New BABAR precision results on $e^+e^- \to \pi^+\pi^-(\gamma)$ for $(g-2)_\mu^{HVP}$ calculations

Bogdan MALAESCU

LPNHE PARIS

anr agence nationale de la recherche AU SERVICE DE LA SCIENCE

cnrs

On behalf of the BaBar collaboration

With the IJCLab & LPNHE team:
Michel DAVIER, Anne-Marie LUTZ, Andres PINTO, Leonard POLAT, Zhiqing ZHANG

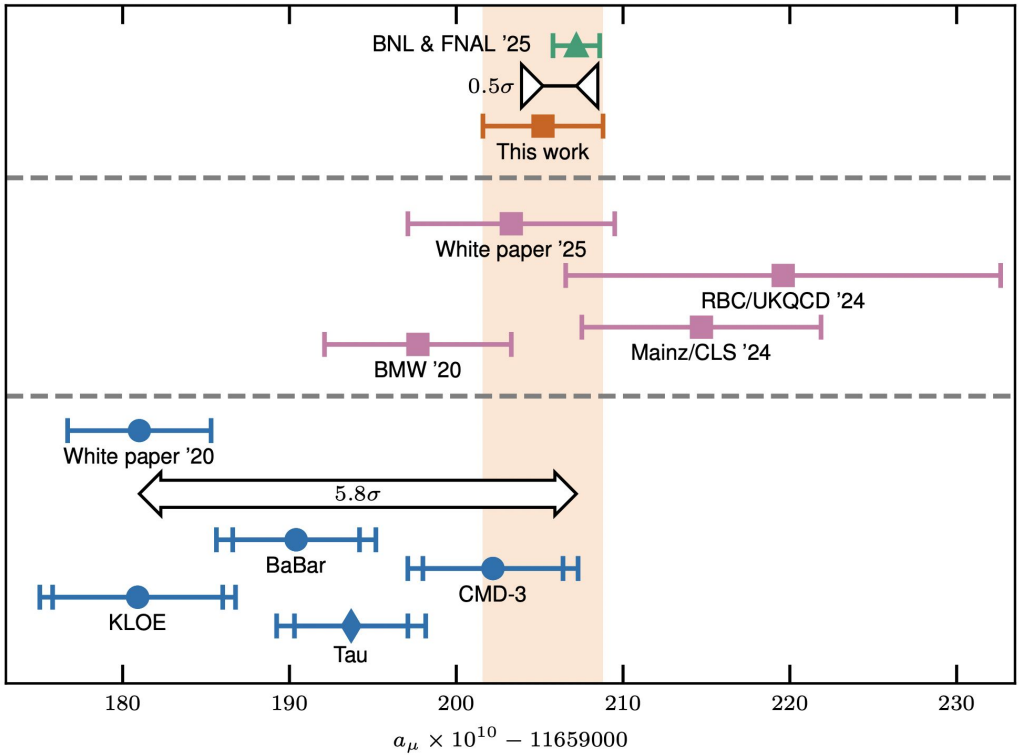


2611.04596

2611.04621

DISCRETE 2026
Bratislava
14/11/2026

Appetizer: Merging dispersive and lattice HVP calculations



Nature 653 (2026) 8114, 373-377

→ *Relatively large spread among data-driven results: tensions in the ρ peak region ($\pi^+\pi^-$ channel)*

→ *Currently most precise prediction, based on improved lattice QCD (BMW) + data-driven inputs (DMZ) at large-t (input data in good agreement at low energy)*

→ *We have an interesting, long standing, multifaceted problem to solve…*

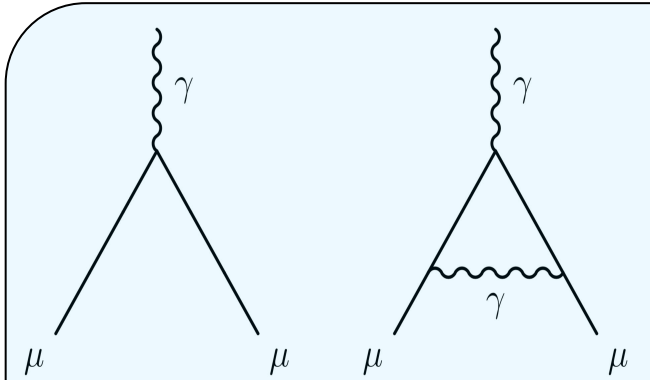
→ *Focusing here on the new BaBar ISR result for the $\pi^+\pi^-$ channel*

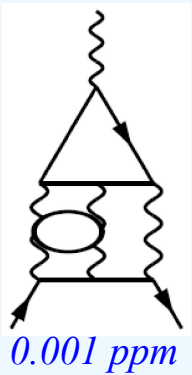
Content of the talk

- Introduction: data-based theoretical prediction for the $(g-2)_\mu$
- BaBar ISR measurement of $e^+e^- \to \pi^+\pi^- (\gamma)$
- Comparison & combination with previous BaBar measurement
- Results on a_μ and comparisons with former measurements
- *Conclusions*

Theoretical prediction

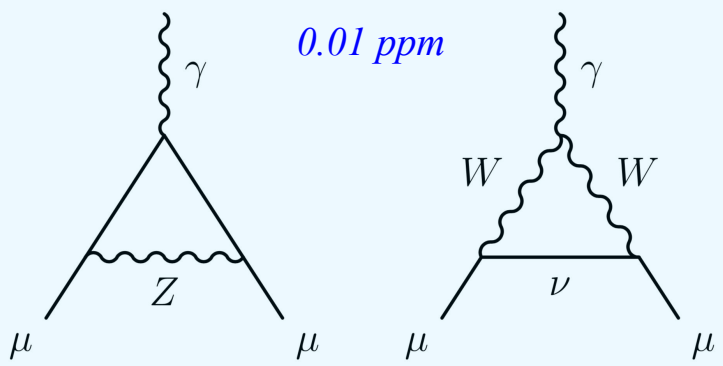
Why is it (so) complicated to compute one number ? (*very precisely*)





0.001 ppm

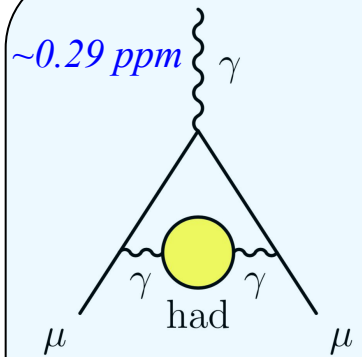
0.01 ppm



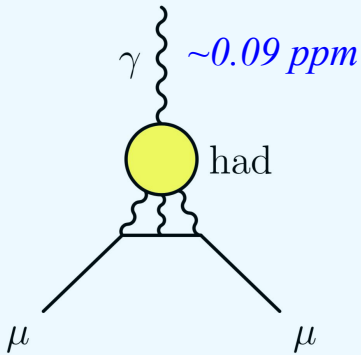
+ Many other diagrams at higher orders…

QED up to $O(\alpha^5)$ (Kinoshita et al.)

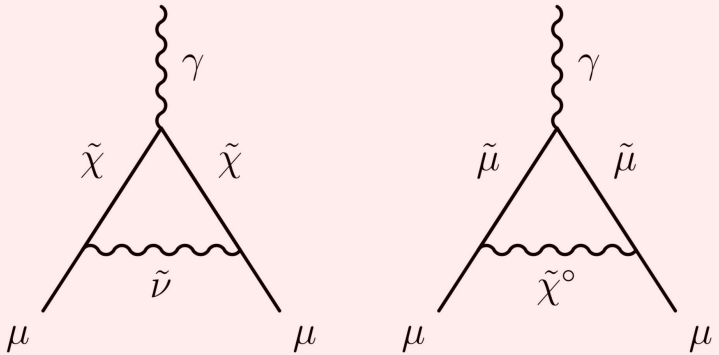
~0.29 ppm



~0.09 ppm



Dominant uncertainties: non-perturbative...



? ? ?

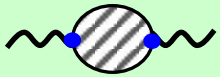
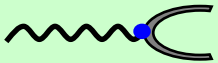
Hadronic Vacuum Polarization and Muon $(g-2)_\mu$

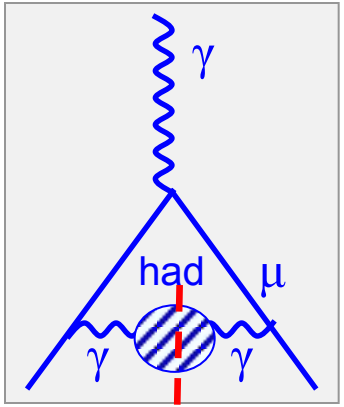
Dominant uncertainty for the theoretical prediction: from lowest-order HVP piece

Cannot be calculated from pQCD (low E-scale), but one can use experimental data on $e^+e^- \to$ hadrons cross section

Born: $\sigma^{(0)}(s) = \sigma(s)\left(\alpha/\alpha(s)\right)^2$

$$12\pi\ \mathrm{Im}\Pi_\gamma(s) = \frac{\sigma^0\ [e^+e^- \to \mathrm{hadrons}\ (\gamma_{FSR})]}{\sigma_{pt}} \equiv R(s)$$

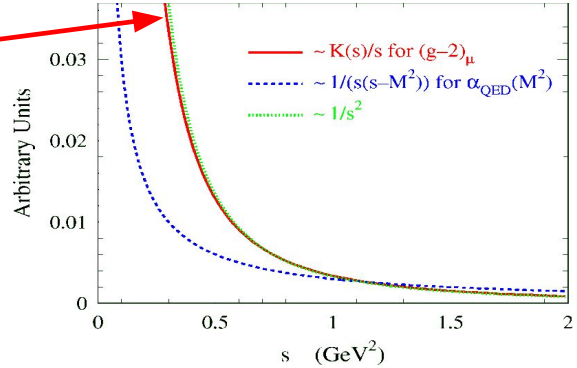
Im[] $\propto$ |  hadrons $|^2$



$$a_\mu^{\mathrm{had}} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds\ \frac{K(s)}{s} R(s)$$

Dispersion relation

Bouchiat and Michel, 1961



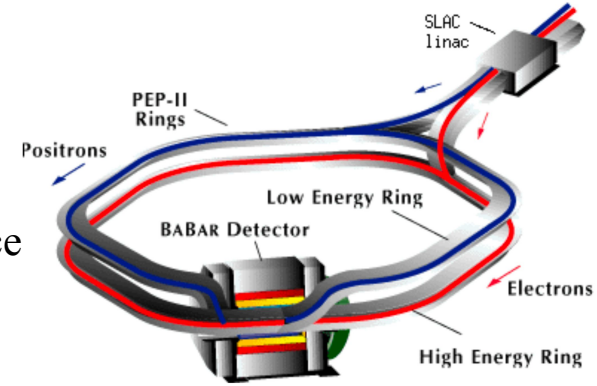
→ Precise $\sigma(e^+e^- \to \mathrm{hadrons})$ measurements at low energy are very important

$\pi\pi$ channel: 73% (~70%) of HVP contribution to a_μ (uncertainty ^{2})

→ Alternatively, one can use hadronic τ decays data + Isospin Breaking corrections

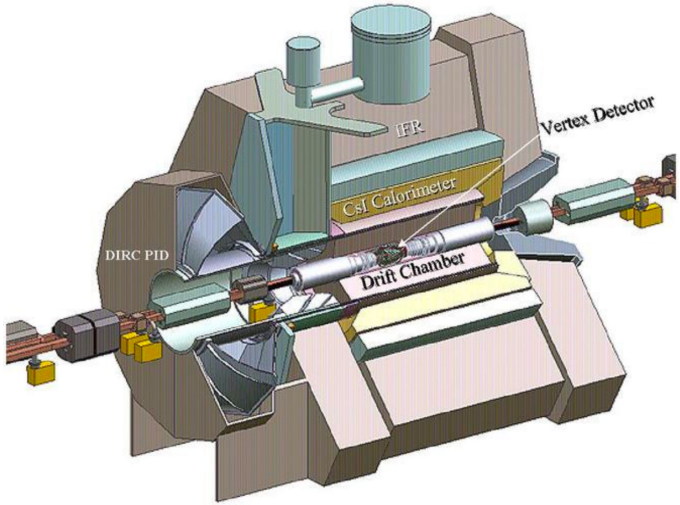
The BaBar experiment

- PEP-II collider(SLAC, USA):
  → Asymmetric e^+(3 GeV) − e^-(9 GeV)
  → From 1999 to 2008 at Υ(4S/3S/2S)
  → Collected **424 fb^{-1} at Υ(4S)** ($\sqrt{s}$=10.58GeV) + **36 fb^{-1}** off-resonance



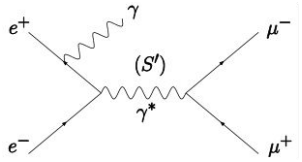
- Studied signals:
  → $e^+e^- \to \pi^+\pi^-\gamma_{ISR}(\gamma_{FSR})$
  → $e^+e^- \to \mu^+\mu^-\gamma_{ISR}(\gamma_{FSR})$
  → Simulated with Phokhara9.1

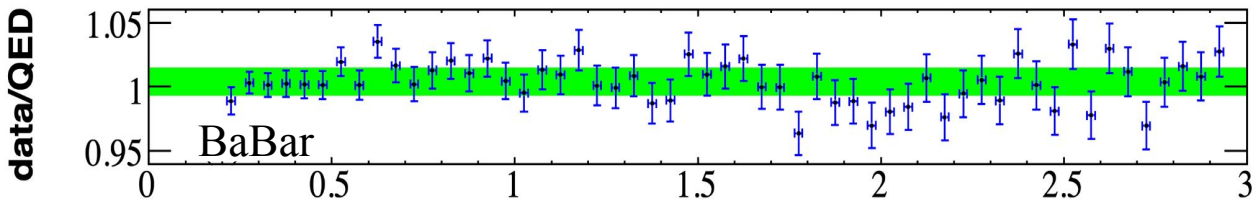
- Backgrounds:
  → $e^+e^-\gamma$, $K^+K^-(\gamma_{ISR})$, $q\bar{q}$ (u, d, s, c), $\tau^+\tau^-$,
  multipion ISR processes



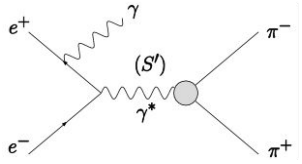
Former BaBar $\pi^+\pi^-$ result: "NLO" cross-section

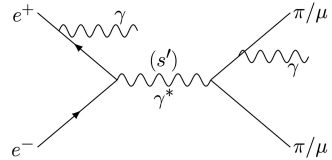
→ BaBar pioneering result: measurement including extra γ , employing the ππ / μμ ratio (cancel syst.)

