\mathbf{P}_i) | \Phi^{S,D}(\mathbf{P}_k) \rangle = (2\pi)^3\ 2E_k\ \delta^3_{V_{S,D}}\ (\mathbf{P}_i - \mathbf{P}_k)$$

Two normalization volumes:

i) source;

ii) Detector.

An example:

J. Phys. G 51, 035202 (2024)

$$\pi^+ + n \to \mu^+ + e^- + p$$

$$\pi^+ \to \mu^+ + \nu_\mu, \quad \nu_\mu \to \nu_e, \quad \nu_e + n \to p + e^-$$

$$\Gamma^{\pi^+ n}_{osc} = \int \frac{d\Phi_\nu(E'_\nu)}{dE'_\nu} \frac{\mathcal{P}_{\nu_\mu\nu_e}(E'_\nu)}{4\pi L^2} \sigma(E'_\nu)\, dE'_\nu$$

$$= \frac{1}{2\pi^2} G^2_\beta \left(\frac{f_\pi}{\sqrt{2}}\right)^2 \frac{m^2_\mu}{m_\pi} E^2_\nu \frac{P_{\nu_\mu\nu_e}(E_\nu)}{4\pi L^2} \left(g^2_V + 3g^2_A\right) p_e E_e$$

$$with$$

$$\mathcal{P}_{\alpha\beta}(E_\nu, L) = \left| \sum_{j=1}^{3} U^*_{\alpha j} U_{\beta j} e^{-i m^2_j L/(2E_\nu)} \right|^2$$

Standard QM approach

New QFT approach (no decoherence, no factorization of two processes)

$$\Gamma^{\pi^+ n}_{QFT} = \frac{1}{2\pi^2} G^2_\beta \left(\frac{f_\pi}{\sqrt{2}}\right)^2 \frac{m^2_\mu}{m_\pi} E^2_\nu \frac{\mathcal{P}^{QFT}_{\mu e}(E_\nu)}{4\pi L^2} \left(g^2_V + 3g^2_A\right) p_e E_e$$

$$with$$

$$\mathcal{P}^{QFT}_{\alpha\beta}(E_\nu) = \frac{1}{2} \sum_{km} U_{\beta k} U^*_{\alpha k}\, U^*_{\beta k} U_{\alpha k}\, e^{i(p_m - p_k)L} \left(1 + \frac{p_k p_m}{E^2_\nu}\right)$$

Is there an entanglement of ν production and detection? (7 momenta in a game)

$$\mu^- + n \to e^- + \overline{\nu}_e + e^- + p$$

$$\mu^- \to \nu_\mu + e^- + \overline{\nu}_e, \quad \nu_\mu \to \nu_e, \quad \nu_e + n \to p + e^-$$

$P_\mu,\ P_{e_s},\ P_{\nu_e},$
$\mathbf{P}_k$
$P_n,\ P_p, P_{e_d}$

Simplification:
$\mathbf{g_V=g_A=1}$
$\mathbf{P_k=P_l}$

source $\int\int \frac{1}{2}\mathrm{Tr}\ \gamma_\rho(1-\gamma_5)\left(\not{P}_{\overline{\nu}_e} - m_\nu\right)\gamma_\lambda(1-\gamma_5)\left(\not{P}_{e_s} + m_e\right) \times$

ν − prop. $\mathrm{Tr}\ \gamma^\sigma \not{P}_k \gamma^\rho(1-\gamma_5)\left(\not{P}_\mu + m_\mu\right)\gamma^\lambda \not{P}_k \gamma^\delta(1-\gamma_5)\left(\not{P}_{e_d} + m_e\right) \times$

detector $\frac{1}{2}\mathrm{Tr}\ \gamma_\sigma(g_V - g_A\gamma_5)\left(\not{P}_n + m_n\right)\gamma_\delta(g_V - g_A\gamma_5)\left(\not{P}_p + m_p\right) \times \frac{2\ 2}{256}$

$$= 16\ 4\ (P_\mu \cdot P_{\overline{\nu}_e})((P_\mu - P_{e_s} - P_k)\cdot P_k) + (P_{e_d}\cdot P_p)(P_n \cdot P_k)$$

Additional terms

$$-8\ 4\ \mathbf{P}_k^2\ (P_\mu\cdot(P_\mu - P_{e_s} - P_k))\ (P_{e_d}\cdot P_p)(P_{e_s}\cdot P_n)$$
$$-2\epsilon_{P_{e_d}P_{e_s}P_\mu P_k}\ ((P_{e_s}\cdot P_p)\ (P_n\cdot P_k) - (P_\mu\cdot P_p)\ (P_n\cdot P_k)$$
$$+(P_p\cdot P_k)\ ((P_{e_s}\cdot P_n) - (P_n\cdot P_k)))$$
$$+2\epsilon_{P_{e_s}P_{e_\mu}P_n P_p}\ (P_{e_d}\cdot P_k)\ \left((P_\mu\cdot P_k) + P_k^2\right)$$
$$+\epsilon_{P_{e_d}P_{e_s}P_p P_k}\ \left(2(P_{e_s}\cdot P_\mu)\ (P_n\cdot P_k) - P_\mu^2\ (P_n\cdot P_k) - (P_\mu\cdot P_n)\ P_k^2\right)$$
$$+\epsilon_{P_{e_d}P_{e_s}P_n P_k}\ \left(2(P_{e_s}\cdot P_\mu)\ (P_p\cdot P_k) - P_\mu^2\ (P_p\cdot P_k)\right.$$
$$\left.-2(P_\mu\cdot P_p)\ (P_p\cdot P_k) + (P_\mu\cdot P_p)\ P_k^2\right)$$
$$+\epsilon_{P_{e_d}P_{e_s}P_\mu P_p}\ (\cdots) + \epsilon_{P_{e_d}P_{e_s}P_\mu P_n}\ (\cdots) + \epsilon_{P_{e_d}P_\mu P_p P_k}\ (\cdots) + \epsilon_{P_{e_d}P_\mu P_n P_k}\ (\cdots)$$
$$+2\epsilon_{P_\mu P_n P_p P_k}\ (\cdots) + \epsilon_{P_{e_s}P_n P_p P_k}\ (\cdots) + \epsilon_{P_{e_s}P_\mu P_p P_k}\ (\cdots) + \epsilon_{P_{e_s}P_\mu P_n P_k}\ (\cdots)$$

source $\int\int \frac{1}{2}\mathrm{Tr}\ \gamma_\rho(1-\gamma_5)\left(\not{P}_{\overline{\nu}_e}-m_\nu\right)\gamma_\lambda(1-\gamma_5)\left(\not{P}_{e_s}+m_e\right)\times$

$\nu-$**prop.** $\mathrm{Tr}\ \gamma^\sigma \not{P}_k\gamma^\rho(1-\gamma_5)\left(\not{P}_\mu+m_\mu\right)\gamma^\lambda \not{P}_k\gamma^\delta(1-\gamma_5)\left(\not{P}_{e_d}+m_e\right)\times$

detector $\frac{1}{2}\mathrm{Tr}\ \gamma_\sigma(g_V-g_A\gamma_5)\left(\not{P}_n+m_n\right)\gamma_\delta(g_V-g_A\gamma_5)\left(\not{P}_p+m_p\right)\ \times$

$$\delta\left(P_{e_s}+P_{\nu_e}-P_Q\right)\frac{d\mathbf{p}_{e_s}}{E_{e_s}}\frac{d\mathbf{p}_{\nu_e}}{E_{\nu_e}}=\frac{1}{4}\,256\times 16\times$$

$$\left\{\ C_g\ (P_\mu\cdot P_k)\ +\ C_{PP}\left(P_\mu^2-(P_\mu\cdot P_k)\right)\ \left((P_\mu\cdot P_k)-P_k^2\right)\ \right)\times$$

$$\left[\left(g_A^2-g_V^2\right)\ m_nm_p\ (P_{e_d}\cdot P_k)+(g_A-g_V)^2\,(P_{e_d}\cdot P_n)\ (P_p\cdot P_k)\right.$$

$$\left.+(g_V+g_A)^2\,(P_{e_d}\cdot P_p)(P_n\cdot P_k)\right]$$

$$-\frac{1}{4}\,256\times 8\times \mathbf{P}_k^2\ \left(\ C_g\ -\ C_{PP}\ \left(P_\mu^2-(P_\mu\cdot P_k)\right)\ \right\}\times$$

$$\left[\left(g_A^2-g_V^2\right)\ m_nm_p\ (P_{e_d}\cdot(P_k-P_\mu))+(g_A-g_V)^2\,(P_{e_d}\cdot P_n)\ (P_p\cdot(P_k-P_\mu))\right.$$

$$\left.+(g_V+g_A)^2\,(P_{e_d}\cdot P_p)(P_n\cdot(P_k-P_\mu))\right]$$

Product of traces under the phase-space integration

$$C_g=\frac{1}{6}\frac{P_Q^2-m_e^2}{P_Q^2}\ \pi\ \frac{\left(P_Q^2-m_e^2\right)^2}{P_Q^2}$$

$$C_{PP}=\frac{1}{3}\frac{P_Q^2+2m_e^2}{(P_Q^2)^2}\ \pi\ \frac{\left(P_Q^2-m_e^2\right)^2}{P_Q^2}$$

$$P_Q^2=(P_\mu-P_\nu)^2=m_\mu^2-2E_\mu E_\nu+p_\mu E_\nu\cos(\theta_{\mu\nu})$$

Entanglement of production and detection processes:
additional term ~ ν–mass squared:
Processes with intermediate-charged lepton processes need to be investigated!