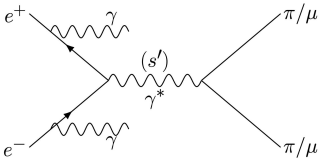


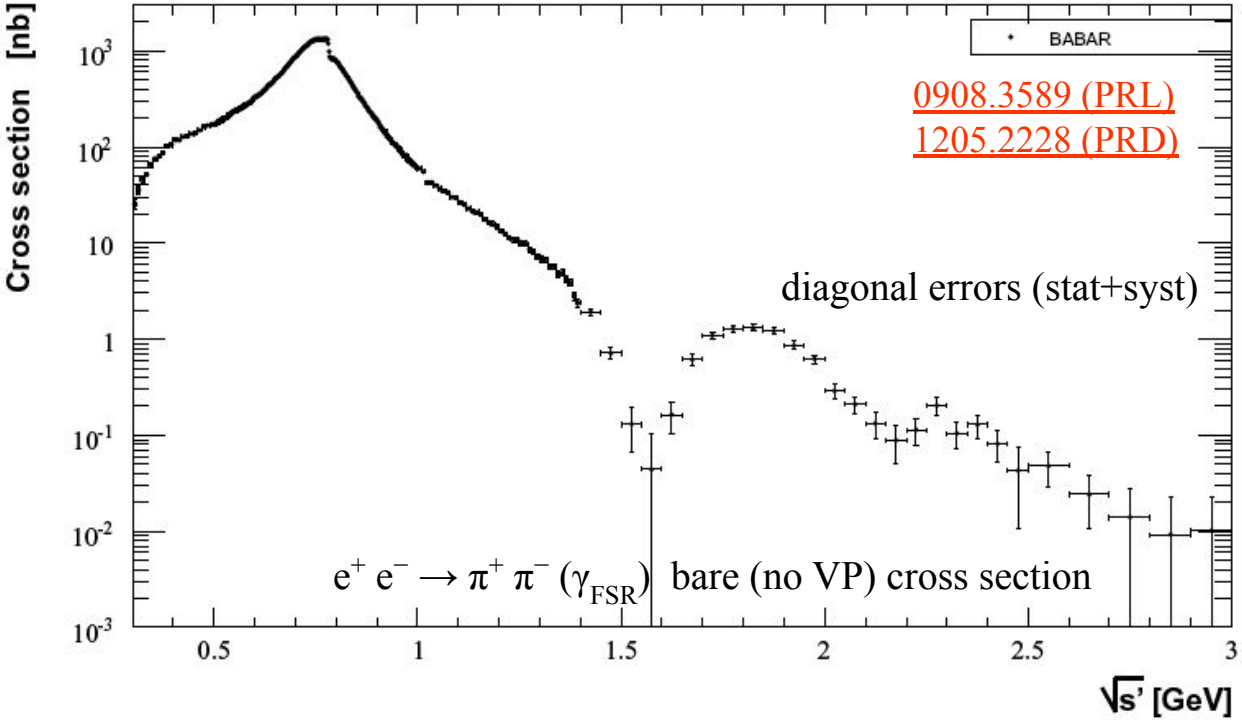


Absolute $\mu^+\mu^-$ cross section agrees with NLO QED within 1.1%

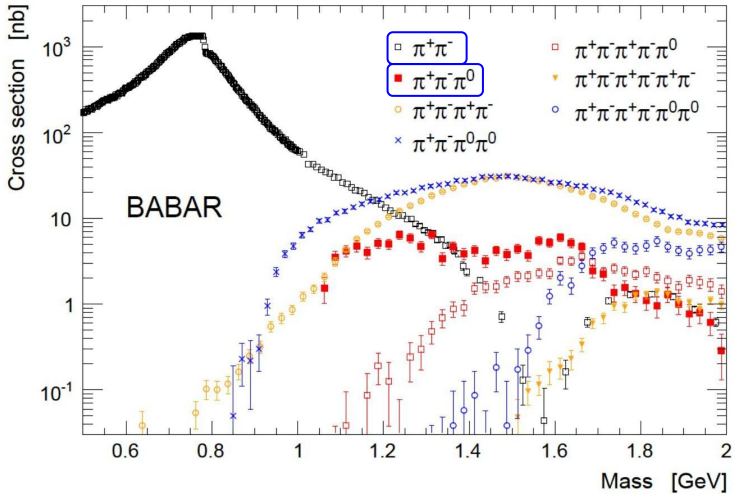


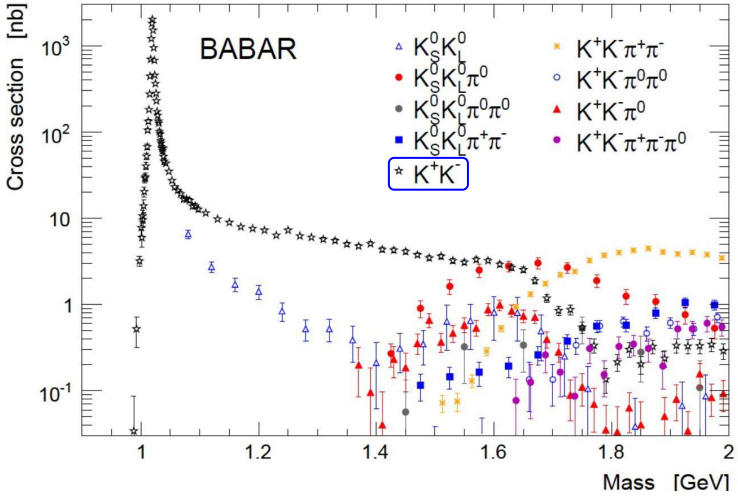


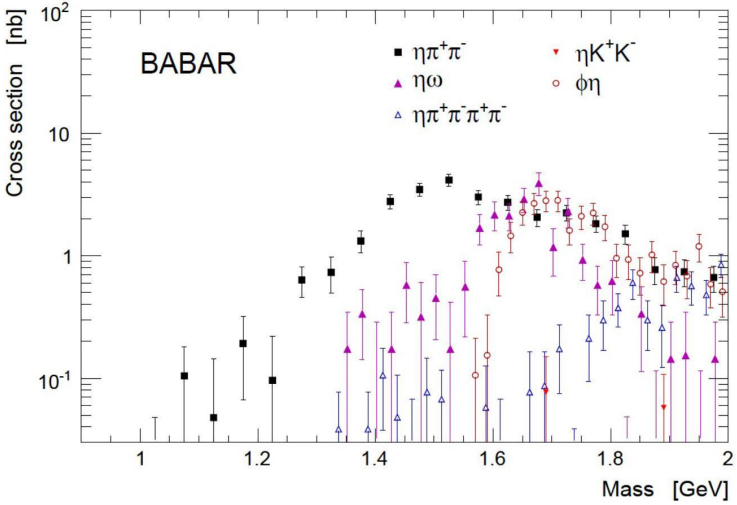




The BaBar ISR program for differential hadronic Xsec measurements



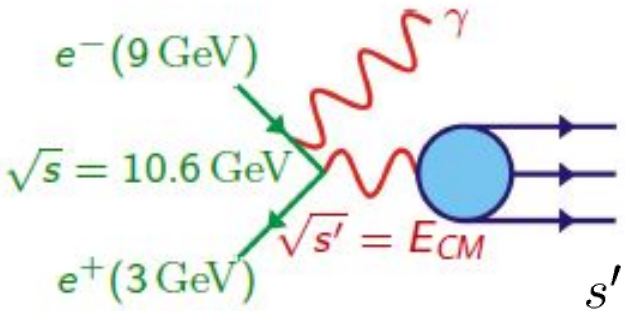




→ Large effort invested by BaBar into ISR measurements

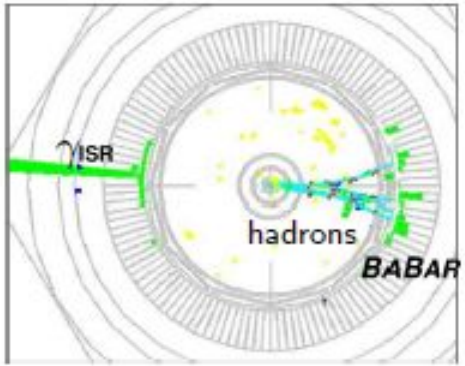
→ Developed *innovative methods* allowing to obtain a large number of *robust and precise measurements*
(only part of them shown here)

The ISR method for the $e^+e^- \to \pi^+\pi^-$ channel at BaBar

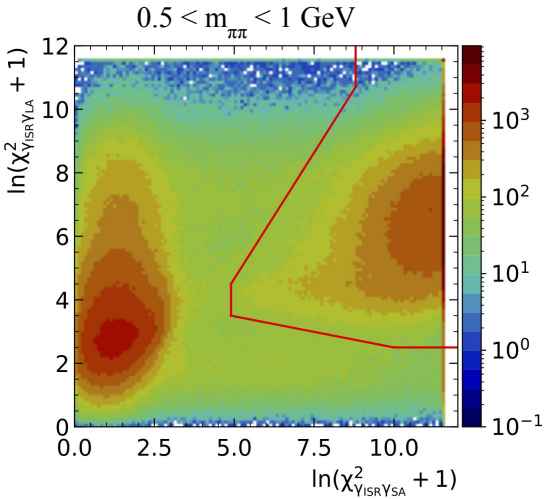


$$x = 2E^*_\gamma/\sqrt{s}$$

$$s' = s(1-x) = \mathrm{M}^2_{\mathrm{hadrons}}$$



- *High energy ISR photon (E^*_γ >4 GeV) detected at large angle*, back-to-back to hadrons
  → Defines √s' and provides strong background rejection
  → High acceptance, large boost to hadrons (measurements from threshold)
- Final state can be hadronic or leptonic (QED)
  → $\mu^+\mu^-\gamma_{ISR}(\gamma_{FSR})$, $\pi^+\pi^-\gamma_{ISR}(\gamma_{FSR})$ (and $K^+K^-\gamma_{ISR}(\gamma_{FSR})$) measured simultaneously
  → $\mu^+\mu^-\gamma(\gamma)$ for ISR luminosity: *add. ISR almost cancels for $\pi\pi\gamma(\gamma)/\mu\mu\gamma(\gamma)$*
- *Kinematic fit* including ISR photon (+ additional ISR/FSR)
  → Removes multihadronic background (BDT-based optimisation) (*Backup*)
  → Improves mass resolution (a few MeV)
- *Data/MC corrections* for efficiencies and acceptance
- Continuous measurement in the energy region of interest for a_μ
  → *Reduced systematic uncertainties* compared to multiple data sets with different colliders and detectors



The 2009 & 2026 BaBar analyses for $e^+e^- \to \pi^+\pi^-$

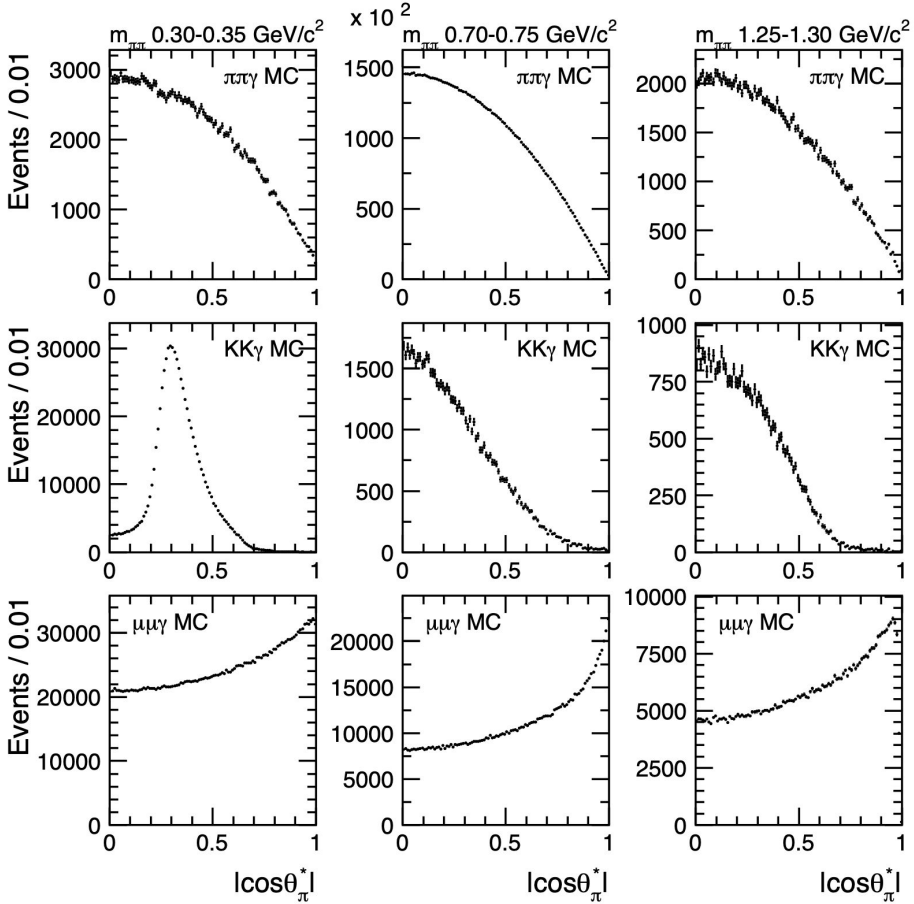
2009 (Phys.Rev.Lett. 103 (2009) 231801, Phys.Rev.D 86 (2012) 032013):

- Data: partial 232 fb−1 (Y(4S))
- π / μ / K separation: particle ID (dominant syst.)
- Track cut: $p > 1$ GeV/c
- Signal MC: AfkQED
- Systematic uncertainty ~ 0.5% for (0.5 - 1 GeV/c^2)

2026 (arXiv:2611.04596 , arXiv:2611.04621):

- Data: full 460 fb^{-1} (×2 larger luminosity)
- π / μ / K separation: angular fit on $|\cos\theta^*|$
  (no PID - larger statistics & reduced syst. unc.)
  New method proposed by M. Davier (2017)

Phokhara, reconstructed-level templates



θ^*: angle between negative charged track and γ_{ISR} in 2-track CM frame

The 2009 & 2026 BaBar analyses for $e^+e^- \to \pi^+\pi^-$

2009 (Phys.Rev.Lett. 103 (2009) 231801, Phys.Rev.D 86 (2012) 032013):

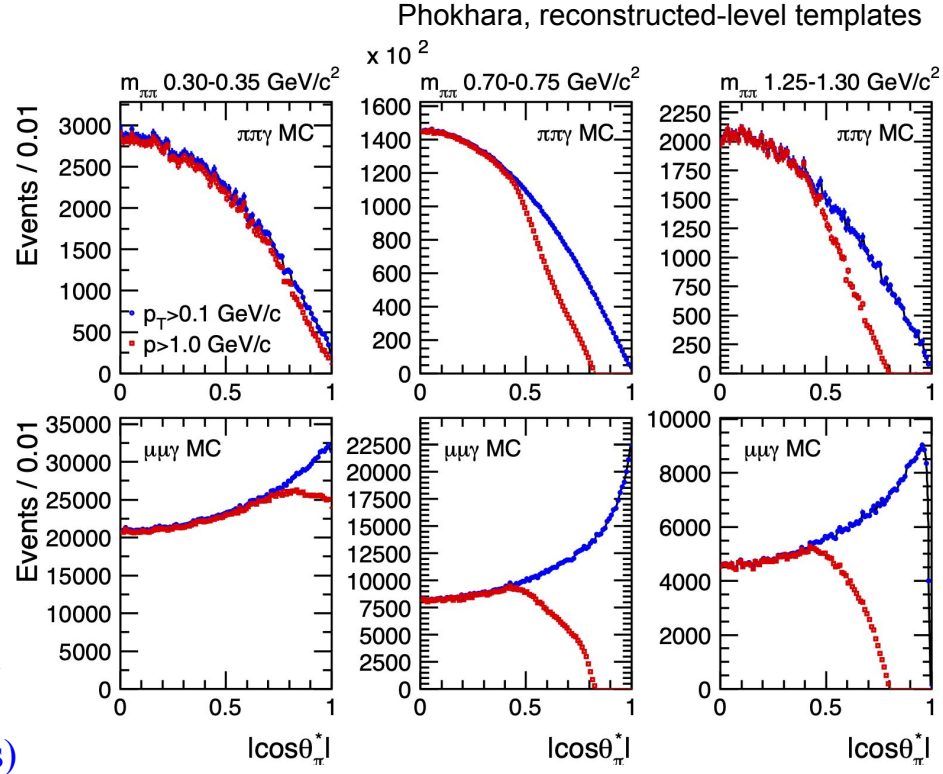
- Data: partial 232 fb−1 (Y(4S))
- π / μ / K separation: particle ID (dominant syst.)
- Track cut: p > 1 GeV/c
- Signal MC: AfkQED
- Systematic uncertainty ~ 0.5% for (0.5 - 1 GeV/c^2)

2026 (arXiv:2611.04596 , arXiv:2611.04621):

- Data: full 460 fb^{-1} (×2 larger luminosity)
- π / μ / K separation: angular fit on $|\cos\theta^*|$ (no PID - larger statistics & reduced syst. unc.)
  New method proposed by M. Davier (2017)
- Track cut: p_T > 0.1 GeV/c, higher statistics
- Signal MC: Phokhara9.1

→ Largely *independent methods*: complementary results

→ Blinding: multiplicative factors applied to mass spectra
3 independent factors for ππ spectra + 3 for μμ
(Tracking & Trigger eff.; Event yields from angular fits)



θ^*: angle between negative charged track and γ_{ISR} in 2-track CM frame

Overview of the 2026 analysis steps

- Kinematic fits of additional radiation → ***2-d χ^2-based selection*** against background

- ***Angular fit*** of data, with data/MC corrections → ππ and μμ mass spectra
  → 6 ***blinding*** factors applied

- Conclusive comparison of μμ spectrum shape vs. simulation
  → ***Unblinding of μμ***: compare normalisation with QED prediction

- Successful QED test
  → μμ spectrum unfolded to $\sqrt{s'}$, i.e. center-of-mass energy of final state, including FSR
  → ***ISR luminosity***

- ππ spectrum unfolded
  → ***cross section*** ($\sqrt{s'}$), blinded contribution to a_μ computed: evaluate uncertainties

- After approval from the collaboration, ππ cross section and contribution to a_μ are unblinded

→ *New independent blinding factors applied for analysis improvements w.r.t. preliminary ('25) result*
(*Backup*)

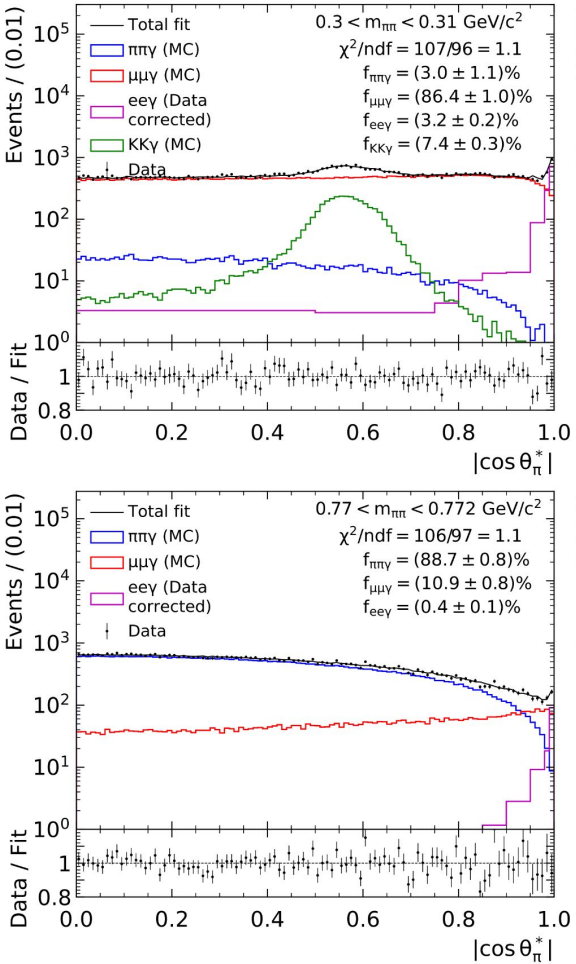
Methodology of angular fits

→ $|\cos\theta^*|$ template fits on background-subtracted data distributions, in bins of $m_{\pi\pi}$
(2 MeV/c^2 mass bins between 0.5-1 GeV/c^2, 10 MeV/c^2 elsewhere)

- Templates for $\pi\pi\gamma$, $\mu\mu\gamma$ and $KK\gamma$ from MC simulation + small data/MC corrections
- Data-driven templates for $ee\gamma$, using cut-based and BDT selections (*Backup*)

- 3 sequential fits for $|\cos\theta^*|$ ranges: [0;1] , [0.9;1] and [0;0.9]
  → Accounting for different fractions of contributions therein, mitigates the systematic uncertainty related to the $ee\gamma$ background

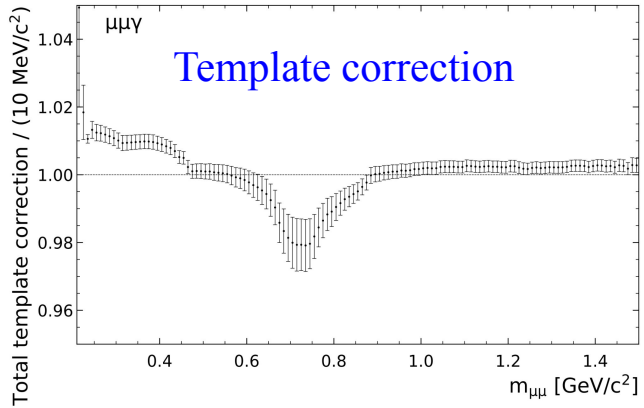
→ Performed closure test of accuracy of the fit, on mix of simulations from different channels:
Confirmed that resulting spectra are consistent with original inputs.

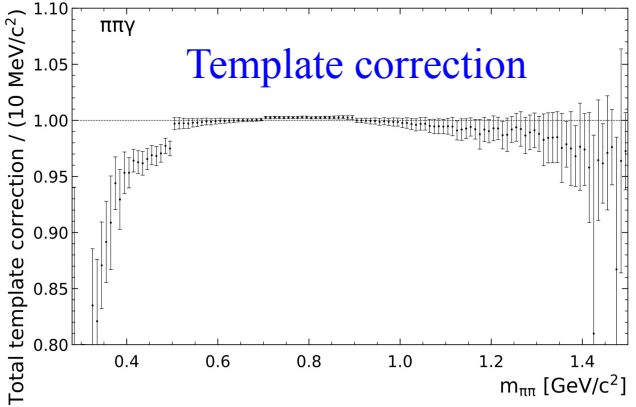


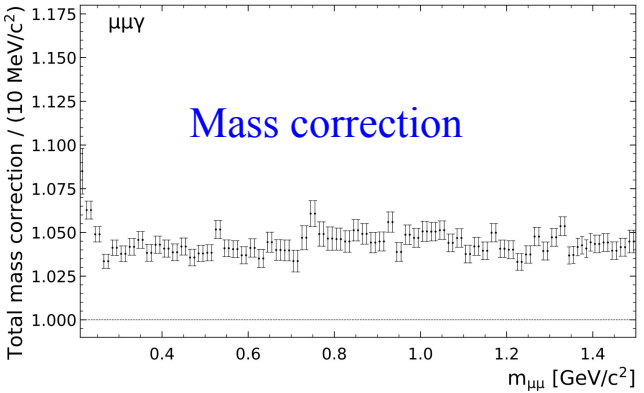
Corrections to simulation

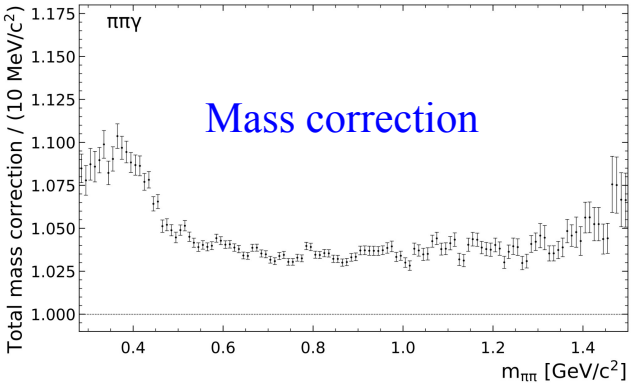
- Efficiency corrections, due to data/MC differences, determined for $|\cos\theta^*|$ templates and fitted mass spectra for e.g. 2-d χ^2 selection, trigger and tracking efficiencies (*corrections initially blinded*)
- Fits have high sensitivity to template shapes, hence to data/MC corrections: *performed extensive studies* to mitigate potentially large systematic uncertainties

$$\varepsilon = \varepsilon_{\rm MC}\left(\frac{\varepsilon^{\rm data}_{\rm trigger}}{\varepsilon^{\rm MC}_{\rm trigger}}\right)\left(\frac{\varepsilon^{\rm data}_{\rm tracking}}{\varepsilon^{\rm MC}_{\rm tracking}}\right)\left(\frac{\varepsilon^{\rm data}_{V_{xy}}}{\varepsilon^{\rm MC}_{V_{xy}}}\right)\left(\frac{\varepsilon^{\rm data}_{\chi^2}}{\varepsilon^{\rm MC}_{\chi^2}}\right)$$





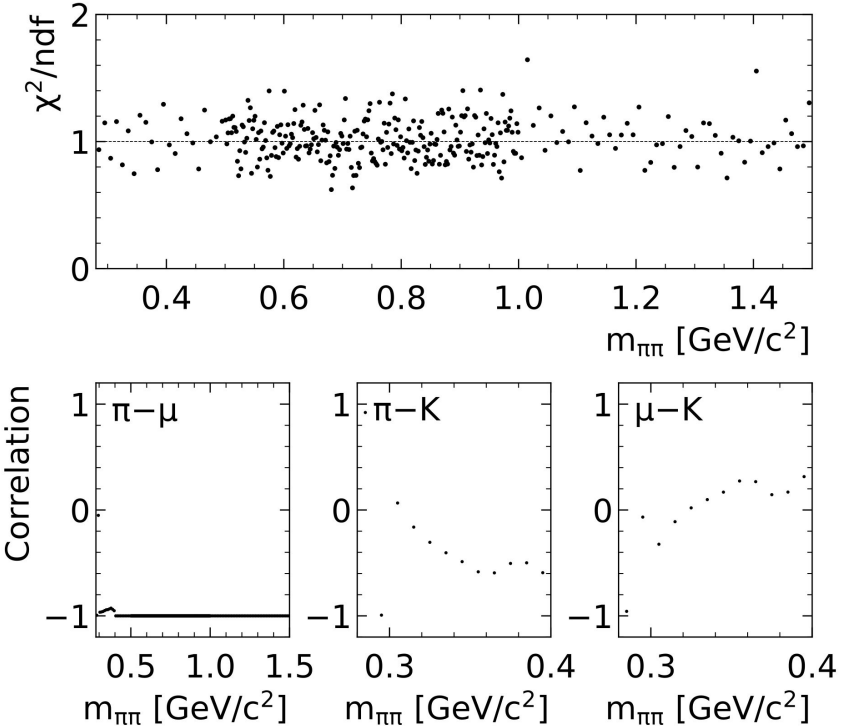


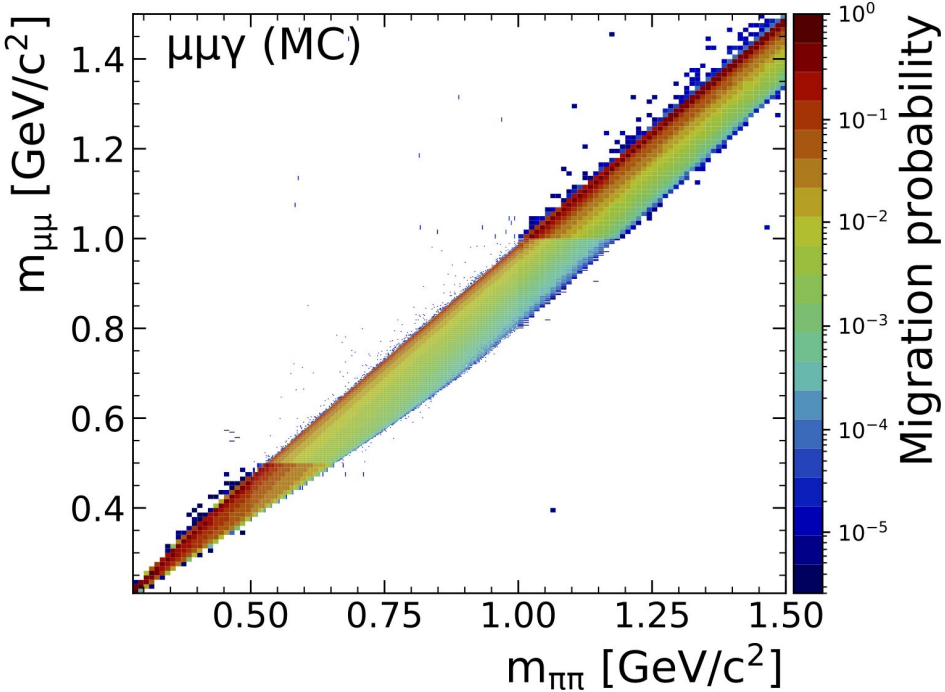


Backup

Extraction of the ππγ and μμγ mass spectra

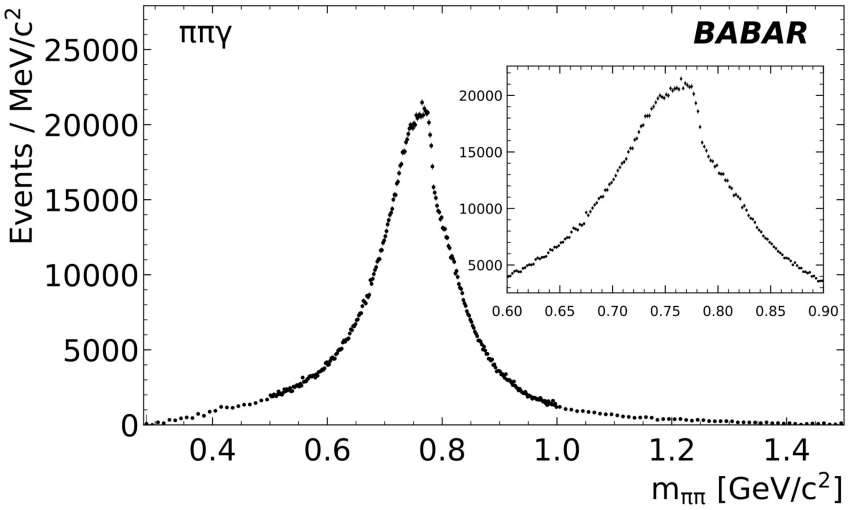
- Mass spectra from template fits, with data/MC corrections and checks of fit quality
- Folding based on transfer matrix for μμγ spectrum: $m_{\pi\pi} \rightarrow m_{\mu\mu}$
- Statistical uncertainties and (anti-)correlations fully propagated employing a bootstrap method (from event- to Xsec-level)

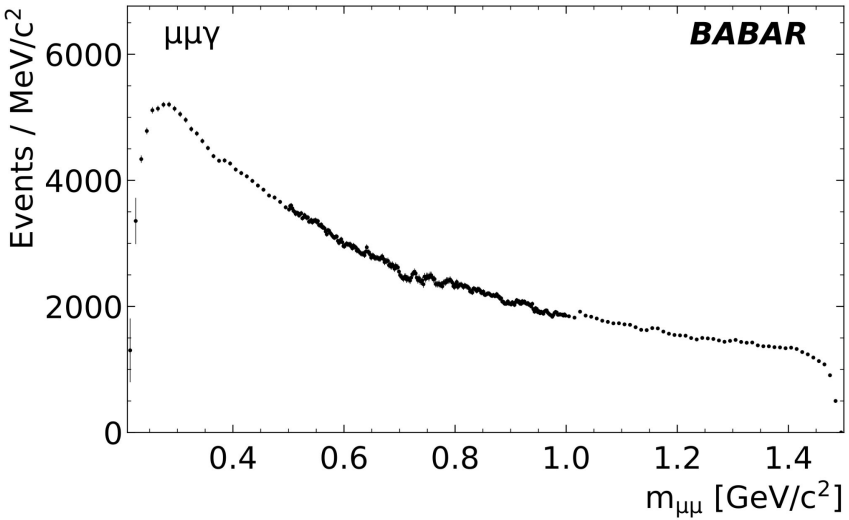




The ππγ and μμγ reconstructed mass spectra

- Normalisation of each mass spectrum *initially blinded*





μμγ spectrum compared to QED

- Measured μμγ spectrum compared to QED prediction derived from Phokhara, corrected for:
  - ISR photon efficiency
  - NLO-NNLO differences
  - Improved vacuum polarisation contribution

- The data / QED ratio fitted with a constant:

$R_{\mu\mu} = 0.9920 \pm 0.0030_{\text{stat}} \pm 0.0031_{\text{syst}}$

Data + stat. uncertainty corrections

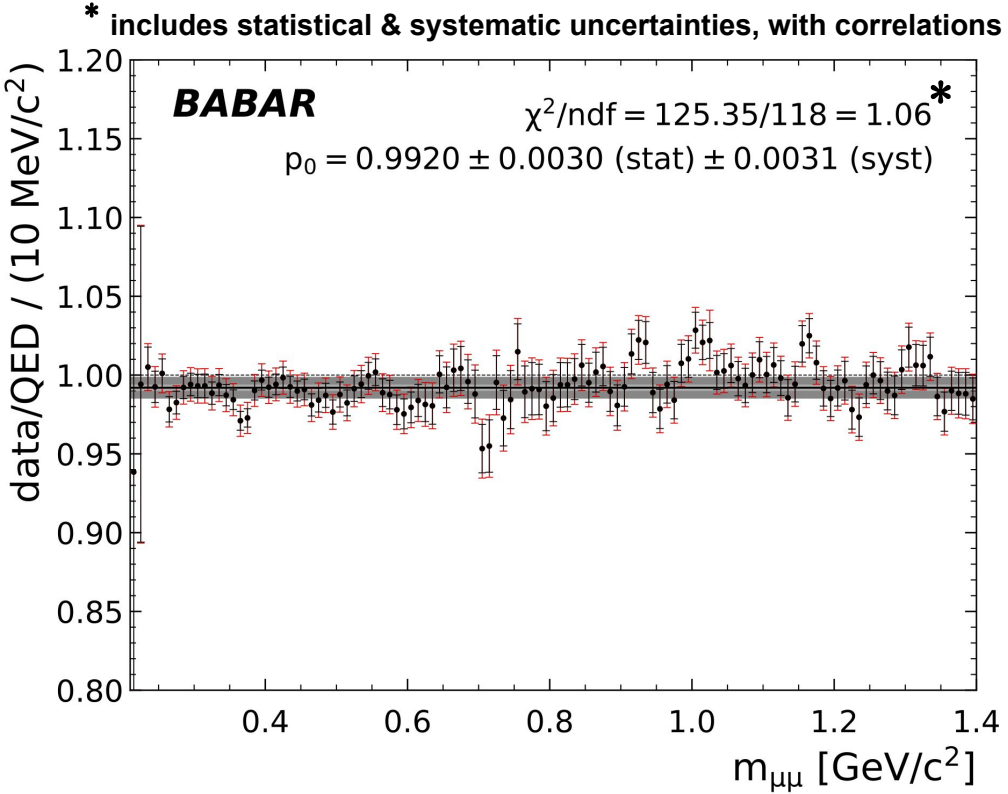
Systematic uncertainties on corrections

(Relevant only for this test. Cancel for Xsec: ππ / μμ)