Separately for production and detection processes

source $\int\int \frac{1}{2}\mathrm{Tr}\ \gamma_\rho(1-\gamma_5)\left(\not{P}_{\overline{\nu}_e}-m_\nu\right)\gamma_\lambda(1-\gamma_5)\left(\not{P}_{e_s}+m_e\right)\times$

$$\delta\left(P_{e_s}+P_{\nu_e}-P_Q\right)\frac{d\mathbf{p}_{e_s}}{E_{e_s}}\frac{d\mathbf{p}_{\nu_e}}{E_{\nu_e}}$$

$$=\frac{1}{2}\,256\times\left(C_g\ (P_\mu\cdot P_k)\ +\ C_{PP}\left(P_\mu^2-(P_\mu\cdot P_k)\right)\ \left((P_\mu\cdot P_k)-P_k^2\right)\ \right)$$

detector $\frac{1}{2}\mathrm{Tr}\ \gamma_\sigma(g_V-g_A\gamma_5)\left(\not{P}_n+m_n\right)\gamma_\delta(g_V-g_A\gamma_5)\left(\not{P}_p+m_p\right)$

$$=\frac{1}{2}\,64\times\left[\left(g_A^2-g_V^2\right)\ m_nm_p\ (P_{e_d}\cdot P_k)+(g_A-g_V)^2\,(P_{e_d}\cdot P_n)\ (P_p\cdot P_k)\right.$$

$$\left.+(g_V+g_A)^2\,(P_{e_d}\cdot P_p)(P_n\cdot P_k)\right]$$

Nuovo Cim. 14, 322 (1937)



neutrino ↔ antineutrino oscillations

Second order process with real intermediate neutrinos

$$S + D \to \ell_\alpha^+ + \ell_\beta^+ + S' + D'$$

$$S \to S' + \ell_\alpha^+ + \nu_\alpha,\ \ \nu_\alpha \to \overline{\nu}_\beta,\ \ \overline{\nu}_\beta + D \to D' + \ell_\beta^+$$

Amplitude proportional to ν–mass

$$T_k^{\alpha\beta} = J_S^\mu(P_S', P_S) J_D^\nu(P_D', P_D) \times \overline{v}(P_\alpha;\lambda_\alpha)\gamma_\mu(1-\gamma_5) m_k \gamma_\nu u(P_\beta;\lambda_\beta)$$

Oscillation probability

$$\mathcal{P}_{\alpha\overline{\beta}}^{\rm QFT}(E_\nu, L) \equiv \left|\langle \nu_\beta | \overline{\nu}_\alpha \rangle\right|^2 = \left| \sum_{j=1}^{3} U_{\alpha j}^* U_{\beta j}^* \frac{m_j}{E_\nu} e^{-i m_j^2 L/(2E_\nu)} \right|^2$$

Replacement:

$$U_{\alpha k} \to U_{\alpha k}^*$$
$$U_{\beta m}^* \to U_{\beta m}$$

Particular process:

$$\pi^+ + p \to \mu^+ + e^+ + n$$

Production rate

$$\Gamma_{QFT}^{\pi^+ p} = \frac{1}{2\pi^2} G_\beta^2 \left(\frac{f_\pi}{\sqrt{2}}\right)^2 \frac{m_\mu^2}{m_\pi} E_\nu^2 \frac{P_{\nu_\mu \overline{\nu}_e}^{\rm QFT}(E_\nu, L)}{4\pi L^2} \left(g_V^2 + 3g_A^2\right) p_e E_e$$

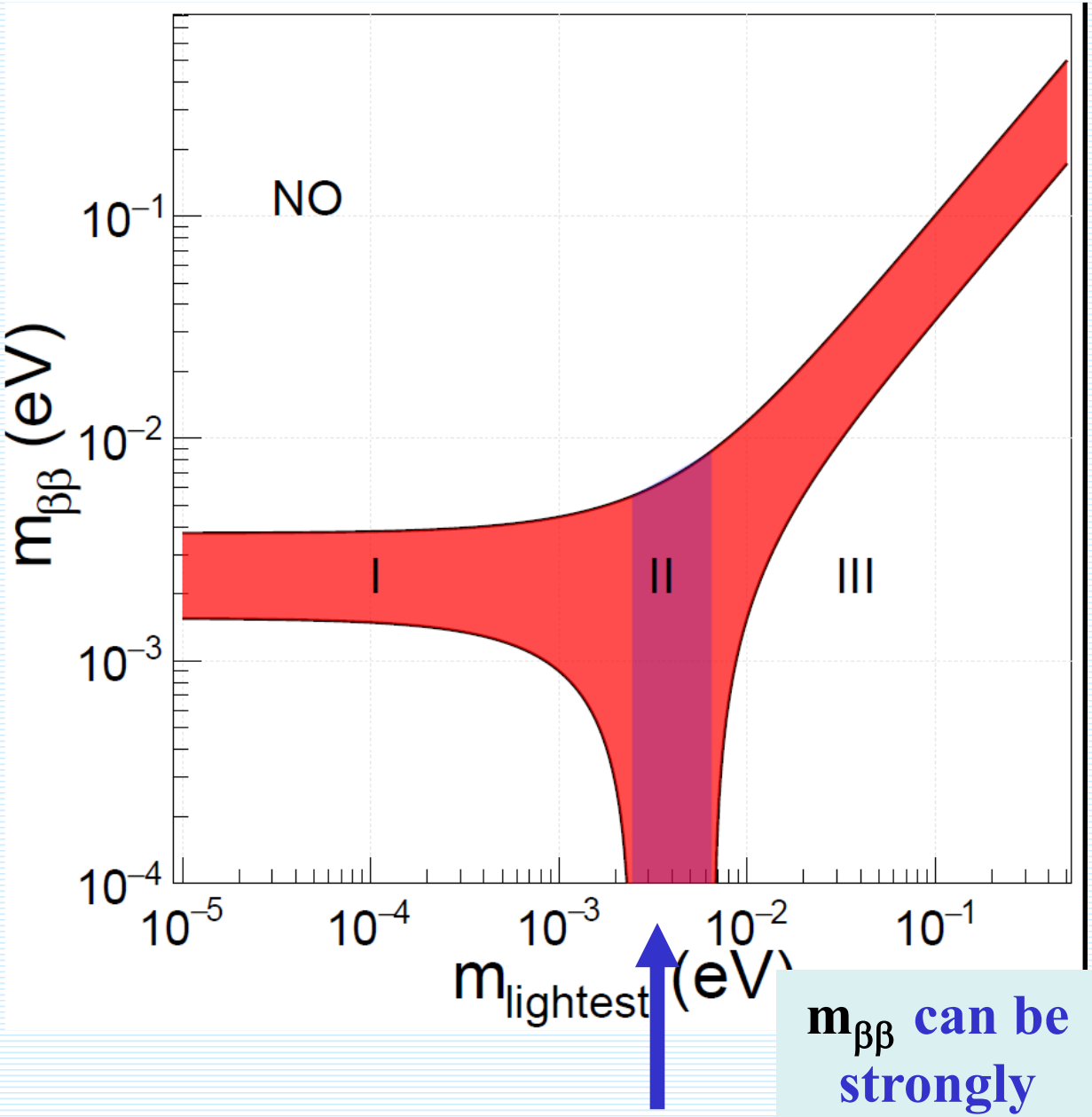
A connection of neutrino-antineutrino oscillation with 0νββ-decay

$$m^{L=0}_{ee} = m_{\beta\beta}$$

$$\mathcal{P}^{\mathrm{QFT}}_{e\bar{e}}(E_\nu, L=0) \equiv |\langle \nu_e | \overline{\nu}_e \rangle|^2 = \left| \sum_{k=1}^{3} U^*_{ek} U^*_{ek} \frac{m_k}{E_\nu} \right|^2 = \frac{(m^{L=0}_{ee})^2}{E_\nu^2} = \frac{(m_{\beta\beta})^2}{E_\nu^2}$$

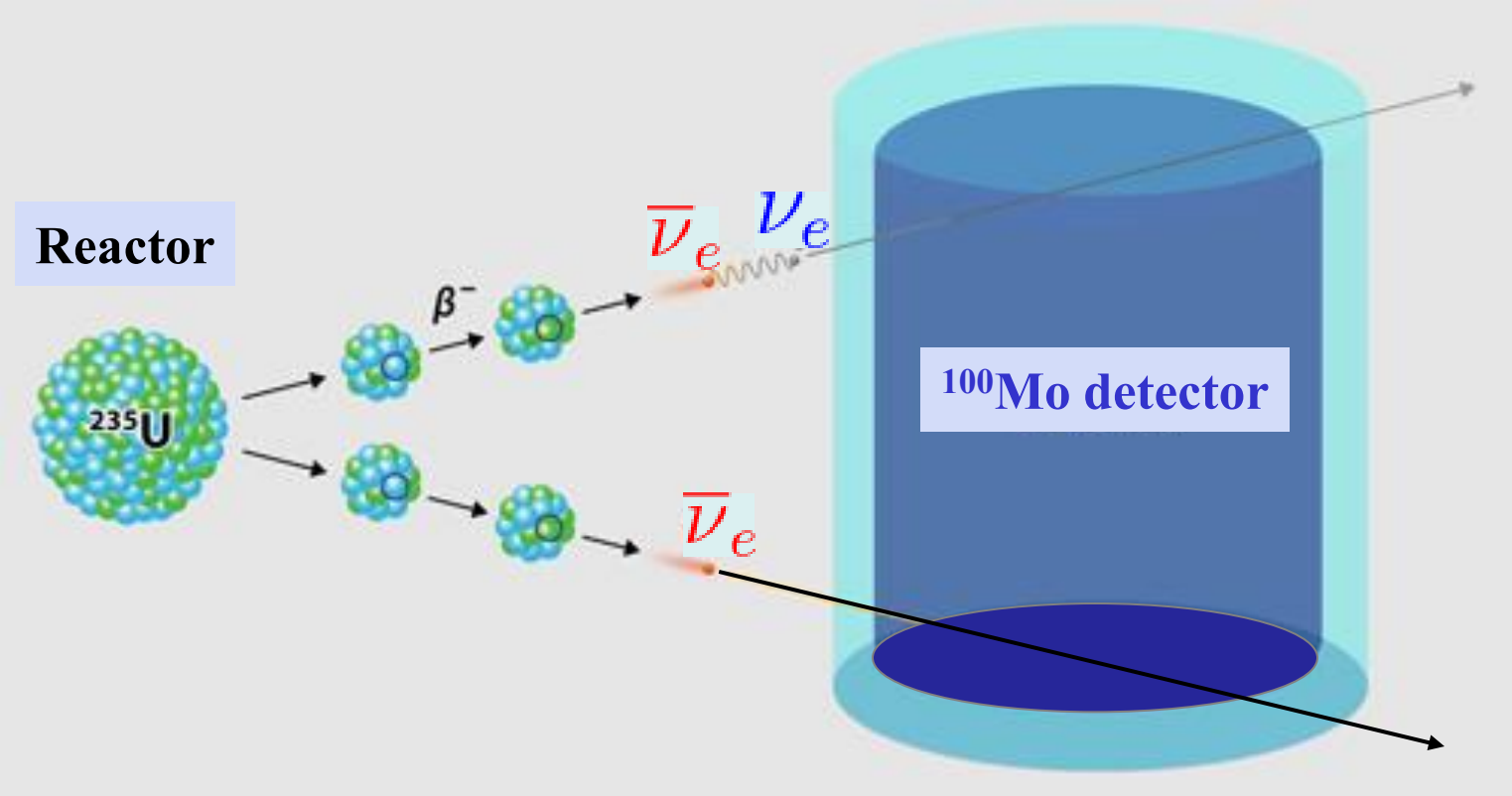


Miriam Chvostálová:
Exploring Majorana phases alongside effective Majorana masses
Monday, 5:20 PM - 5:40 PM



$m_{\beta\beta}$ can be strongly \suppressed (?!)

Oscillations of reactor antineutrinos to neutrinos



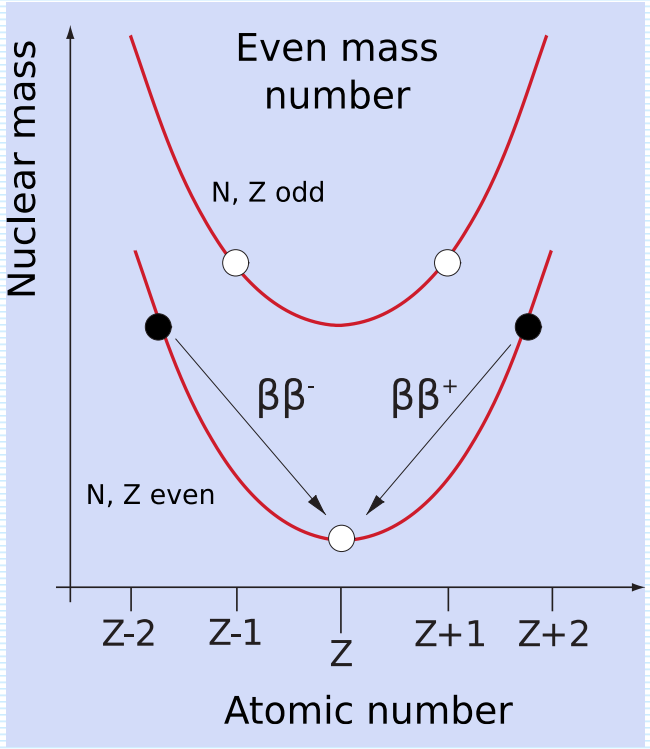
$$\nu_e + {}^{100}Mo \rightarrow {}^{100}Tc^* + e^-$$
$$^{100}Tc^* \rightarrow {}^{100}Tc + \gamma's$$
$$^{100}Tc \rightarrow {}^{100}Ru + e^- + \overline{\nu}_e$$

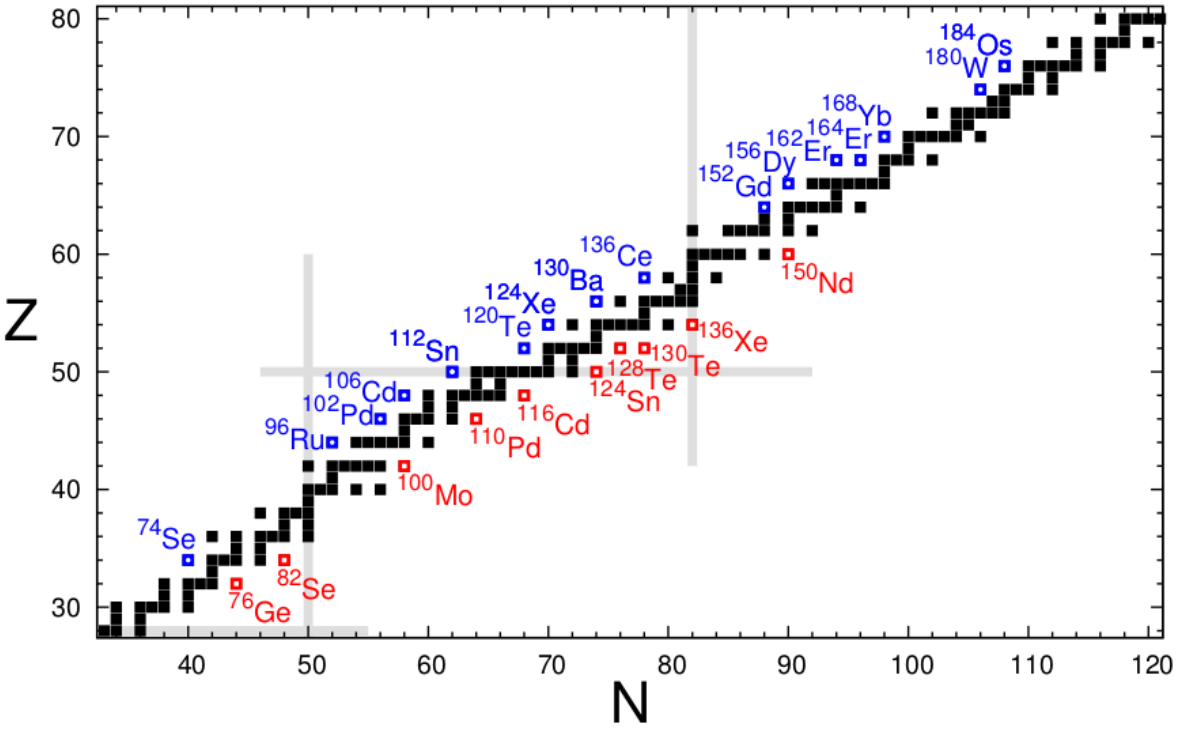
M_T = 1 ton
W = 1 Gwatt
L = 10 m
F(Z,E_e) B_{GT} = 5 x 0.33
μ ≈ 100 g mol^{-1}

$$R_L^{\bar{\nu}\nu} = 5.33 \times 10^{-13} \left(\frac{M_T}{1\text{kg}}\right) \left(\frac{1\text{g}\cdot\text{mol}^{-1}}{\mu}\right) \times$$
$$\left(\frac{m_{\beta\beta}}{1\text{eV}}\right)^2 \left(\frac{\text{W}}{1\text{GWatt}}\right) \left(\frac{1\text{m}}{\text{L}}\right)^2 F(Z,E_e) B_{GT} \left(\frac{\text{events}}{\text{s}}\right)$$

$$R_L^{\bar{\nu}\nu} \approx 10^{-6} \left(\frac{m_{\beta\beta}}{1\text{eV}}\right)^2 \left(\frac{\text{events}}{\text{year}}\right)$$

Nuclear double-β decay
(even-even nuclei, pairing int.)







Phys. Rev. 48, 512 (1935)

Two-neutrino double-β decay – LN conserved

$(A,Z) \rightarrow (A,Z+2) + e^- + e^- + \nu_e + \nu_e$

Goepert-Mayer – 1935. 1st observation in 1987



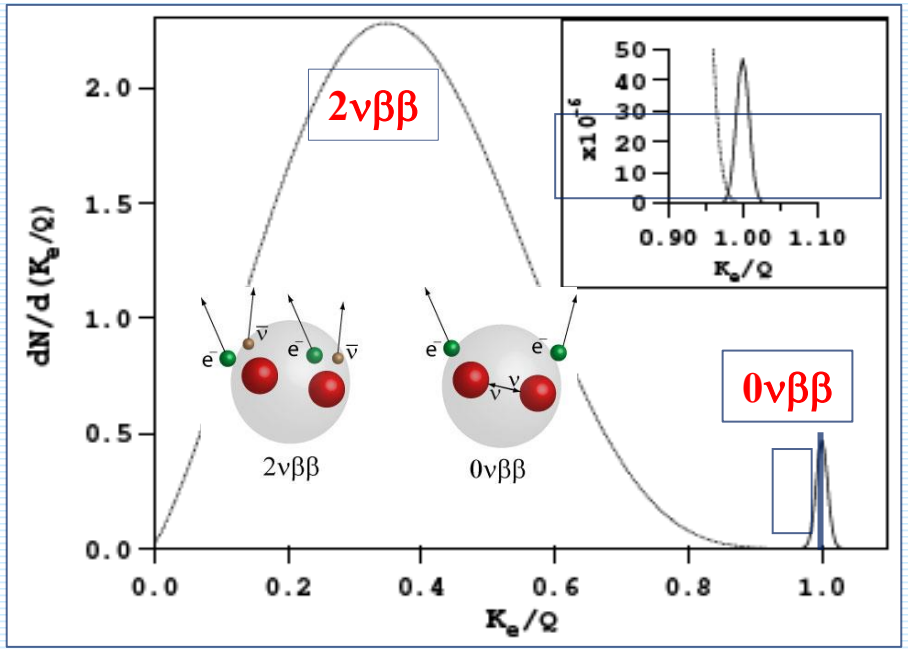
Nuovo Cim. 14, 322 (1937) Phys. Rev. 56, 1184 (1939)

Neutrinoless double-β decay – LN violated

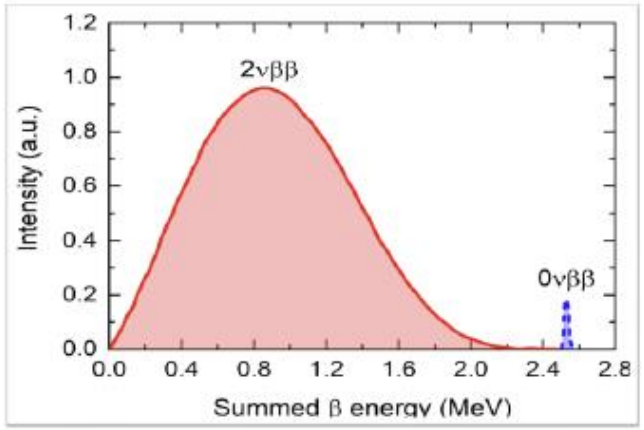
$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$ (Furry 1937)

Not observed yet. Requires massive Majorana ν's





0νββ experiments – a worldwide competition of ideas and underground physics technologies



GERDA (^{76}Ge)

$T_{1/2}^{0\nu} > 1.8 \times 10^{26}$yrs

Phys.Rev.Lett.125(2020)252502

GUORE (^{130}Te)

$T_{1/2}^{0\nu} > 2.2 \times 10^{25}$yrs

Nature 604(2022)53

CUPID-0 (^{82}Se)

$T_{1/2}^{0\nu} > 2.4 \times 10^{24}$yrs

Phys.Rev.Lett.120(2018)232502

MAJORANA (^{76}Ge)