$\pm\ 0.0033_{\gamma\ \text{ISR}} \pm 0.0043_{\text{lumi}\ ee}$

ISR photon data/MC efficiency

Uncertainty on e^+e^- luminosity

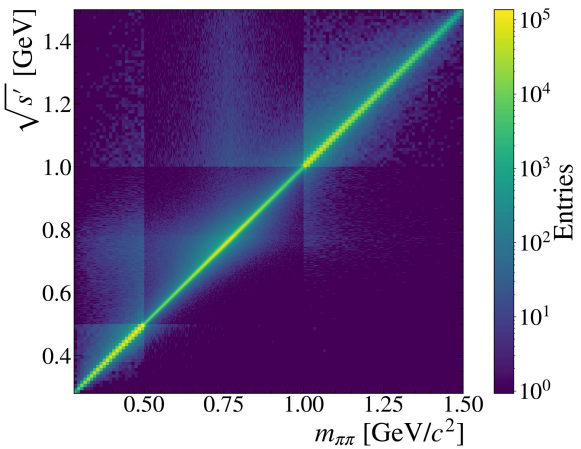


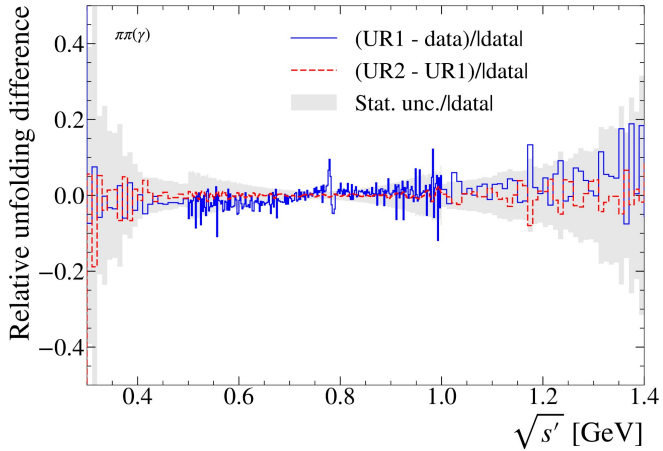
- Compatible with unity within 0.69% (better precision than for 2009: 1.1%)
- Validates ππ / μμ separation

Unfolding and uncertainties (1)

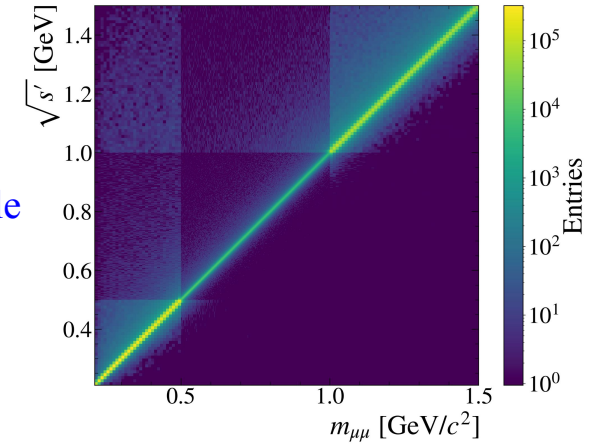
- Iterative unfolding method (IDS) based on transfer matrix $m_{ll} \rightarrow \sqrt{s'}$
  Iterations correct for data-MC shape differences
- UR1 (with original MC) vs. UR2 (truth-MC reweighting, *nominal*):
  Close to original data;
  main correction in ρ-ω region
  + effect of coherent migrations
- Additional iterations have negligible effect → Unfolded spectrum stable

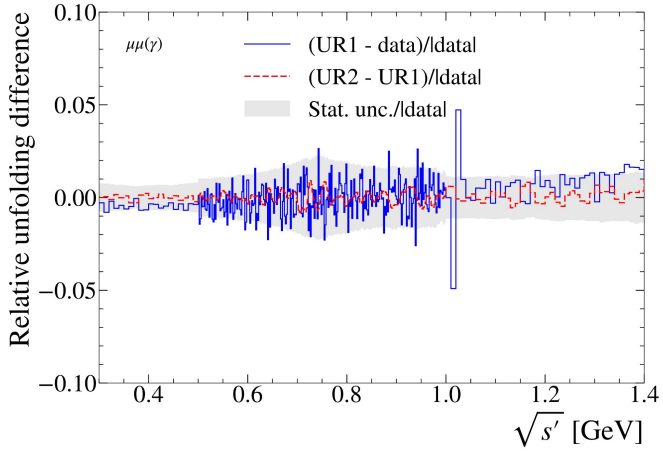
$\pi\pi(\gamma)$





$\mu\mu(\gamma)$



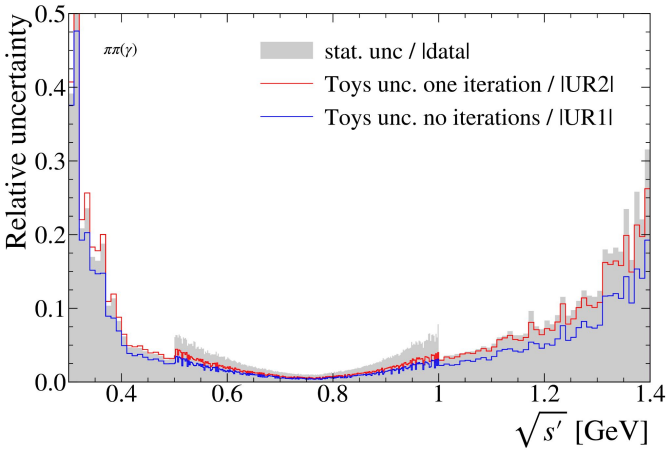


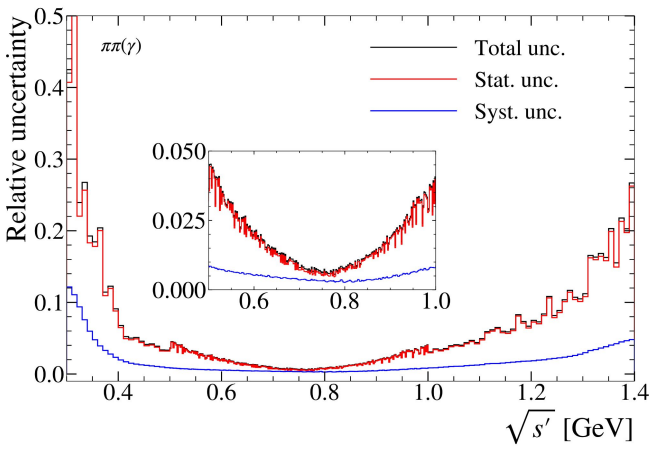
Unfolding and uncertainties (2)

- Data-driven closure test to evaluate potential residual uncertainty: *negligible methodological bias*
- Statistical uncertainties, from data and MC, and their (anti-)correlations fully propagated employing a bootstrap method
- Systematic uncertainties propagated through $\pm 1\sigma$ variations

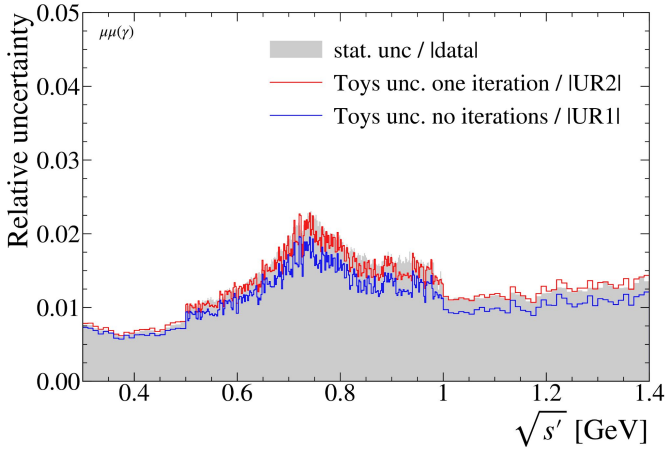
Backup

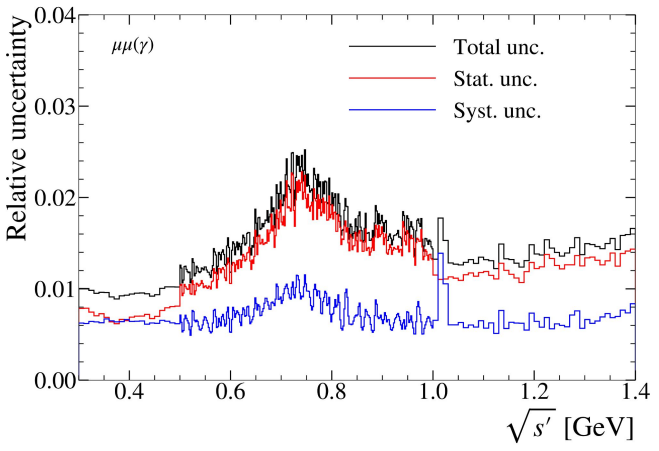
$\pi\pi(\gamma)$





$\mu\mu(\gamma)$





Determination of the effective ISR luminosity

- The Effective ISR Luminosity is obtained from the unfolded μμγ spectrum $dN^{\mathrm{ISR}}_{\mu\mu}/d\sqrt{s'}$

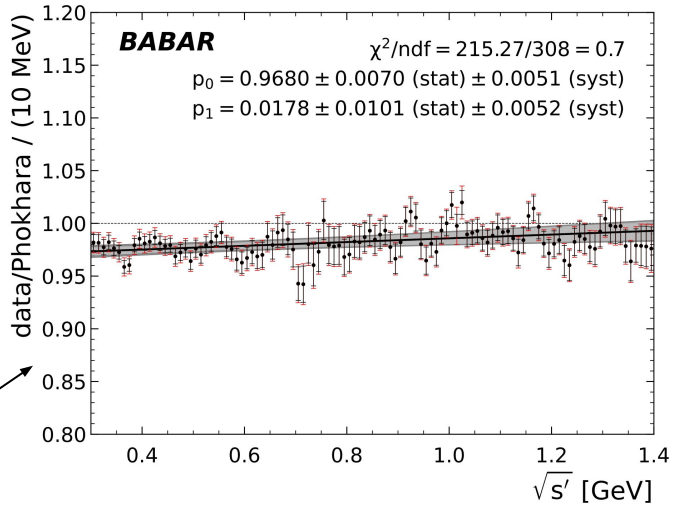
$$\frac{dL^{\mathrm{eff}}_{\mathrm{ISR}}}{d\sqrt{s'}} = \frac{dN^{\mathrm{ISR}}_{\mu\mu}/d\sqrt{s'}}{\epsilon_{\mu\mu}(\sqrt{s'})\,\sigma^{0}_{\mu\mu}(\sqrt{s'})}$$

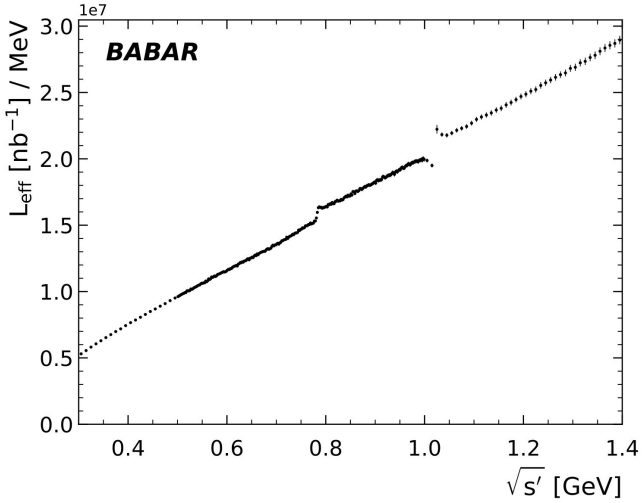
$\epsilon_{\mu\mu}(\sqrt{s'})$ → Acceptance of the selection for muons

$\sigma^{0}_{\mu\mu}(\sqrt{s'})$ → Bare μμ cross-section

$(dN^{\mathrm{ISR}}_{\mu\mu}/d\sqrt{s'})/\epsilon_{\mu\mu}(\sqrt{s'})$ replaced with:

$$\underbrace{\frac{dN^{\mathrm{MC\ gen}}_{\mu\mu}}{d\sqrt{s'}}}_{\text{Phokhara spectrum at generation level}} \times \underbrace{(1 - f_{\mathrm{LO\ FSR}})}_{\text{Removes small LO FSR contribution}} \times \underbrace{\boxed{f_{\mu\mu}(\sqrt{s'})}}_{\text{Polynomial fit to data/MC +systematic}}$$



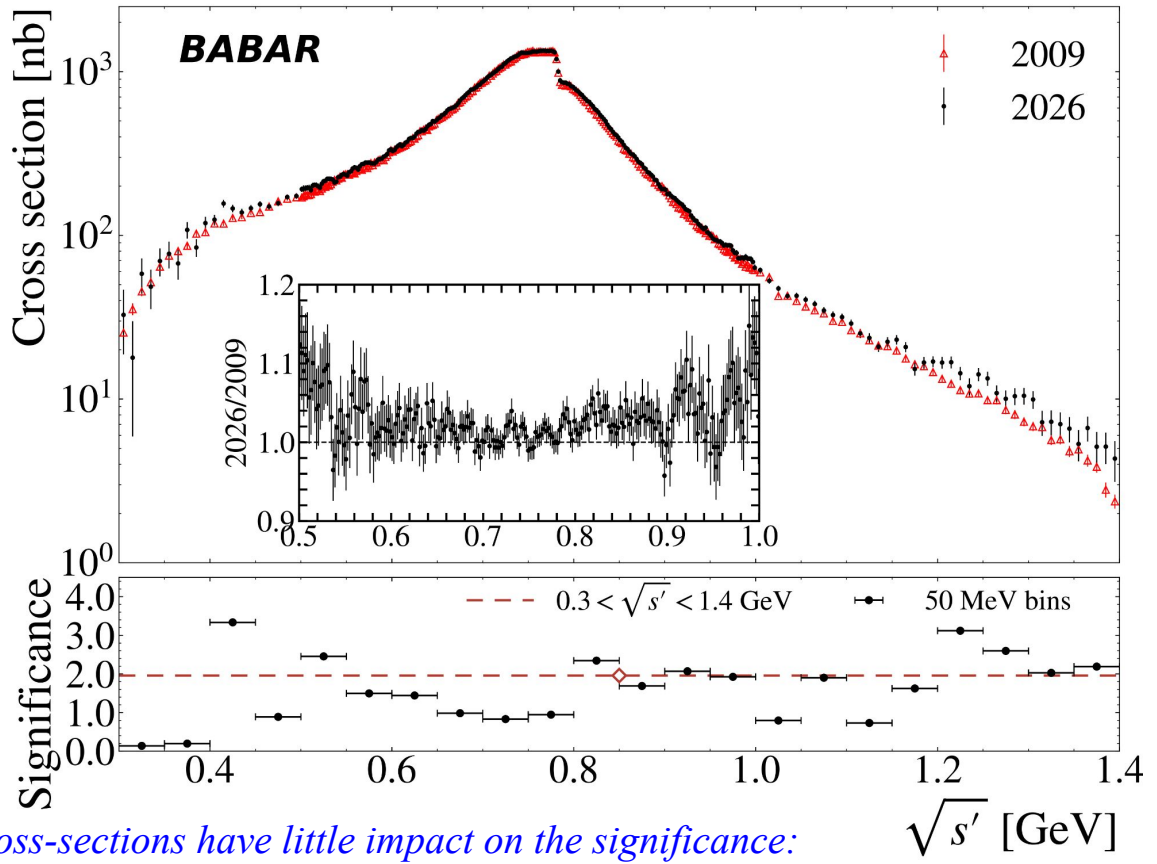


BaBar 09 + BaBar 26 Cross-Sections Comparison

$$\sigma^0_{\pi\pi}(\sqrt{s'}) = \frac{dN_{\pi\pi}/d\sqrt{s'}}{\epsilon_{\pi\pi}(\sqrt{s'})\; dL^{\rm eff}_{\rm ISR}/d\sqrt{s'}}$$

→ Computing the statistical significance of the difference among a_μ integrals for the two measurements:

- locally (scan of 50 MeV ranges) or more globally
- taking into account correlations
- studied impact of uncertainties on correlations



→ *Statistical correlations among 09 & 26 cross-sections have little impact on the significance:*
~1% on Rho peak, ~up to 3-4% at intermediate masses; much less at lower / higher masses

→ *Systematic correlations have little impact ($\leq 0.2\sigma$) on the significance of the 09 - 26 differences*