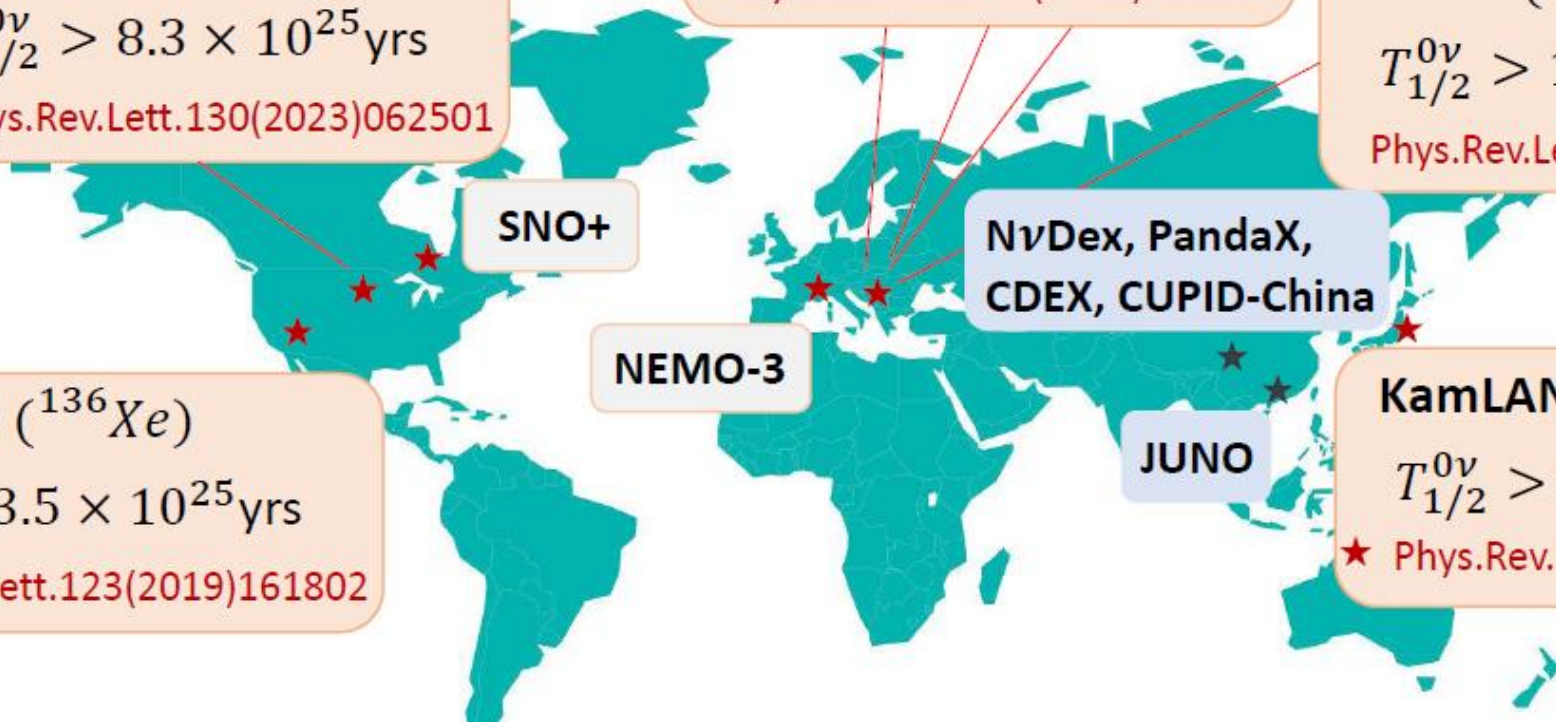
$T_{1/2}^{0\nu} > 8.3 \times 10^{25}$yrs

Phys.Rev.Lett.130(2023)062501

CUPID (^{100}Mo)

$T_{1/2}^{0\nu} > 1.5 \times 10^{24}$yrs

Phys.Rev.Lett.126(2021)181802



SNO+

NEMO-3

NνDex, PandaX, CDEX, CUPID-China

JUNO

KamLAND-Zen (^{136}Xe)

$T_{1/2}^{0\nu} > 2.3 \times 10^{26}$yrs

★ Phys.Rev.Lett.130(2023)051801

EXO-200 (^{136}Xe)

$T_{1/2}^{0\nu} > 3.5 \times 10^{25}$yrs

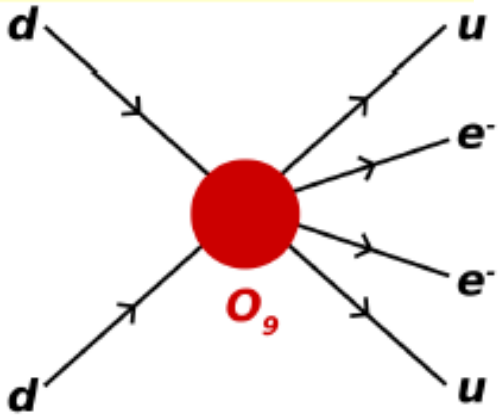
Phys.Rev.Lett.123(2019)161802

Beyond the SM physics

$$\mathcal{L} = \mathcal{L}^{(4)}_{SM} + \frac{1}{\Lambda}\sum_i c_i^{(5)}\mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2}\sum_i c_i^{(6)}\mathcal{O}_i^{(6)} + O(\frac{1}{\Lambda^3})$$

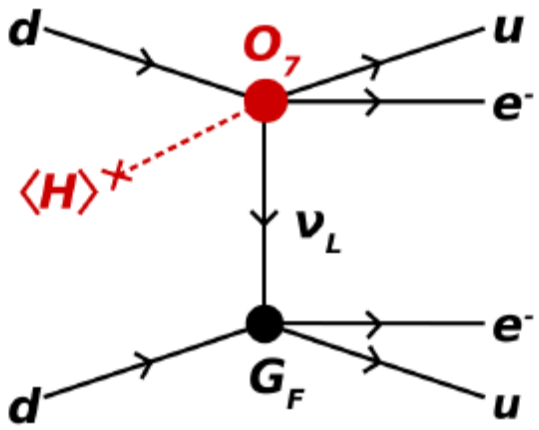
Amplitude for (A,Z)→(A,Z+2)+2e⁻ can be divided into:

short range: d=9 (d=11)



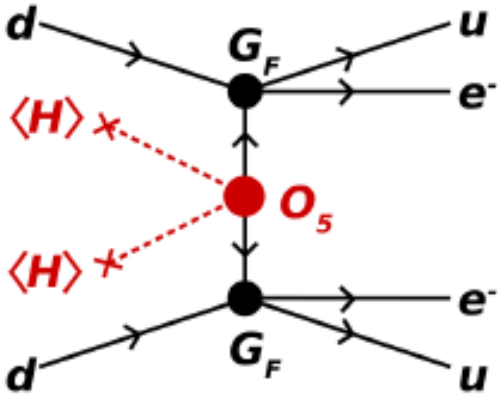
+

long range: d=7



+

mass mechanism: d=5



$$\mathcal{O}_W \propto \frac{c_{ij}}{\Lambda}(L_i H)(L_j H)$$

Weinberg, 1979

$$\mathcal{O}_2 \propto LLLe^cH$$
$$\mathcal{O}_3 \propto LLQd^cH$$
$$\mathcal{O}_4 \propto LL\bar{Q}\bar{u}^cH$$
$$\mathcal{O}_8 \propto L\bar{e}^c\bar{u}^cd^cH$$

Babu, Leung: 2001
de Gouvea, Jenkins: 2007

$$\mathcal{O}_5 \propto LLQd^cHHH^\dagger$$
$$\mathcal{O}_6 \propto LL\bar{Q}\bar{u}^cHH^\dagger H$$
$$\mathcal{O}_7 \propto LQ\bar{e}^c\bar{Q}HHH^\dagger$$
$$\mathcal{O}_9 \propto LLLe^cLe^c$$
$$\mathcal{O}_{10} \propto LLLe^cQd^c$$
$$\mathcal{O}_{11} \propto LLQd^cQd^c$$
$$\cdots\cdots$$

Valle

$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$

0νββ-decay

(LNV at ≈ GUT scale, exchange of three light ν)

$$\left(T^{0\nu}_{1/2}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 g_A^4 \left|M^{0\nu}_{\nu}\right|^2 G^{0\nu}$$

?

Phase space factor well understood

$$m_{\beta\beta} = \left|c^2_{13}c^2_{12}e^{i\alpha_1}m_1 + c^2_{13}s^2_{12}e^{i\alpha_2}m_2 + s^2_{13}m_3\right|$$

NME must be evaluated using tools of nuclear theory

Constraint from cosmology

$\Sigma = m_1 + m_2 + m_3$

< 0.90 eV

< 0.26 eV (Planck coll.)

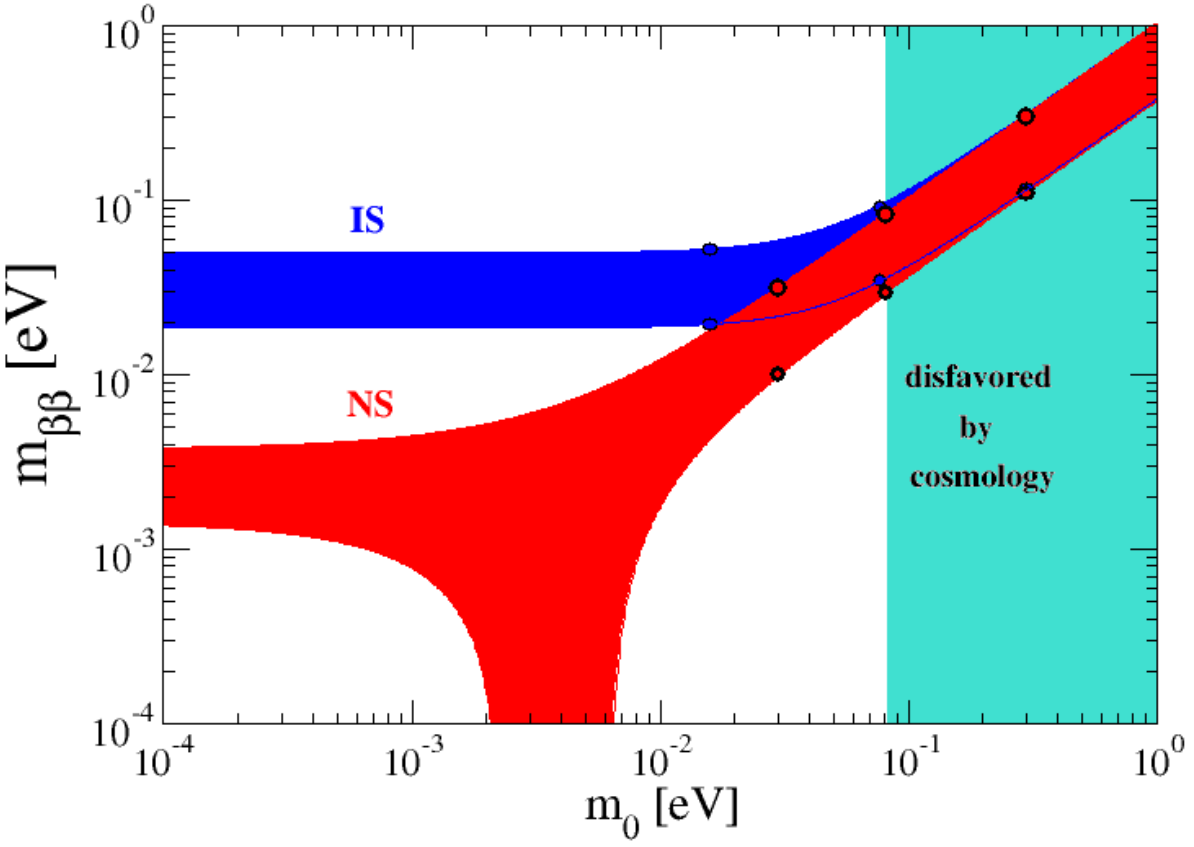
< 0.12 eV

Contrary, the constraint from 0νββ-decay (KLZ)

$m_{\beta\beta} < 0.036\text{-}0.156$ eV

implies

$\Sigma < 0.12$ eV



Quark Condensate Seesaw Mechanism for Neutrino Mass

PRD 103, 015007 (2021)

This operator contributes to the Majorana-neutrino mass matrix due to chiral symmetry breaking via the light-quark condensate.

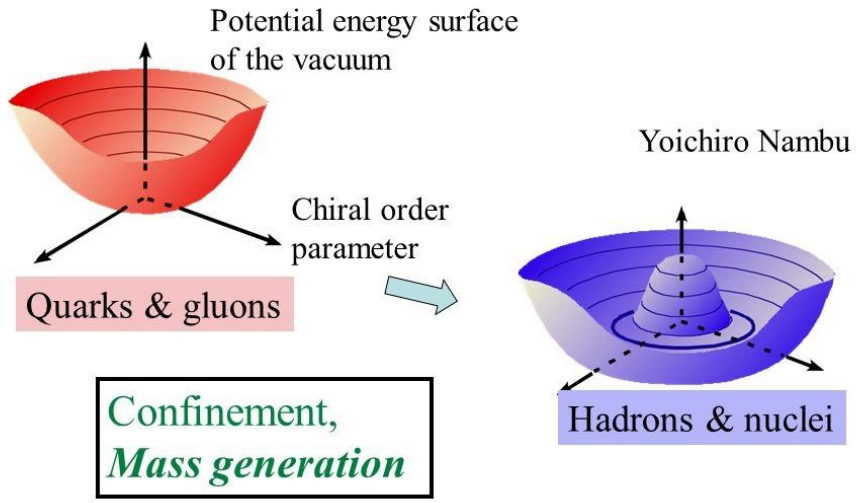
The SM gauge-invariant effective operators

$$\mathcal{O}_7^{u,d} = \frac{\tilde{g}_{\alpha\beta}^{u,d}}{\Lambda^3}\,\overline{L_\alpha^C}\,L_\beta\,H\left\{(\overline{Q}\,u_R),\,(\overline{d_R}\,Q)\right\}$$

After the EWSB and ChSB one arrives at the Majorana mass matrix of active neutrinos

$$\begin{aligned} m_{\alpha\beta}{}^{\nu} &= g_{\alpha\beta}\,v\,\frac{\langle\overline{q}q\rangle}{\Lambda^3} \\ &= g_{\alpha\beta}\,v\left(\frac{\omega}{\Lambda}\right)^3 \\ g_{\alpha\beta} &= g_{\alpha\beta}^{u} + g_{\alpha\beta}^{d}, \quad v/\sqrt{2} = \langle H^0\rangle \\ \omega &= -\langle\overline{q}q\rangle^{1/3}, \quad \langle\overline{q}q\rangle^{1/3} \approx -283\,\mathrm{MeV} \end{aligned}$$

Spontaneous breaking of ***chiral (χ) symmetry***



Λ ~ a few TeV
we get the neutrino mass in the sub-eV ballpark

The genuine QCSS scenario

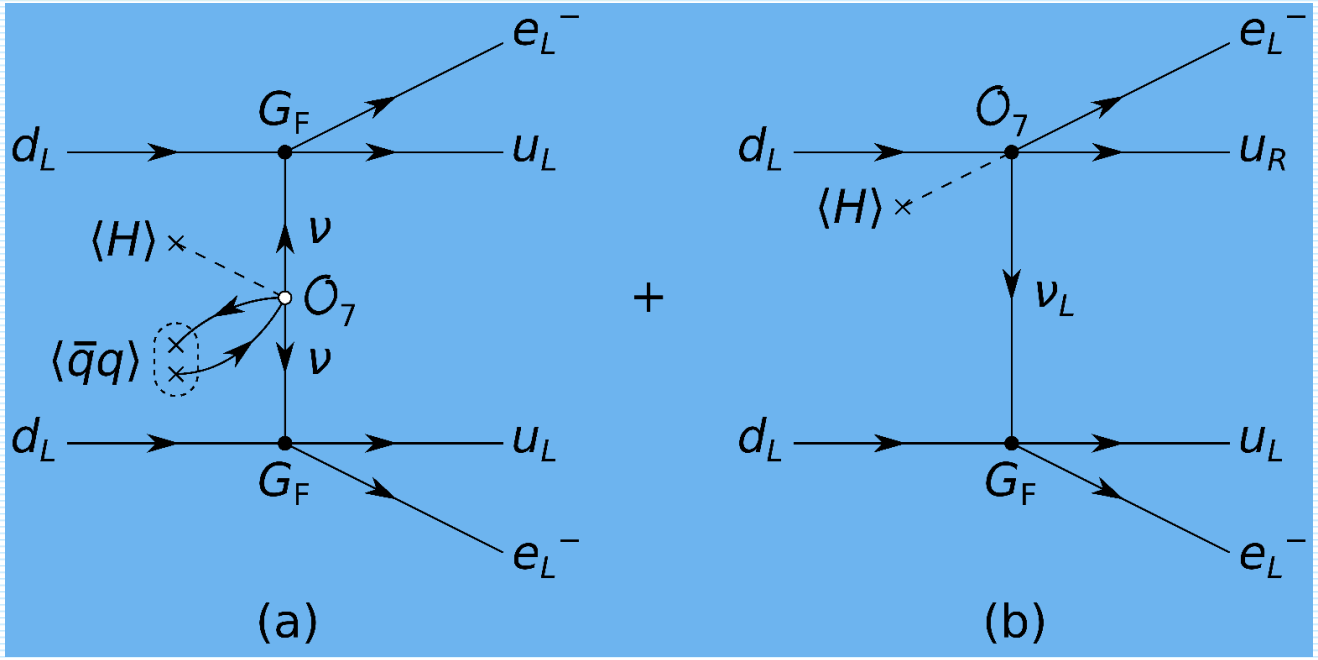
(predicts NH and ν-mass spectrum)

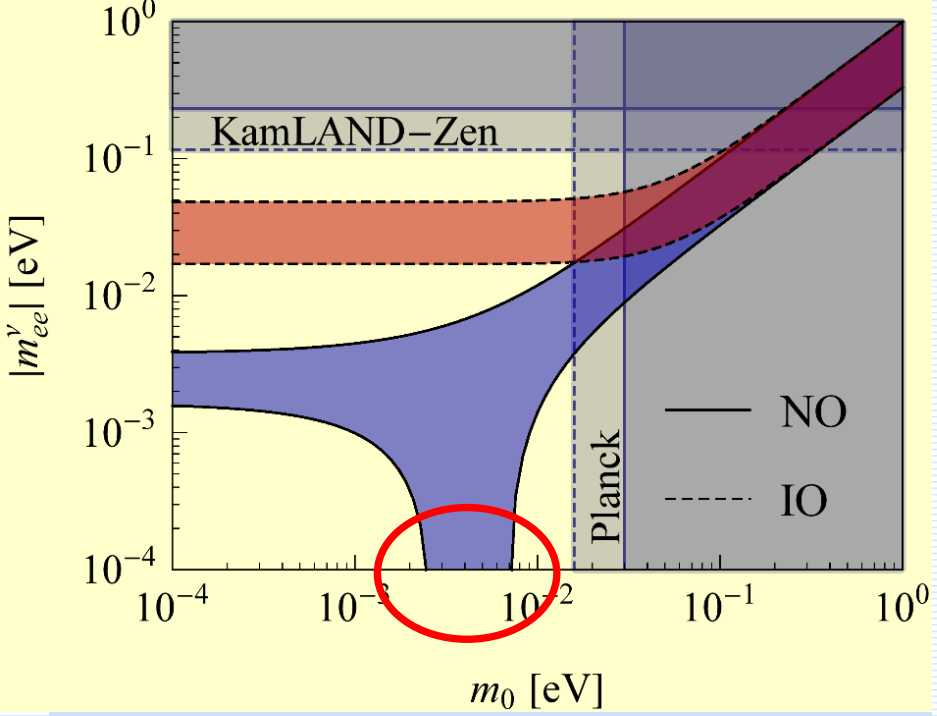
$$\mathcal{L}_7 = \frac{1}{\sqrt{2}} \sum_{\alpha\beta} \frac{v}{\Lambda^3} \overline{\nu^C_{\alpha L}} \nu_{\beta L} \left(g^u_{\alpha\beta} \overline{u_L} u_R + g^d_{\alpha\beta} \overline{d_R} d_L \right) + \text{H.c.}$$

$$m^\nu_{\alpha\beta} = -\frac{g_{\alpha\beta}}{\sqrt{2}} v \frac{\langle \bar{q} q \rangle}{\Lambda^3} = \frac{g_{\alpha\beta}}{\sqrt{2}} v \left(\frac{\omega}{\Lambda} \right)^3$$

(a) PRL 112, 142503 (2014).

(b) PLB 453, 194 (1999).





Neutrino spectrum (NH) !!!