→ *If using flat kernel: enhancement by 0.2σ for [0.3;1.4] GeV and (much) less for narrower intervals*

Backup

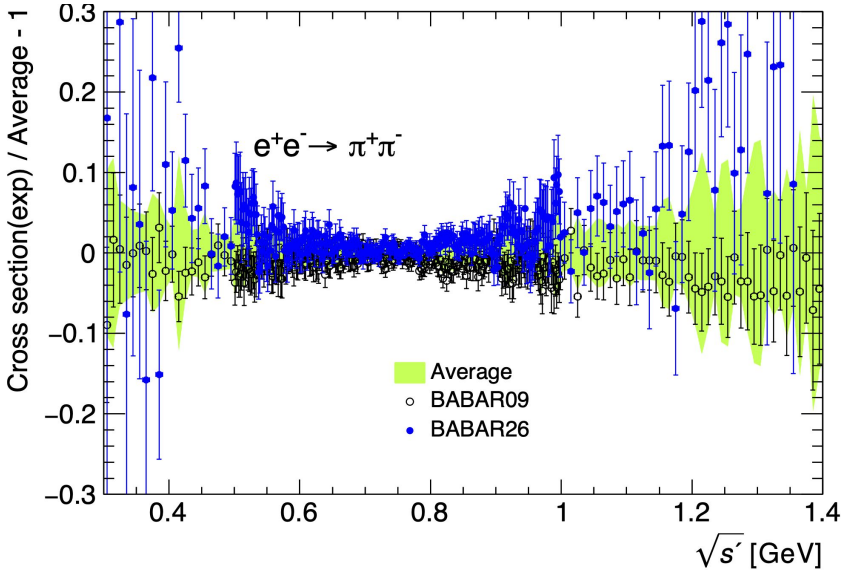
BaBar 09 + BaBar 26 Comparison & Combination

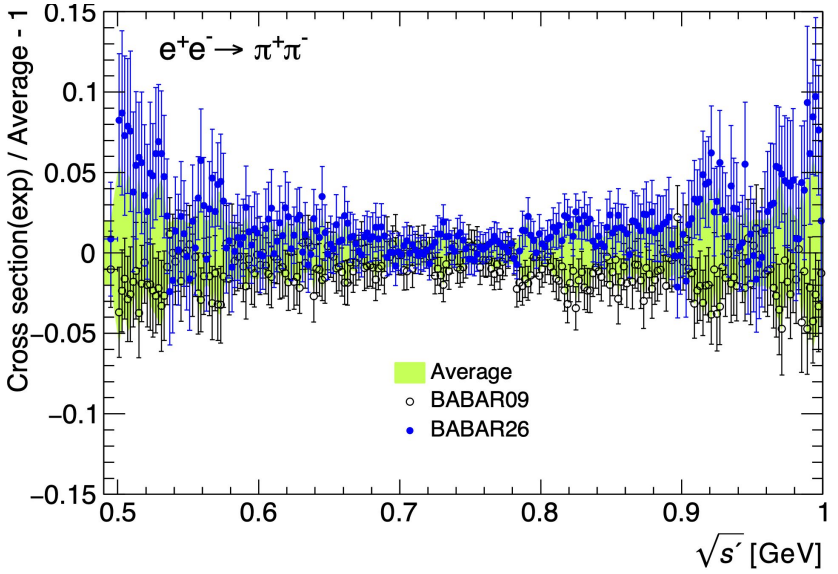
→ Using the (variable) binning of the measurements for the integrals and the combination

→ Full propagation of *uncertainties* and *correlations* from input measurements, to final results, employing several consistent methods (e.g. pseudo-experiments)

- *Among data bins of a given measurements*
- *Among measurements* (*Backup*)

Employ bootstrap replicas, covariance matrices and/or detailed split of uncertainties in sub-components: systematic correlations for tracking, trigger, background subtraction, kinematic fits





Backup

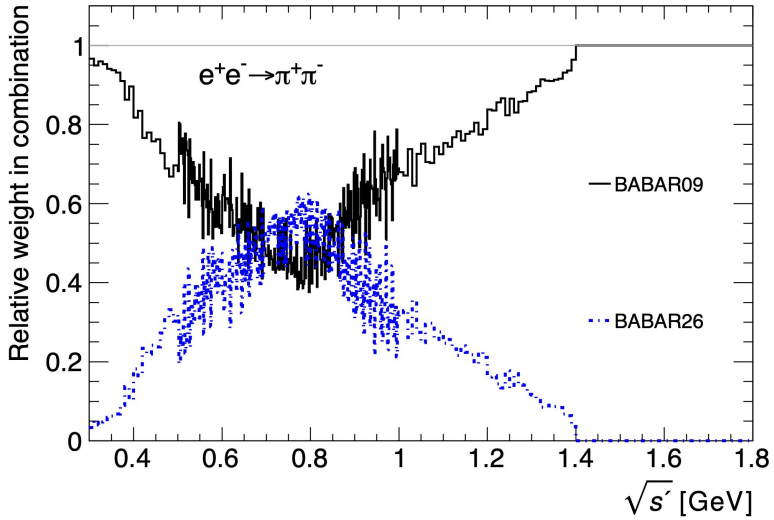
BaBar 09 + BaBar 26 Comparison & Combination

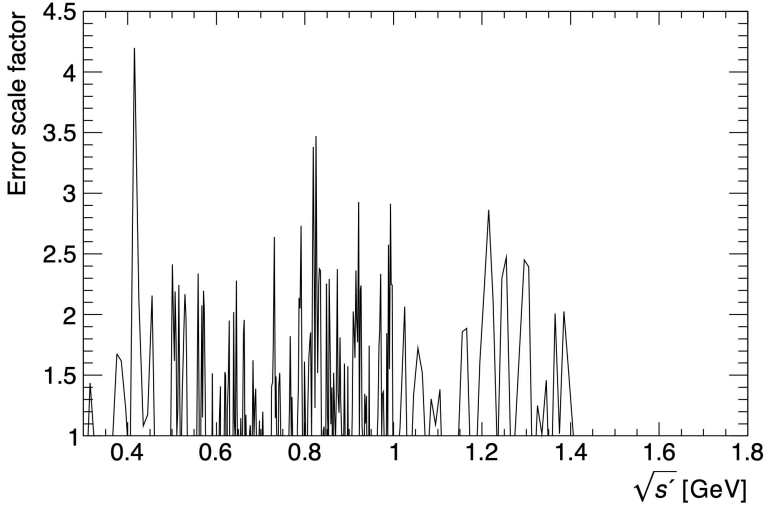
→ Average weights derived through a local χ^2 minimisation:

- Accounting for relevant correlations among data bins and among different measurements
- Driven by the relative precision of the measurements in different mass regions
- General features stable w.r.t. variations of the combination procedure

→ χ^2 /ndof employed to test locally the level of agreement between input measurements

- Scale uncertainties in bins with χ^2 /ndof > 1 (PDG): *locally conservative*

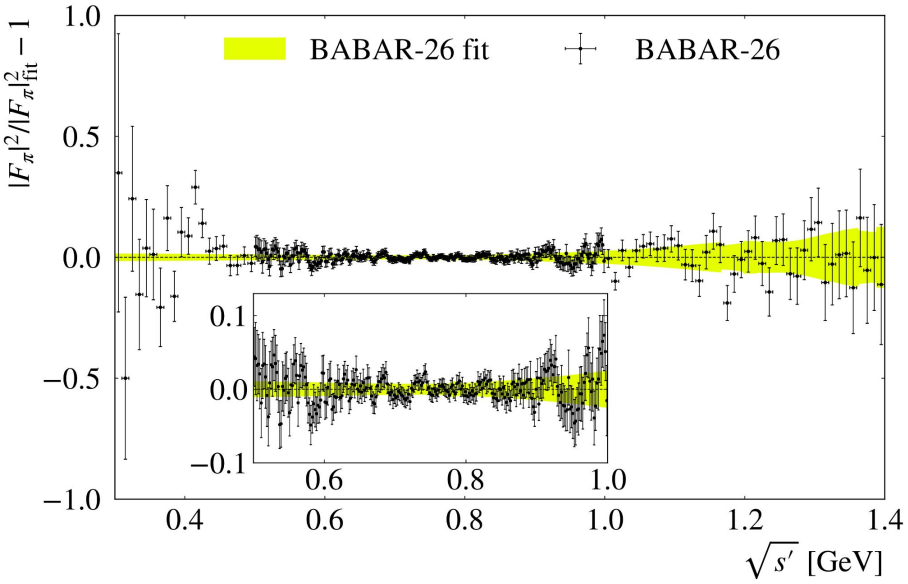


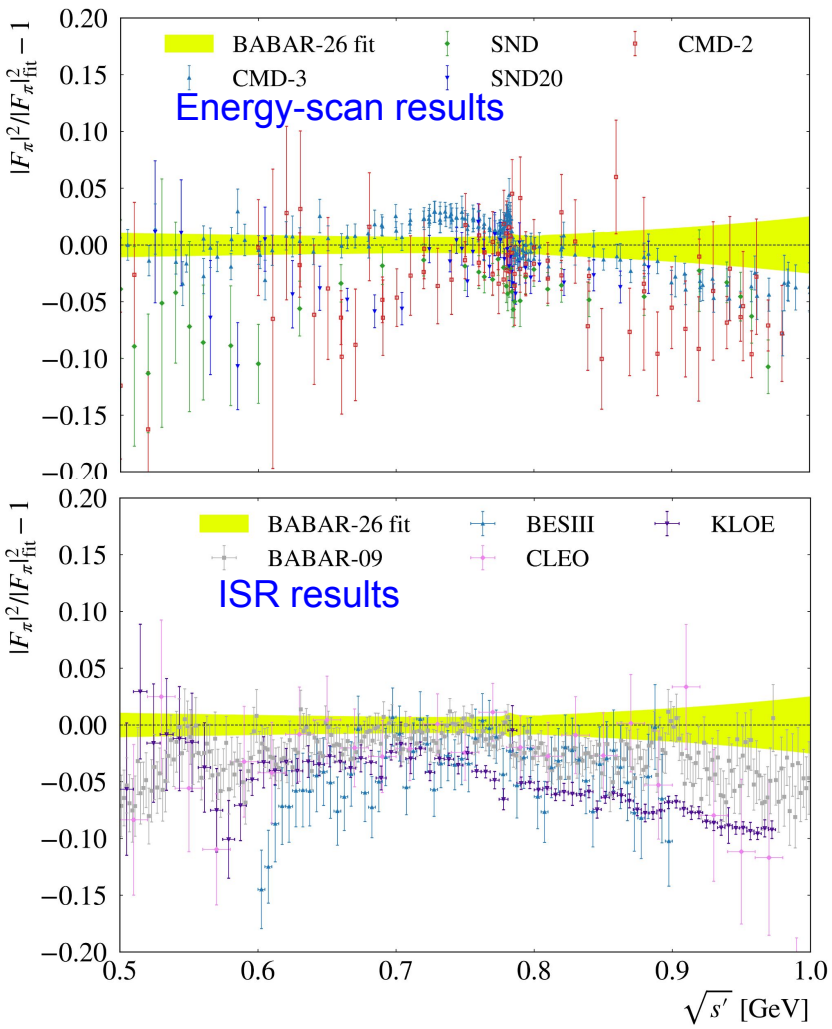


→ The *uncertainties on correlations* have *little impact* for:
the averaging weights, the local χ^2/ndf, the integrals for the combined spectrum and their uncertainties

Comparison with other measurements

→ Using a VDM fit of the BaBar '26 form factor as reference for the comparison



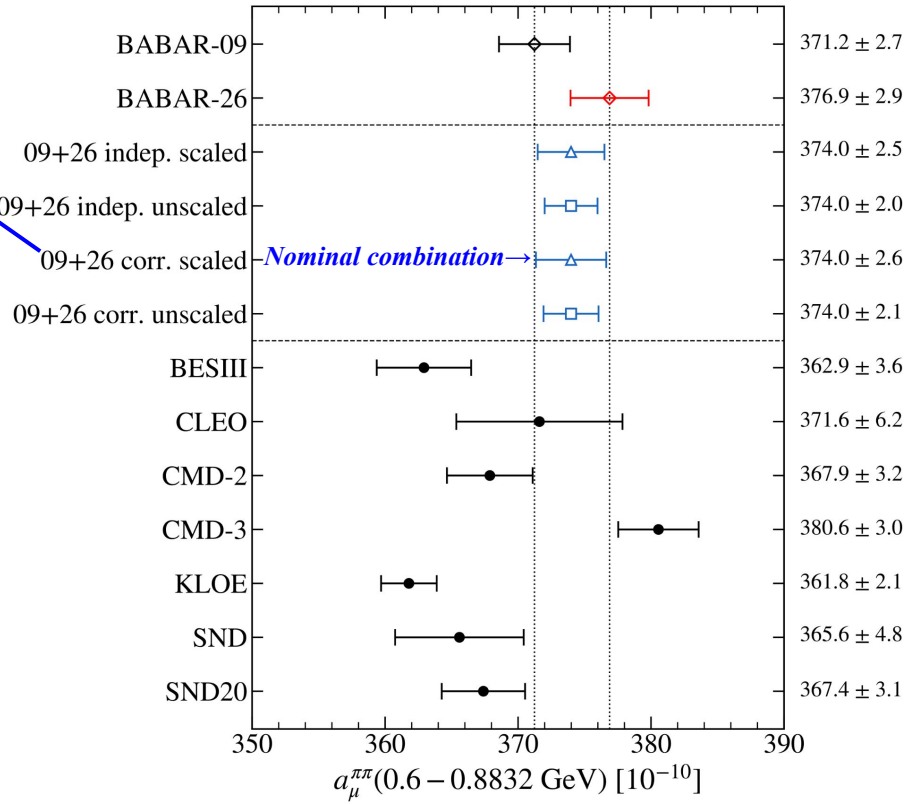


Hadronic vacuum polarization contribution to a_μ

TABLE I. Contributions to $a_\mu^{\pi\pi}$ per energy ranges, in units of 10^{-10}. The first and second uncertainties are statistical and systematic, respectively.

Energy range (GeV)	Present result	BABAR 2009	Combination 2009+2026
0.3–0.5	60.7(1.7)(1.4)	57.6(0.63)(0.55)	58.2(0.86)(0.78)
0.5–1.0	454.6(2.6)(2.5)	445.9(2.1)(2.5)	449.7(2.1)(2.5)
1.0–1.4	10.6(0.16)(0.34)	9.7(0.10)(0.08)	9.9(0.12)(0.14)
Total	525.9(3.5)(3.8)	513.3(2.2)(3.1)	517.8(2.4)(3.3)

→ The new BaBar '26 measurement provides a result with interesting precision, in particular for the ρ peak region



→ Employing the most conservative '09 + '26 combination (with correlations and rescaled uncertainties) as *nominal*

→ Comparison with other measurements: common mass region on ρ peak

Conclusion

New BaBar '26 measurement, largely independent of the BaBar '09 result, provides interesting new insights for the situation of the ππ channel

Backup

Lepton Magnetic Anomaly: from Dirac to QED

- Magnetic dipole moment of a charged lepton: $\vec{\mu} = g\frac{e}{2m}\vec{s}$

  Dirac (1928) $g_e=2$ $a_e=0$

- "anomaly" = deviation w.r.t. Dirac's prediction: $a = \frac{g-2}{2}$

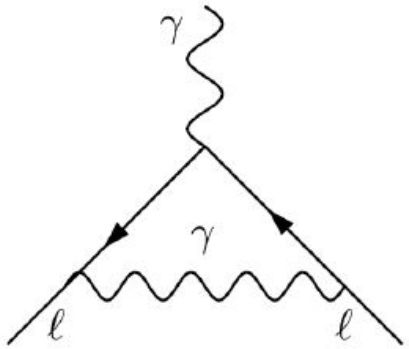
anomaly discovered:

Kusch-Foley (1948) $a_e = (1.19 \pm 0.05)\ 10^{-3}$

and explained by O(α) QED contribution:

Schwinger (1948) $a_e = \alpha/2\pi = 1.16\ 10^{-3}$

first triumph of QED



⇒ a_e sensitive to quantum fluctuations of fields

More Quantum Fluctuations

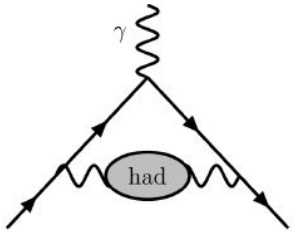
$$a = a^{\mathrm{QED}} + a^{\mathrm{had}} + a^{\mathrm{weak}}$$ + ? a new physics ?

typical contributions:

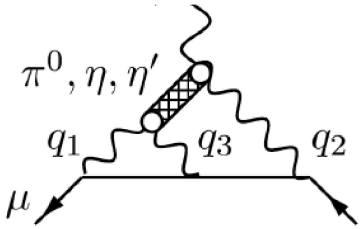
QED up to O(α^5) (Kinoshita et al.)