(PRD 103, 015007 (2021))

2 meV < m_1 < 7 meV

9 meV < m_2 < 11 meV

50 meV < m_2 < 51 meV

Prediction for m_β

9 meV < m_β < 12 meV

Prediction for cosmology (Σ)

62 meV < $m_1 + m_2 + m_3$ < 69 meV

The 0νββ-decay within L-R symmetric theories (interpolating formula)

(D-M mass term, see-saw, V-A and V+A int., exchange of heavy neutrinos)

PRD 98, 015003 (2018)

$$[T^{0\nu}_{1/2}]^{-1} = \eta^2_{\nu N}\ C_{\nu N}$$

6x6 matrix: 15 angles, 10+5 CP phases
3x3 matrix: 3 angles, 1+2 CP phases

$$\mathcal{U} = \begin{pmatrix} U & S \\ T & V \end{pmatrix}$$

Effective LNV parameter (interpolating formula)

$$\eta^2_{\nu N} = \left| \sum_{j=1}^{3} \left(U^2_{ej}\ \frac{m_j}{m_e} + S^2_{ej}\ \frac{\langle p^2 \rangle_a}{\langle p^2 \rangle_a + M^2_j}\ \frac{M_j}{m_e} \right) \right|^2 + \lambda^2 \left| \sum_{j=1}^{3} \left(T^2_{ej}\ \frac{m_j}{m_e} + V^2_{ej}\ \frac{\langle p^2 \rangle_a}{\langle p^2 \rangle_a + M^2_j}\ \frac{M_j}{m_e} \right) \right|^2$$

$$\nu_{eL} = \sum_{j=1}^{3} \left(U_{ej} \nu_{jL} + S_{ej} (N_{jR})^C \right),$$

$$\nu_{eR} = \sum_{j=1}^{3} \left(T^*_{ej} (\nu_{jL})^C + V^*_{ej} N_{jR} \right)$$

$$\langle p^2 \rangle = m_p m_e \frac{M'^{0\nu}_N}{M'^{0\nu}_\nu}$$

Assumptions:
i) the see-saw structure
ii) mixing between different generations is neglected

$$\mathcal{U}_{\rm PMNS} = \begin{pmatrix} U_{\rm PMNS} & \zeta\ \mathbf{1} \\ -\zeta\ \mathbf{1} & U^\dagger_{\rm PMNS} \end{pmatrix}$$

$$m_i M_i \simeq m^2_D$$

$$M^{\rm R}_{\beta\beta} = \lambda\ \frac{\langle p^2 \rangle_a}{m^2_D} \left| \sum_{j=1}^{3} (U^\dagger_0)^2_{ej}\ m_j \right|$$

$M^R_{\beta\beta}$ depends on "Dirac" CP phase δ unlike "Majorana" CP phases α_1 and α_2

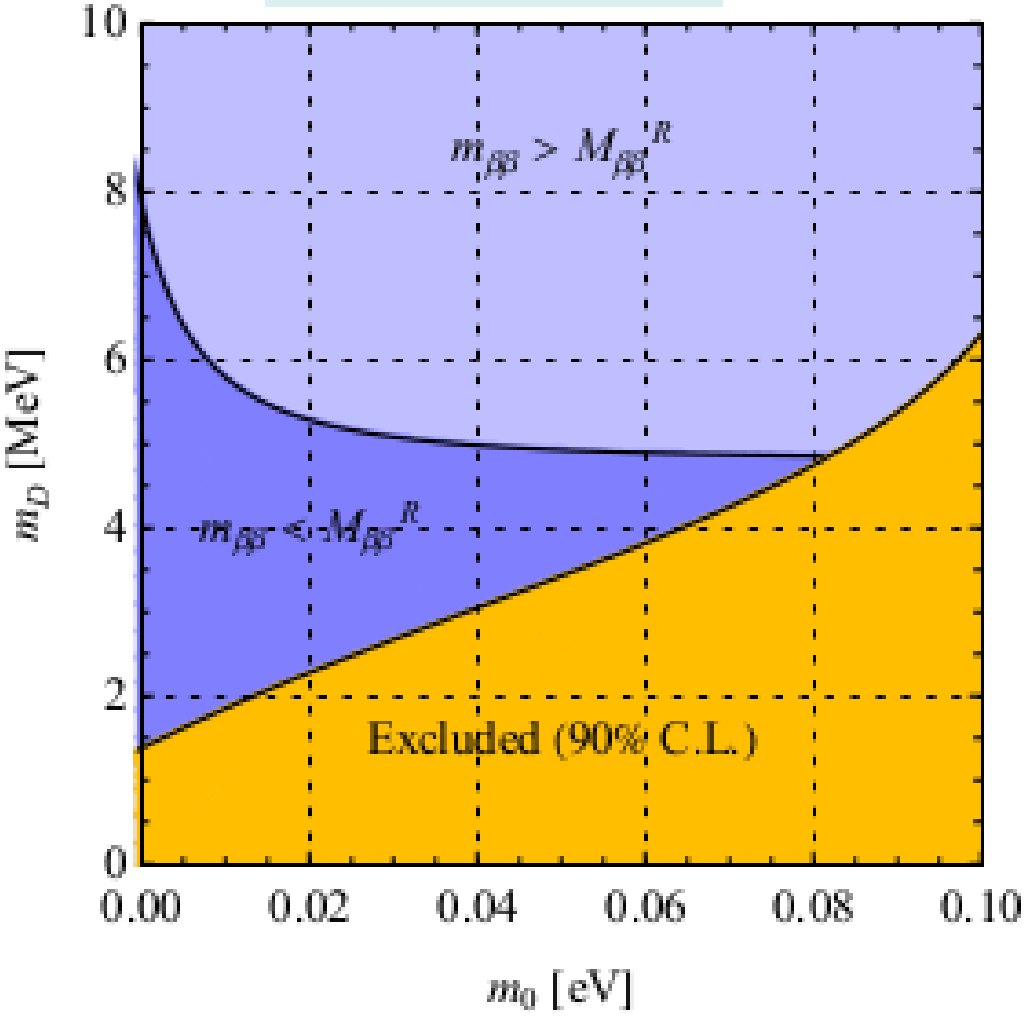
$$\eta^2_{\nu N} = \frac{1}{m_e^2}\left(m^2_{\beta\beta} + (M^R_{\beta\beta})^2\right)$$

See-saw I scenario

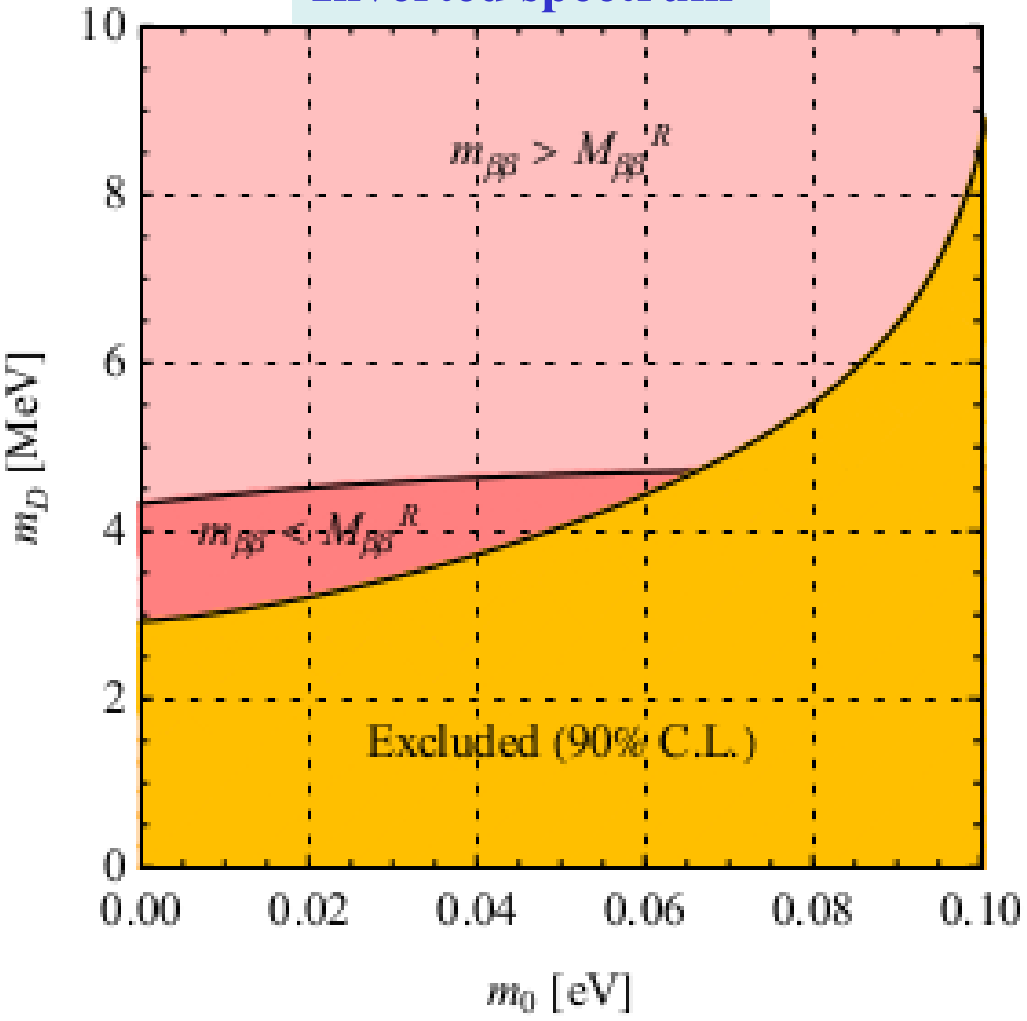
$$m_i M_i \simeq m_D^2$$

$$M^{\rm R}_{\beta\beta} = \lambda \frac{\langle p^2\rangle_a}{m_D^2}\left|\sum_{j=1}^{3}(U_0^\dagger)^2_{ej}\ m_j\right|$$

Normal spectrum



Inverted spectrum



Six Quasi-Dirac neutrinos and 0νββ-decay

Symmetry 12, 1310 (2020).