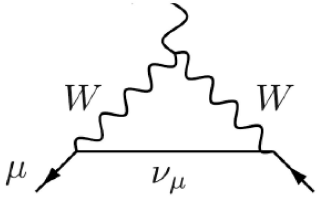
Hadrons vacuum polarization



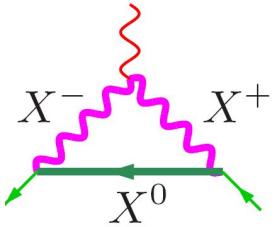
light-by-light (dispersive & lattice QCD)



Electroweak



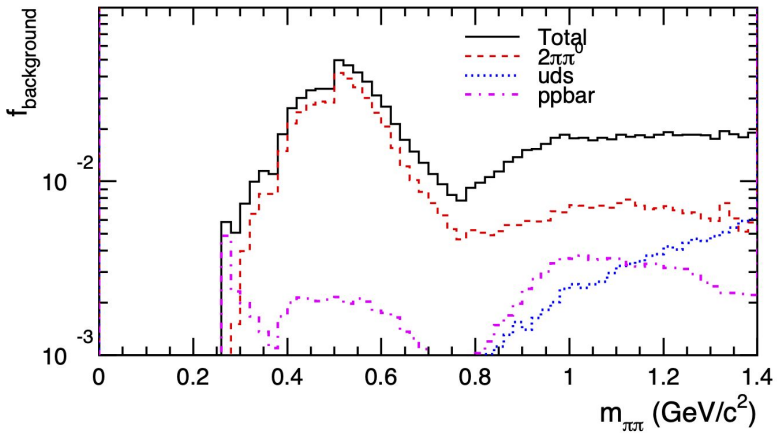
new physics at high mass scale

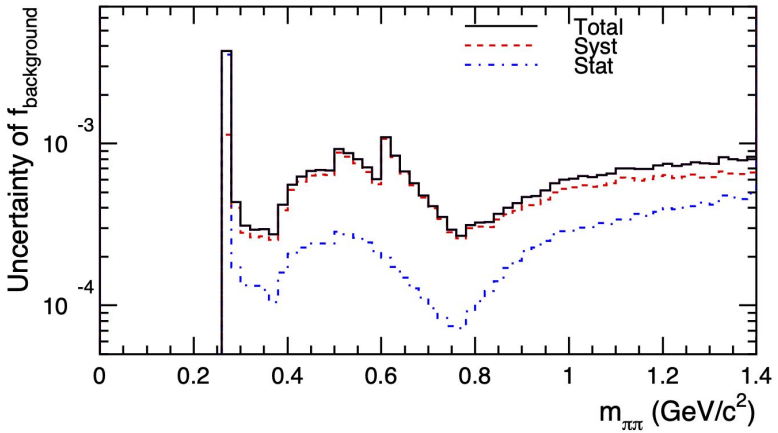


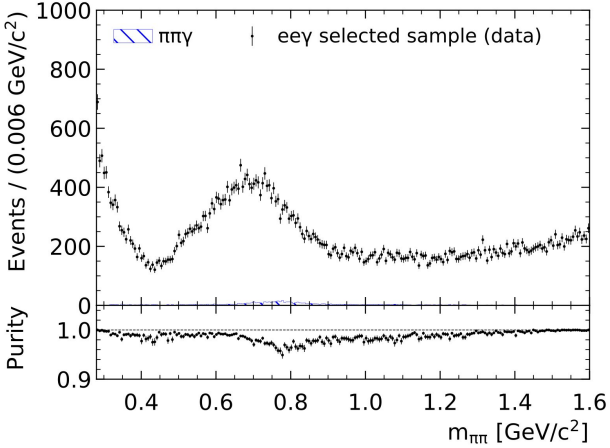
$\delta a_l \propto \frac{m_l^2}{M^2}$ ⇒ a_μ much more sensitive to high scales

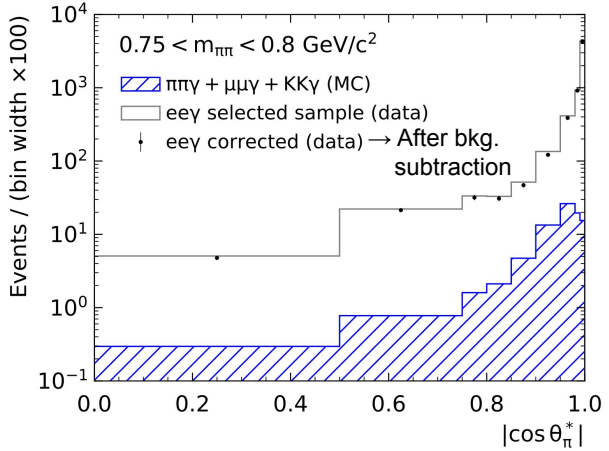
Background contributions

- Small residual multihadronic background after 2d-χ^2 selection
- Data-driven templates for eeγ background, using cut-based and BDT selections





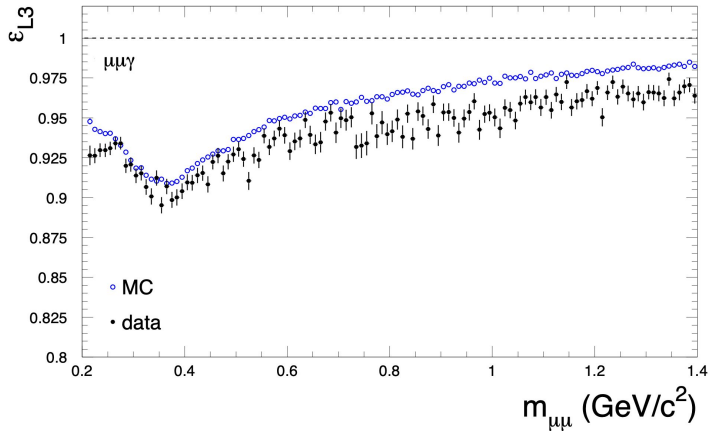


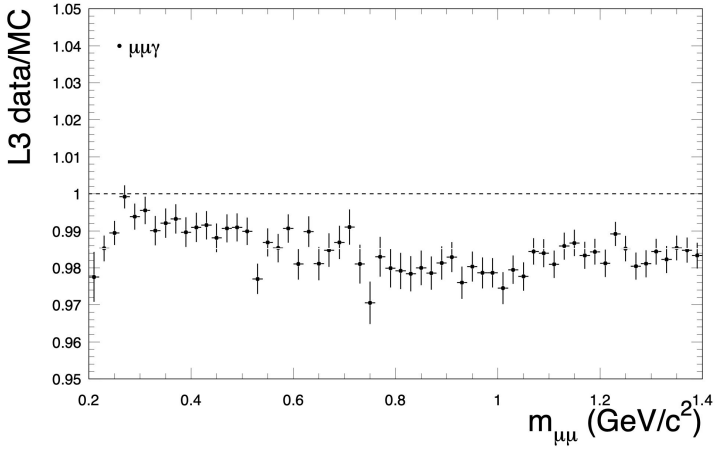


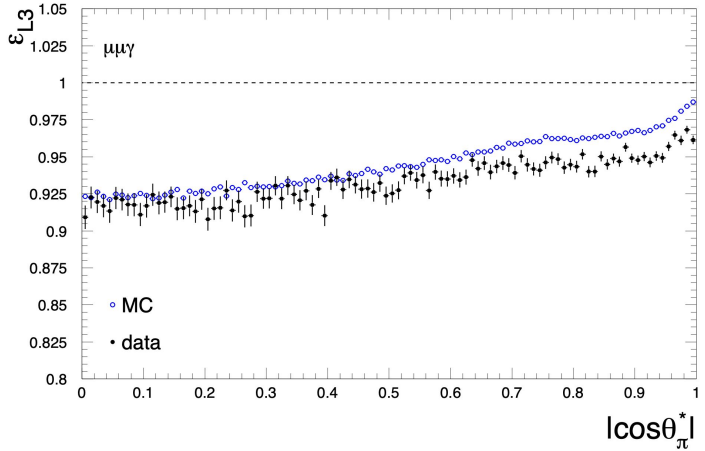
Back

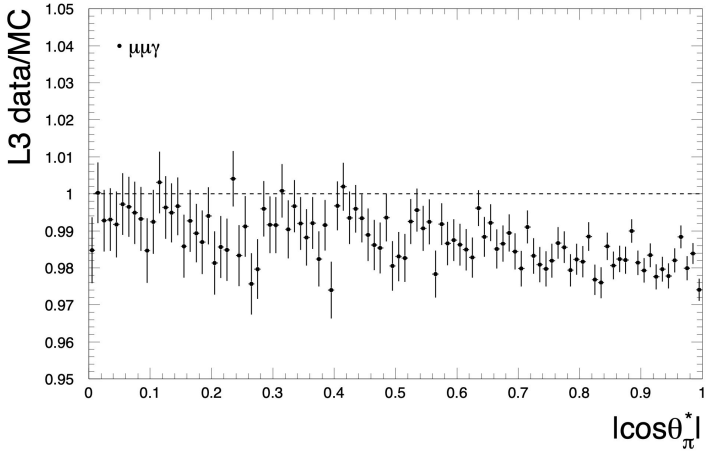
Efficiency corrections: Trigger L3 for μμ

- Data-driven determination of the efficiency corrections





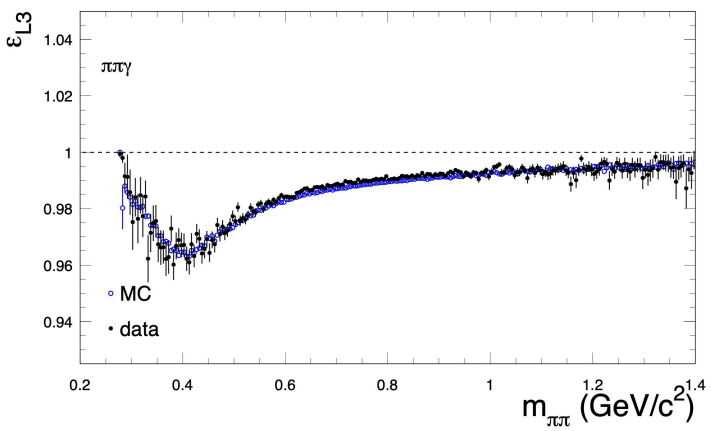


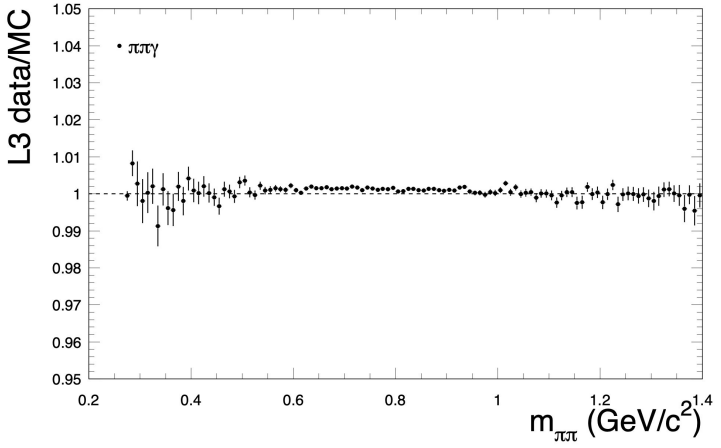


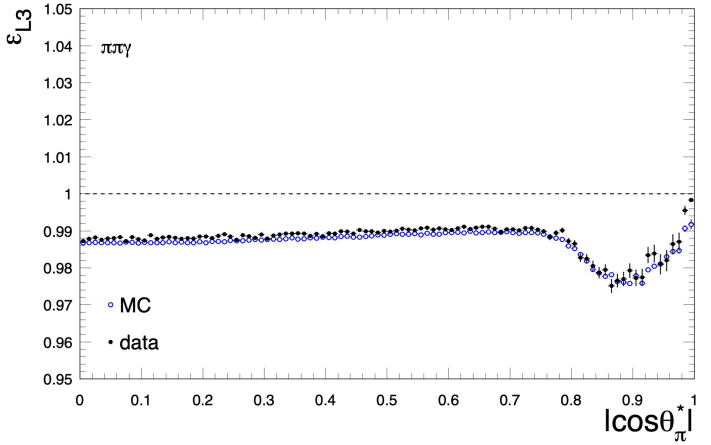
Back

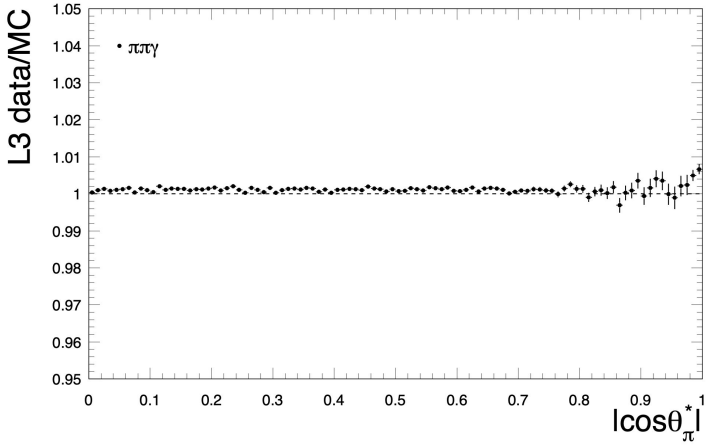
Efficiency corrections: Trigger L3 for ππ