M_D - 3x3 complex matrix (18 real numb.)
$M_{L,R}$ - 3x3 symmetric matrix (12 real numb.)
(42 parameters)

Dirac-Majorana mass term

$$\mathcal{L}_m = \frac{1}{2}\left(\overline{\nu_L} \quad \overline{\nu_R^c} \right) \mathcal{M} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + h.c.$$

$$\mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix}$$

$|M_{L,R}| \ll |M_D|$

$$\mathcal{U}^T \, \tilde{\mathcal{M}} \, \mathcal{U} = \mathcal{M}$$

6 eigenvalues:
3 Dirac masses $m_{1,2,3}$, 3 Majorana mass splitting $\varepsilon_{1,2,3}$

$$m_i^{\pm} = \pm m_i + \epsilon_i$$

Diagonalization: 6x6 unitary mixing matrix
(15 mixing angles plus 15 phases)

$$\mathcal{U} = \mathcal{X} \cdot \mathcal{A} \cdot \mathcal{S}$$

Product of 3 unitary matrices.
A and S mix exclusively active and sterile neutrino flavors, each given by 3 angles and 3 phases.

$$\mathcal{A} \equiv \begin{pmatrix} U^T & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathcal{S} \equiv \begin{pmatrix} 1 & 0 \\ 0 & V^{\dagger} \end{pmatrix}$$

$$\mathcal{X} = \begin{pmatrix} 1 & X^{\dagger} \\ -X & 1 \end{pmatrix} + O(X^2)$$

X given by 9 angles and 9 phases, small parameters.

Quasi-Dirac ν a simplified scenario

(limits on neutrino masses)

The survival probability of electron antineutrinos

$$\mathcal{U}_{\rm QD} = \frac{1}{\sqrt{2}} \begin{pmatrix} U & U \\ -V^* & V^* \end{pmatrix}$$

$$m_i^{\pm} = \pm m_i + \epsilon \quad (\epsilon > 0)$$

$$P_{\bar\nu_e\to\bar\nu_e}(\epsilon \neq 0) = P_{\bar\nu_e\to\bar\nu_e}(\epsilon = 0) - \frac{\epsilon^2 L^2}{E^2}\left[c_{13}^4 c_{12}^4 m_1^2 + c_{13}^4 s_{12}^4 m_2^2 + s_{13}^4 m_3^2\right]$$
$$-\frac{\epsilon^2 L^2}{4E^2}\left[4\, c_{13}^4 s_{12}^2 c_{12}^2 \Sigma m_{21}^2 \cos\frac{\Delta m_{21}^2 L}{2E} + 4 s_{13}^2 c_{13}^2 c_{12}^2 \Sigma m_{31}^2 \cos\frac{\Delta m_{31}^2 L}{2E}\right.$$
$$\left. +4\, s_{13}^2 c_{13}^2 s_{12}^2 \Sigma m_{32}^2 \cos\frac{\Delta m_{32}^2 L}{2E}\right] + \mathcal{O}(\epsilon^4)\,,$$

Tritium β-decay

$$m_\beta = \sqrt{m_1^2 c_{12}^2 c_{13}^2 + m_2^2 c_{13}^2 s_{12}^2 + m_3^2 s_{13}^2 + \epsilon^2}$$
$$= m_\beta^{(0)}\left(1 + \frac{1}{2}\left(\epsilon/m_\beta^{(0)}\right)^2 + \ldots\right)$$

Cosmology

$$\frac{1}{2}\sum_{i=1}^{6}\left|\tilde{\mathcal{M}}_{ii}\right| = \sum_{i=1}^{3} m_i$$

Restriction from Daya-Bay data (3σ):

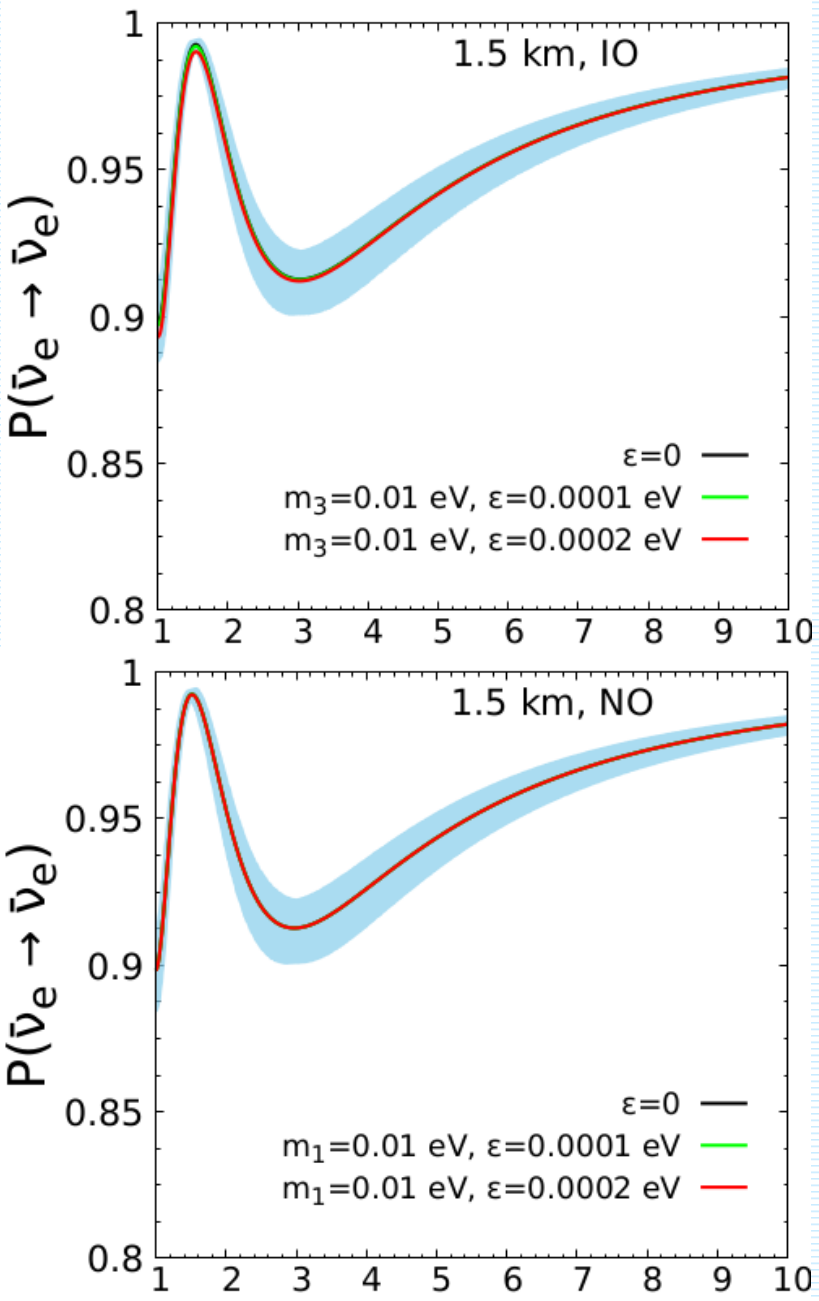
Survival probabilities with non-zero ε are the same 3ν cases.

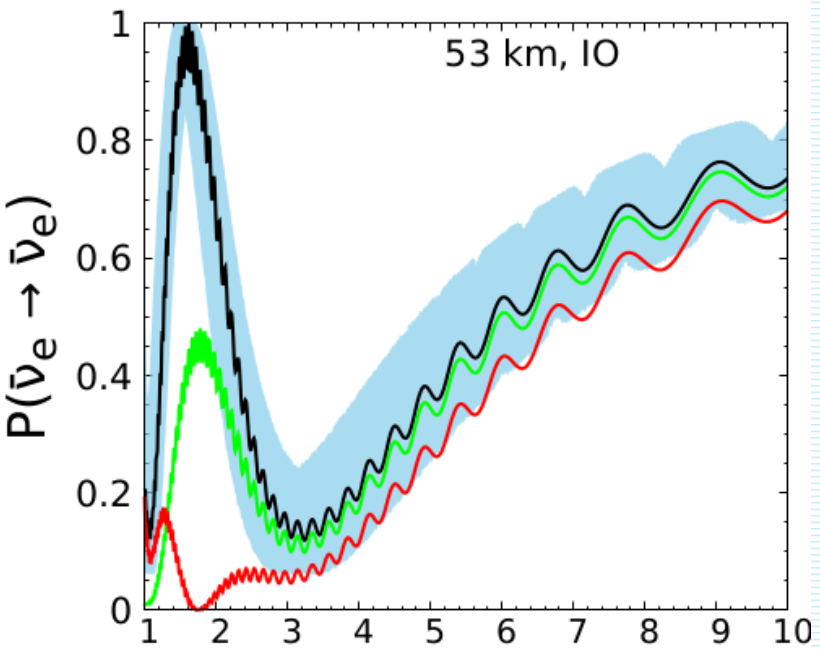
0νββ-decay

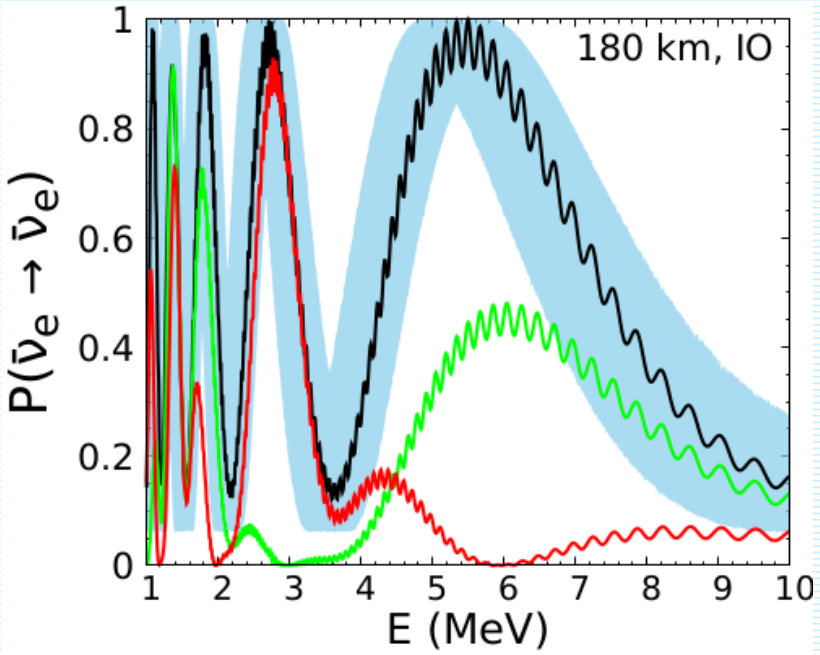
$$m_{\beta\beta} = \left[M_L\right]_{ee}$$
$$= \epsilon\,\left|c_{12}^2 c_{13}^2 + e^{2i\alpha_{21}} c_{13}^2 s_{12}^2 + e^{2i\alpha_{31}} s_{13}^2\right|$$