- Data-driven determination of the efficiency corrections



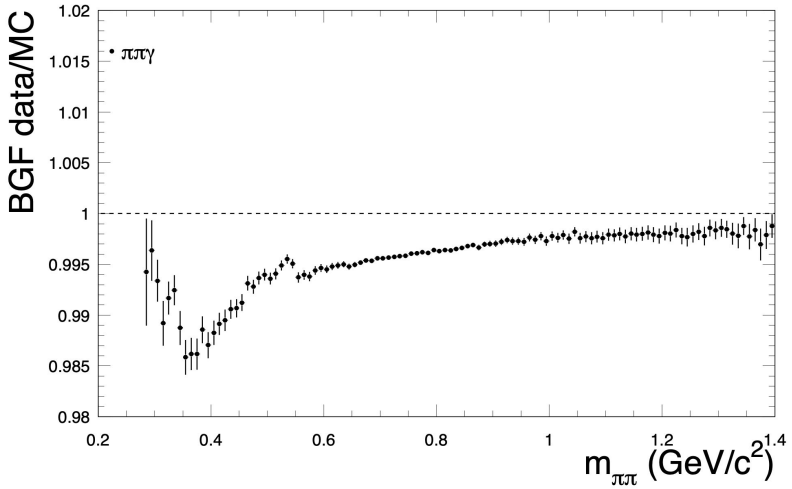


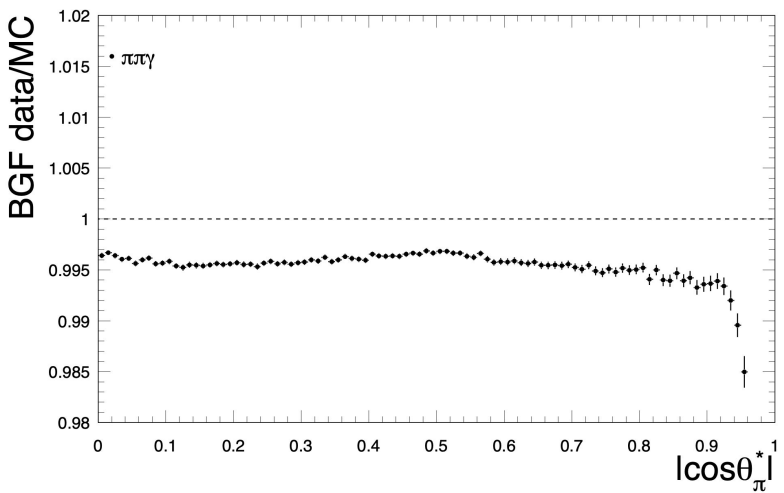




Efficiency corrections: Trigger BGFilter for ππ

- Data-driven determination of the efficiency corrections

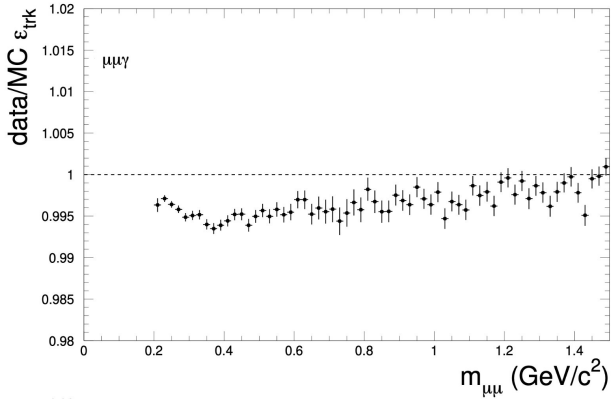


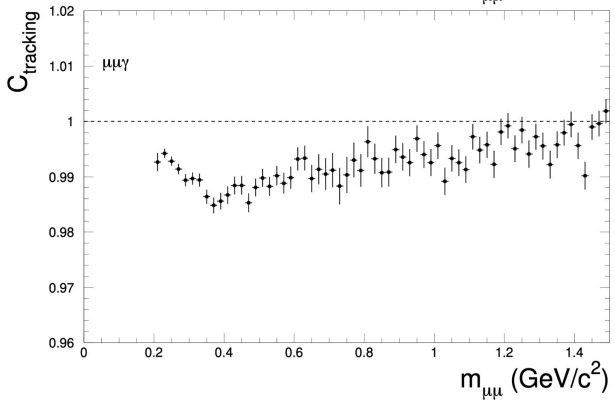


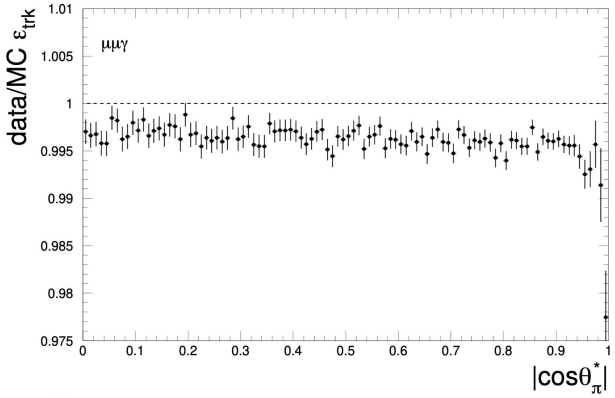
Efficiency corrections: Tracking for μμ

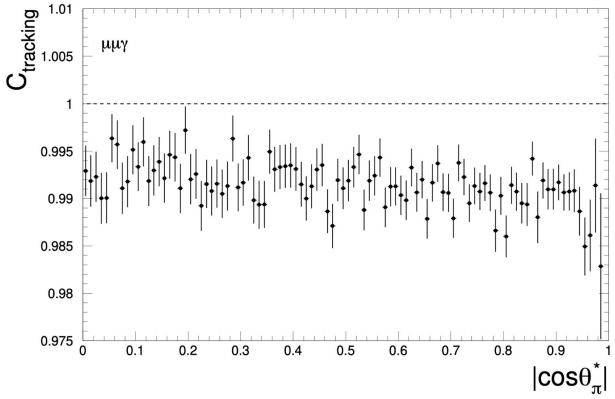
- Data-driven determination of the efficiency corrections
  ε: single-track efficiency
  f_0: probability for losing the tracks in a correlated way
  f_3: probability for having an extra reconstructed good track

$$C_{\text{tracking}} = \frac{[(1 - f_0 - f_3)\epsilon^2]_{\text{data}}}{[(1 - f_0 - f_3)\epsilon^2]_{\text{MC}}}$$





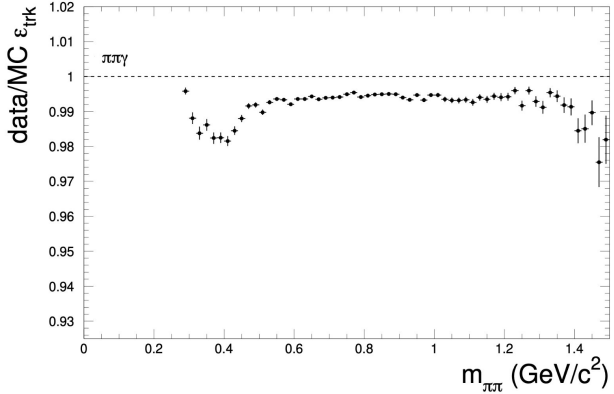


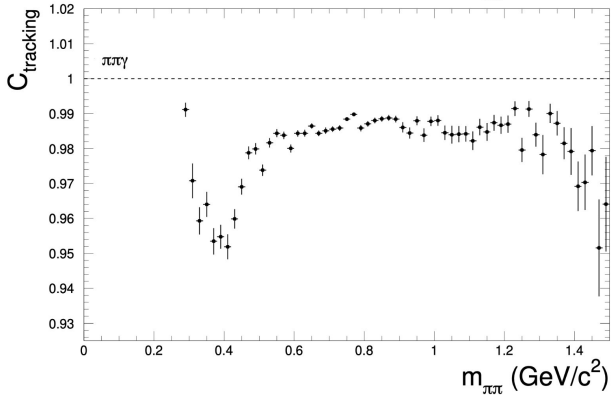


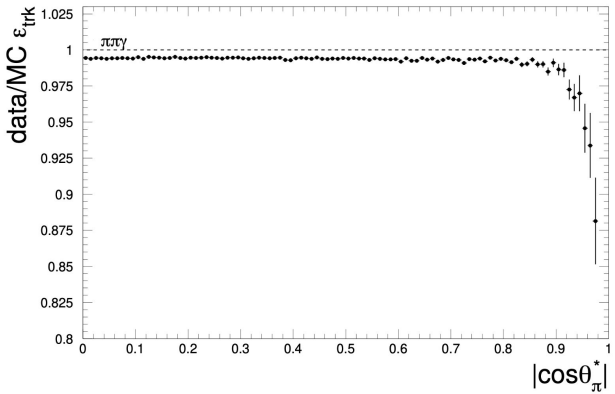
Efficiency corrections: Tracking for ππ

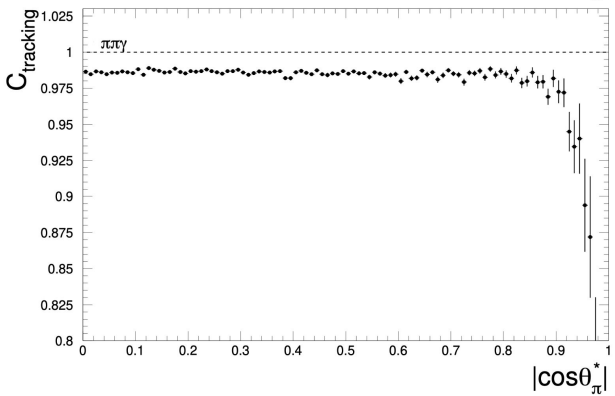
- Data-driven determination of the efficiency corrections
  ε: single-track efficiency
  f_0: probability for losing the tracks in a correlated way
  f_3: probability for having an extra reconstructed good track

$$C_{\text{tracking}} = \frac{[(1 - f_0 - f_3)\epsilon^2]_{\text{data}}}{[(1 - f_0 - f_3)\epsilon^2]_{\text{MC}}}$$

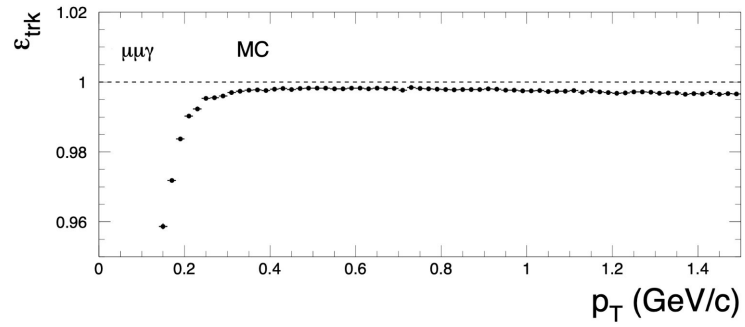


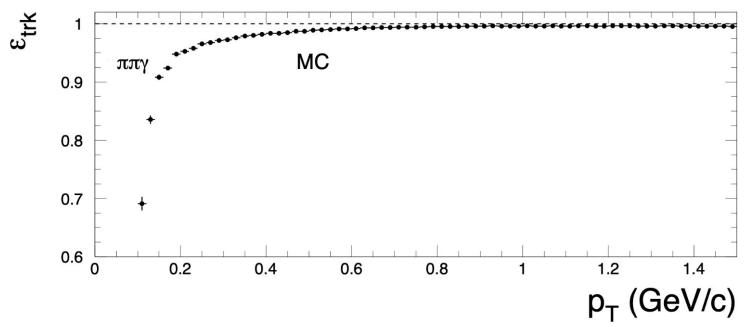


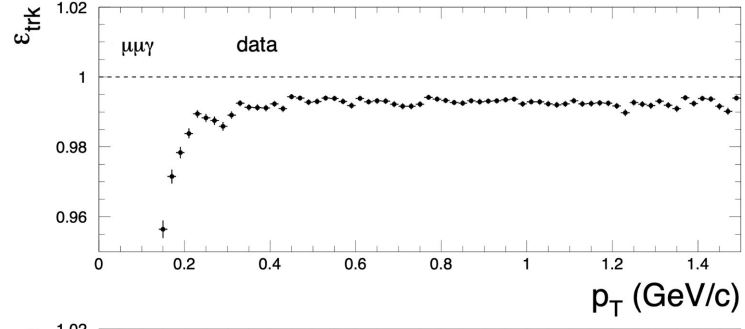


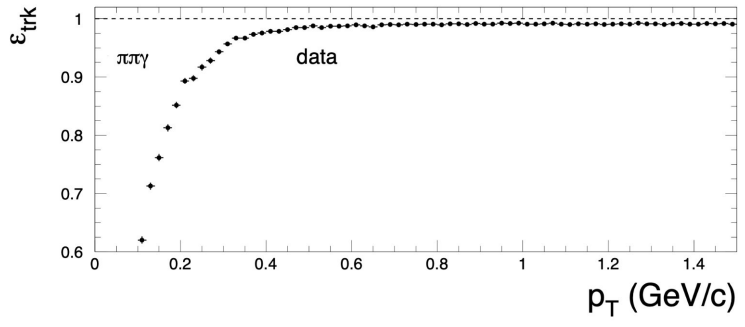


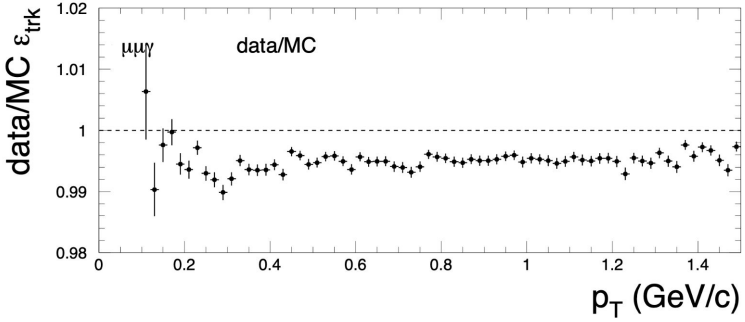
Efficiency corrections: Tracking at low-p_T