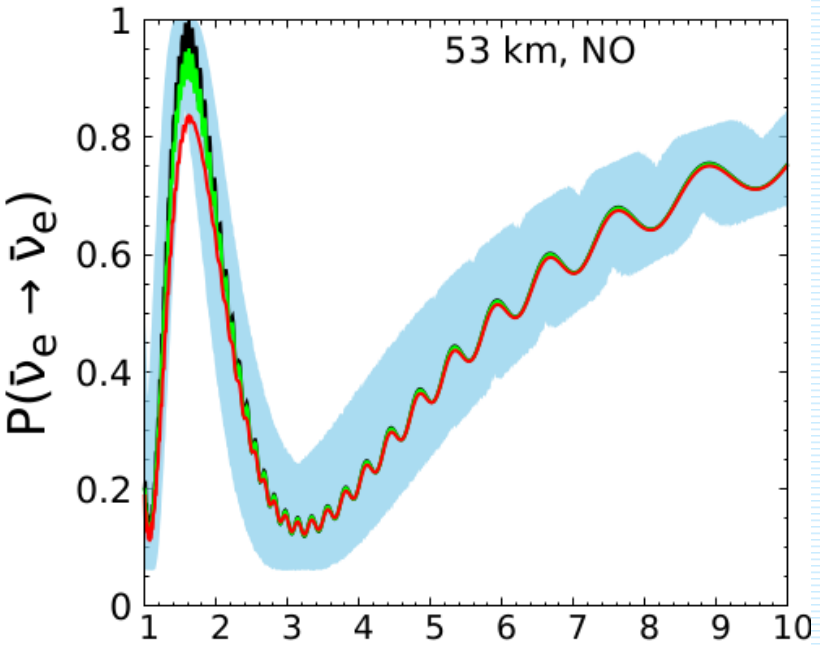
$$m_{\beta\beta} \lesssim 30 \text{ meV} \quad \text{for} \quad \text{NO}$$
$$\lesssim 1 \text{ meV} \quad \text{for} \quad \text{IO}$$

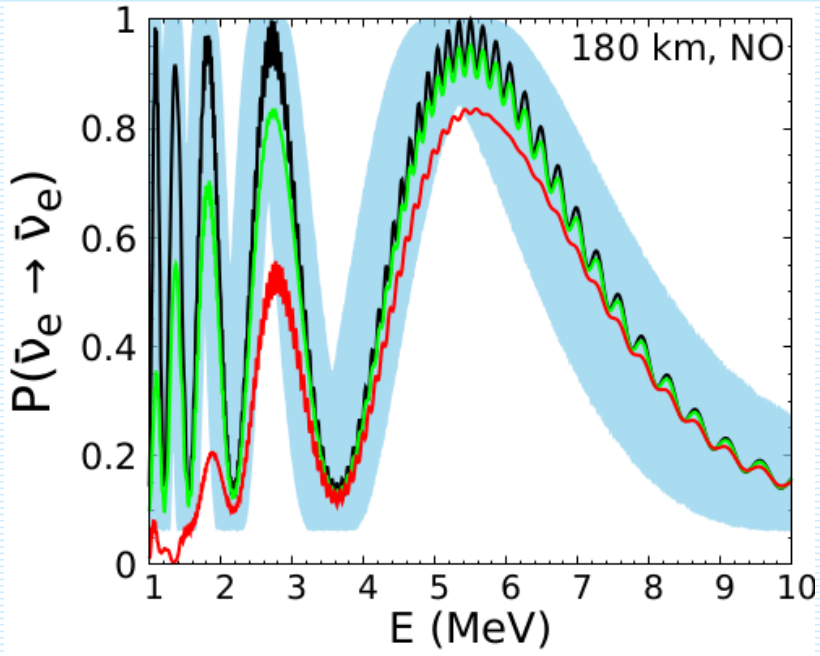
Quasi-Dirac neutrino oscillations at different distances











Towards fixing parameters of ν mass matrix

Phys. Atom. Nucl. 86 (2023) 5, 709-724
Phys. Part. Nucl. Lett. 21 (2024) 1, 1-4

Fitting 9 parameters: 6 of neutrino mixing matric plus 3 masses, assumption NH or IH

$$U^{PMNS} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\ -c_{23}s_{12} - e^{i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}c_{23}s_{13} & -e^{i\delta}c_{23}s_{12}s_{13} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Fitting 9 parmeters of 3x3 Majorana neutrino mass matrix

$$\mathbb{M}_+ = \begin{vmatrix} M^{\mathrm{M}}_{ee} & M^{\mathrm{M}}_{e\mu}\, e^{i\phi_{e\mu}} & M^{\mathrm{M}}_{e\tau}\, e^{i\phi_{e\tau}} \\ M^{\mathrm{M}}_{e\mu}\, e^{i\phi_{e\mu}} & M^{\mathrm{M}}_{\mu\mu} & M^{\mathrm{M}}_{\mu\tau}\, e^{i\phi_{\mu\tau}} \\ M^{\mathrm{M}}_{e\tau}\, e^{i\phi_{e\tau}} & M^{\mathrm{M}}_{\mu\tau}\, e^{i\phi_{\mu\tau}} & M^{\mathrm{M}}_{\tau\tau} \end{vmatrix}$$

Neutrino flavor states are projected onto mass states with Frobenius covariants

$$\mathbb{M}^{\dagger}_{+} = \mathbb{M}^{*}_{+} = \mathbb{M}_{-}$$

All the processes can be rewritten with Frobenius covariants (instead of mixing matrices)

$$\mathbb{F}_{r\pm} \equiv |r\pm\rangle\langle r\pm| = \prod_{s\neq r} \frac{\mathbb{M}_{\pm}\mathbb{M}_{\mp} - \lambda_s}{\lambda_r - \lambda_s}$$

$$p(\lambda) = \det ||\lambda - \mathbb{M}_{\pm}\mathbb{M}_{\mp}|| = 0$$

Tri-bimaximal mixing model of Majorana neutrinos

Frobenius covariants are matrix-valued projection operators constructed from a mass matrix. They isolate the individual mass eigenstates without explicitly writing the diagonalizing matrix.

ν mass matrix

$$\mathbb{M}_L = \begin{bmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{bmatrix}$$

$$x = \frac{2}{3}m_1 + \frac{1}{3}m_2$$
$$y = -\frac{1}{3}m_1 + \frac{1}{3}m_2$$
$$v = -\frac{1}{2}m_1 + \frac{1}{2}m_3$$

Frobenius covariants

$$\mathbb{F}_{1\pm} = \begin{bmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 1/6 & 1/6 \\ -1/3 & 1/6 & 1/6 \end{bmatrix}$$

$$\mathbb{F}_{2\pm} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

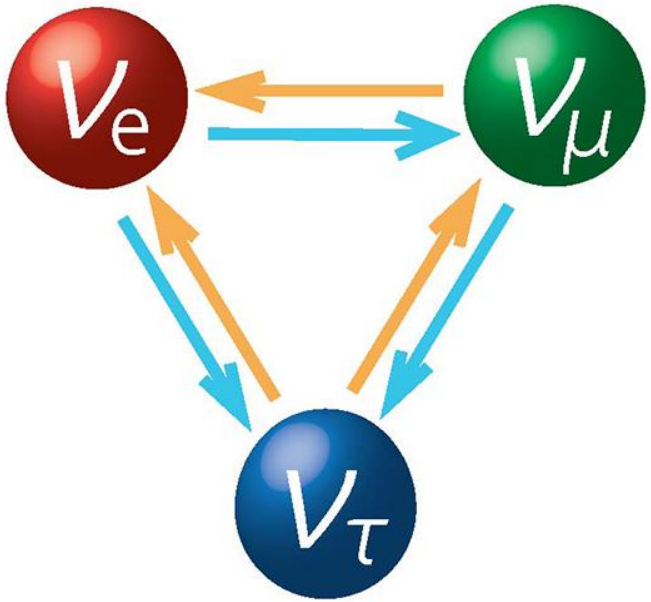
$$\mathbb{F}_{3\pm} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$$

tri-bimaximal mixing matrix

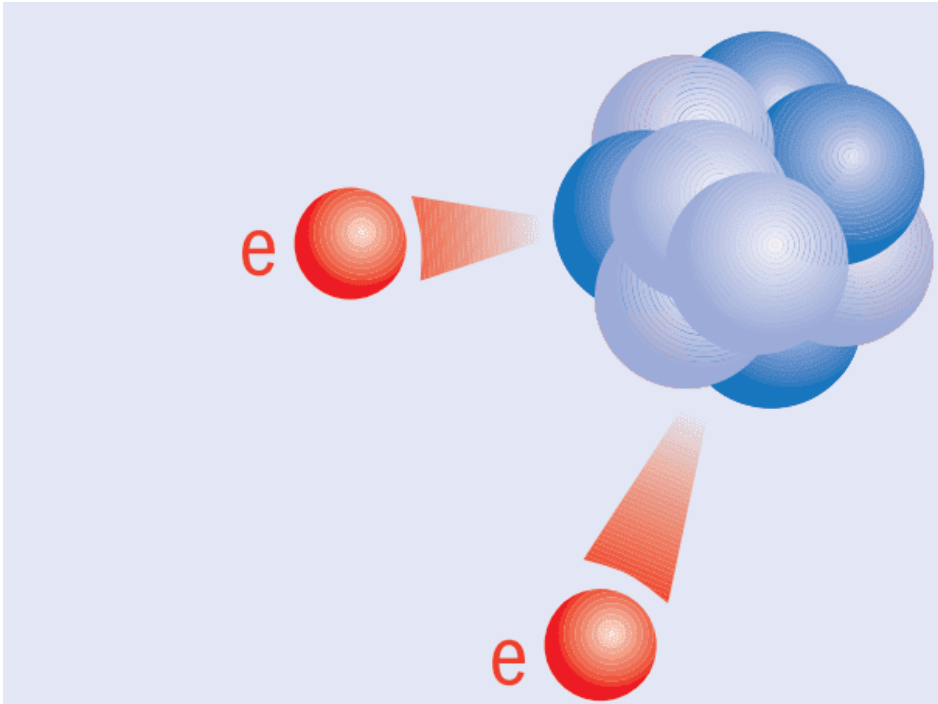
$$\mathbb{U} = \begin{bmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{bmatrix}$$

Fedor Simkovic


Time flies when
you are having fun.
Albert Einstein
quotefancy


ν_e
ν_μ
ν_τ


THANK YOU!


e
e