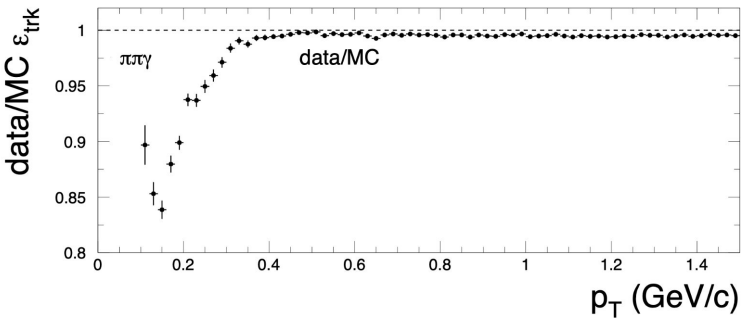




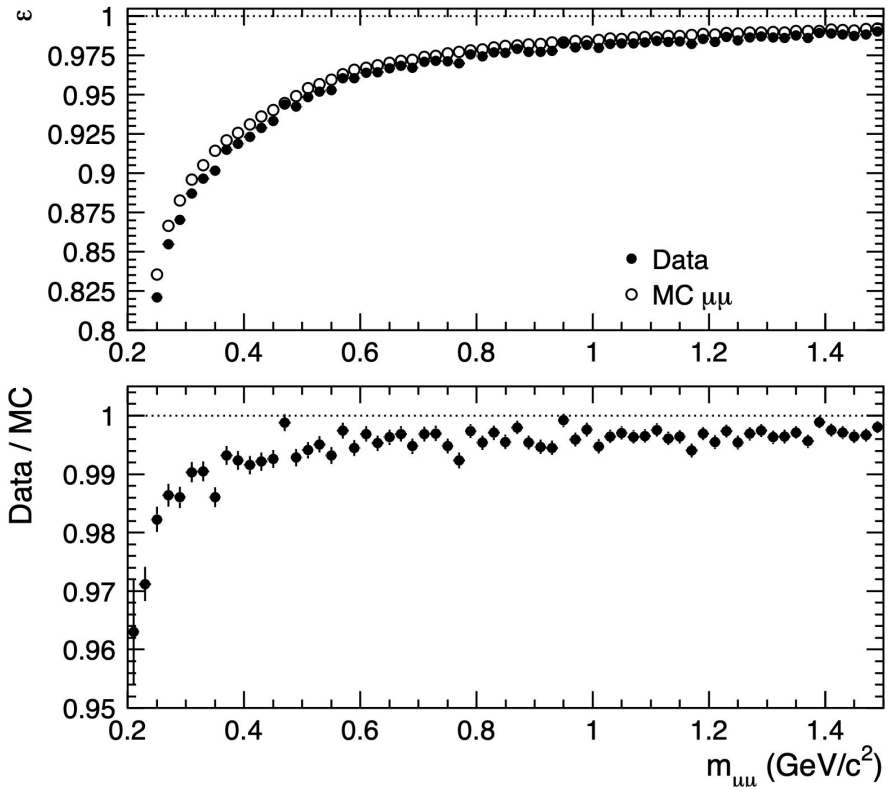


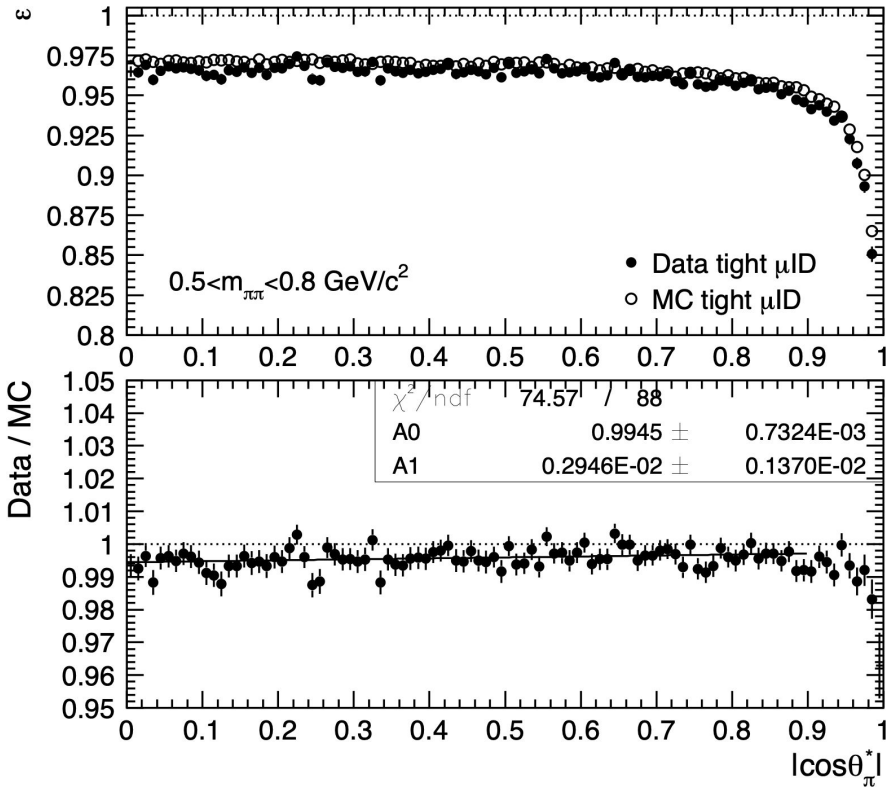




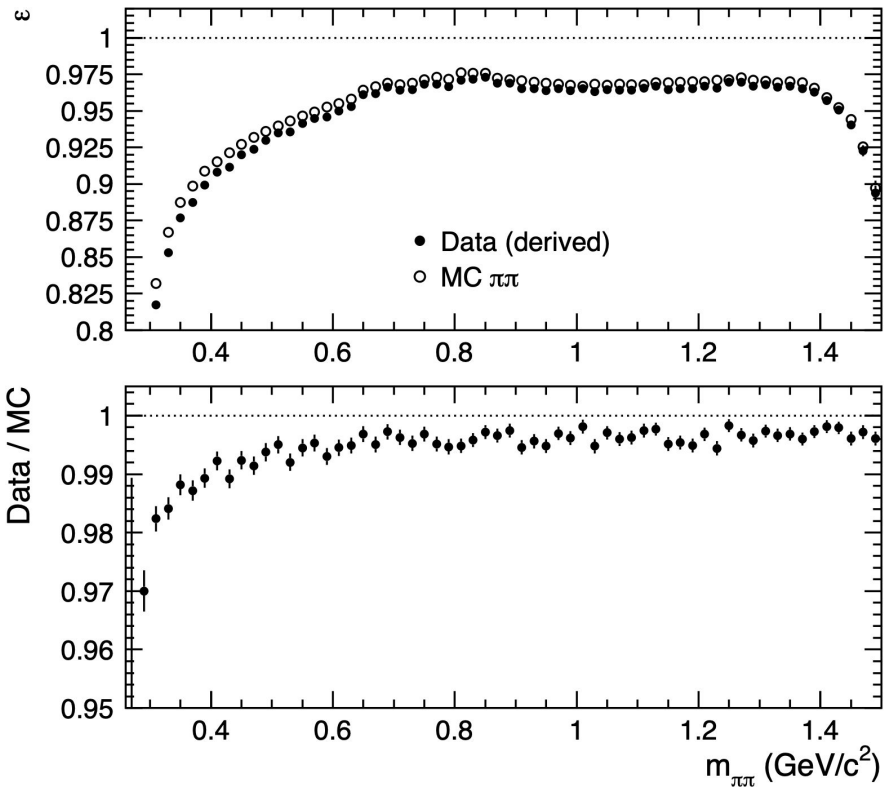


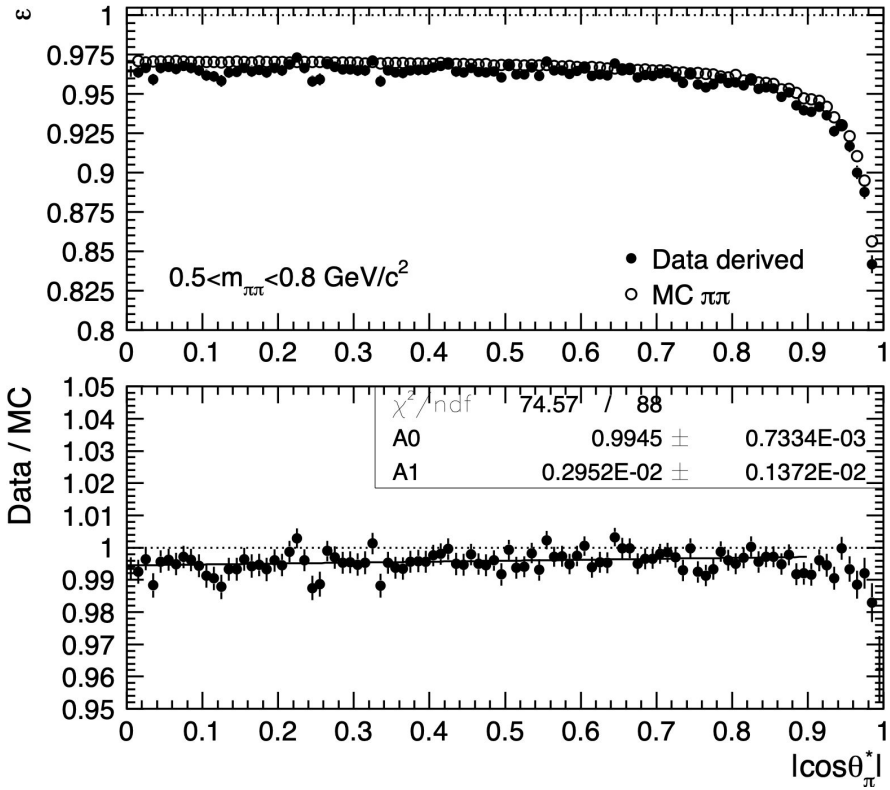
Efficiency corrections: Vertex position cut "Vxy" for μμ



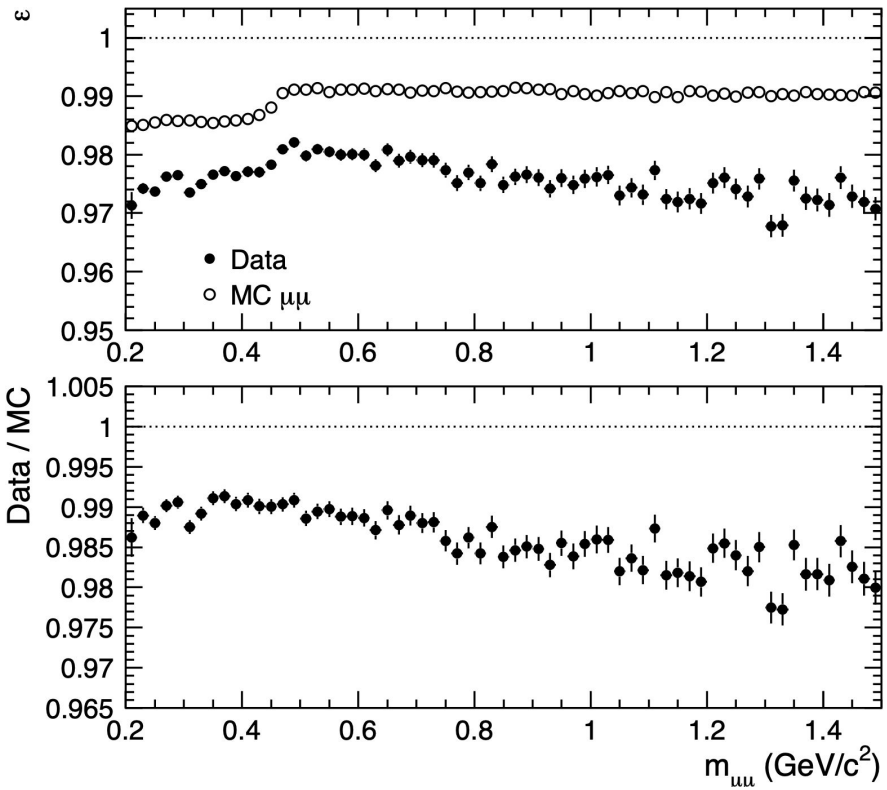


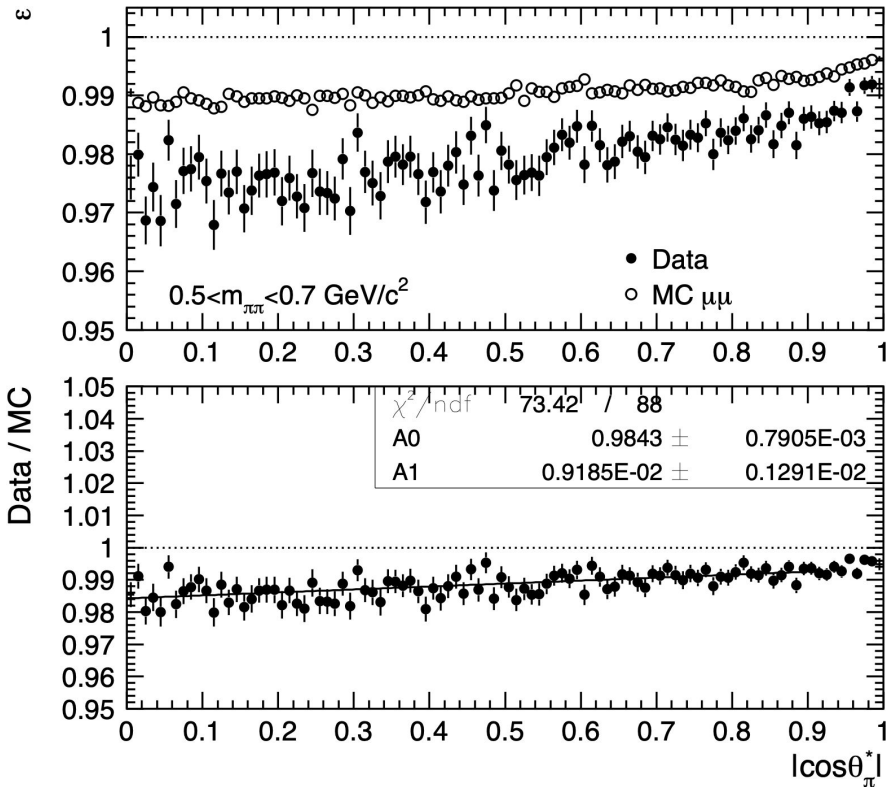
Efficiency corrections: Vertex position cut "Vxy" for ππ





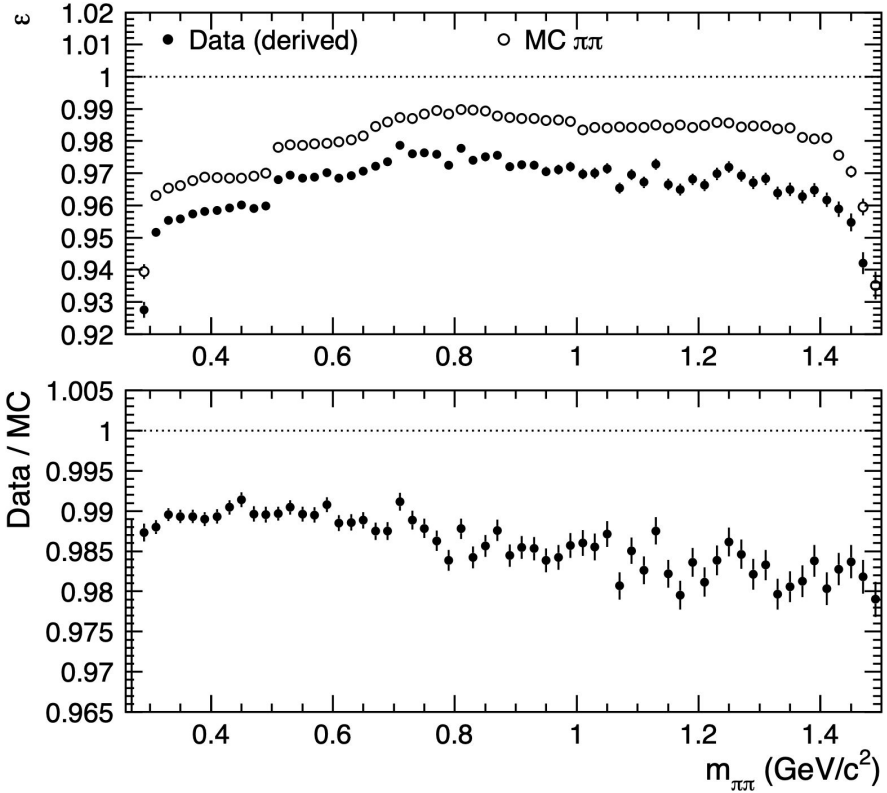
Efficiency corrections: χ^2 selection for $\mu\mu$

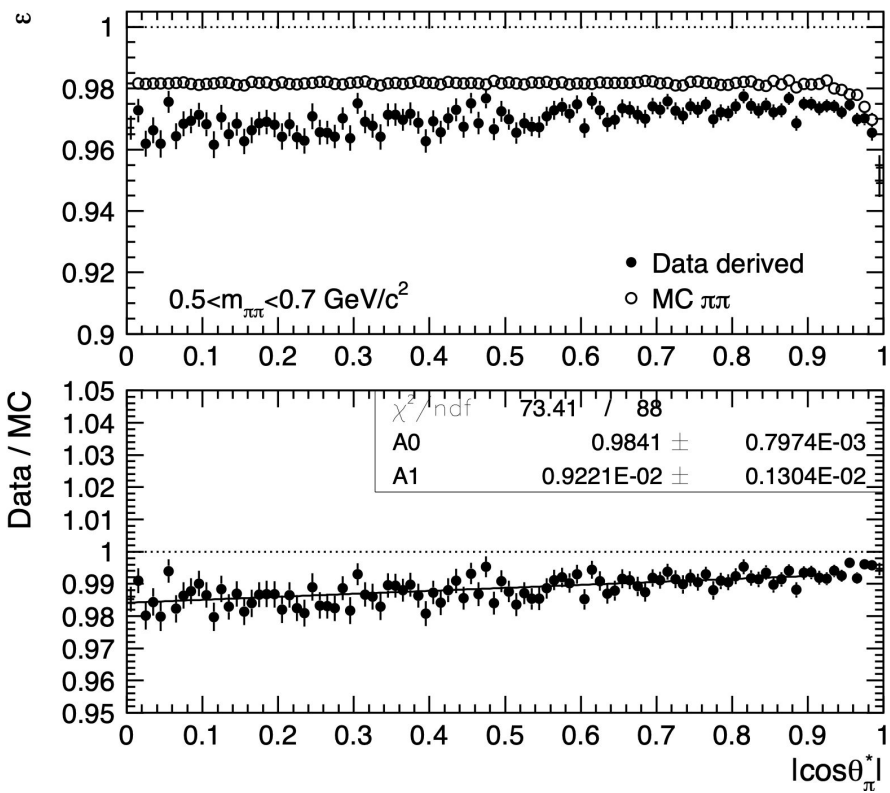




Efficiency corrections: χ^2 selection for $\pi\pi$

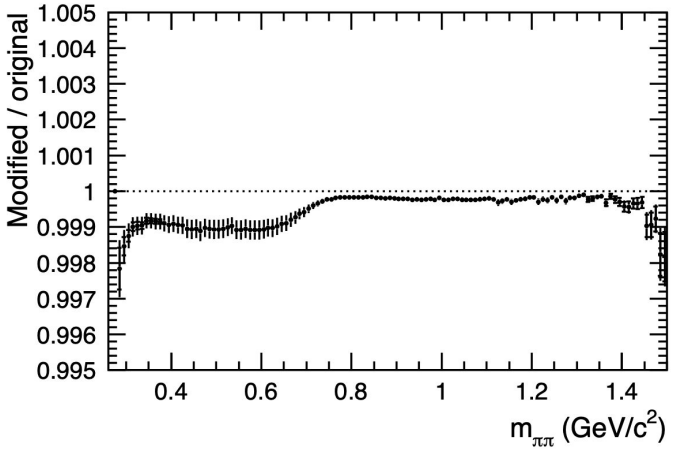
$$\epsilon_{\chi^2}^{\pi\pi\gamma,\text{data}} = \epsilon_{\chi^2}^{\mu\mu\gamma,\text{data}} + \epsilon_{\chi^2}^{\pi\pi\gamma,\text{MC}} - \epsilon_{\chi^2}^{\mu\mu\gamma,\text{MC}}$$

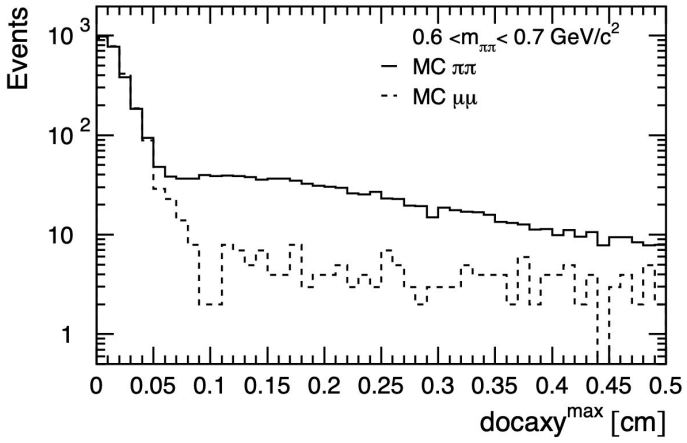




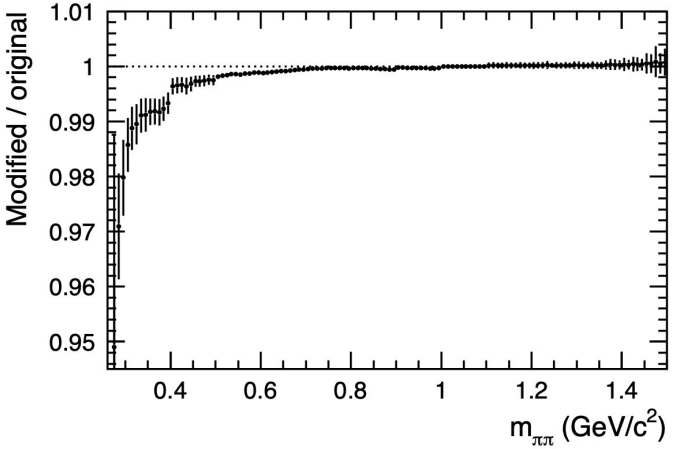
Efficiency corrections: χ^2 selection for $\pi\pi$ - fake γ and 2^{nd} interactions

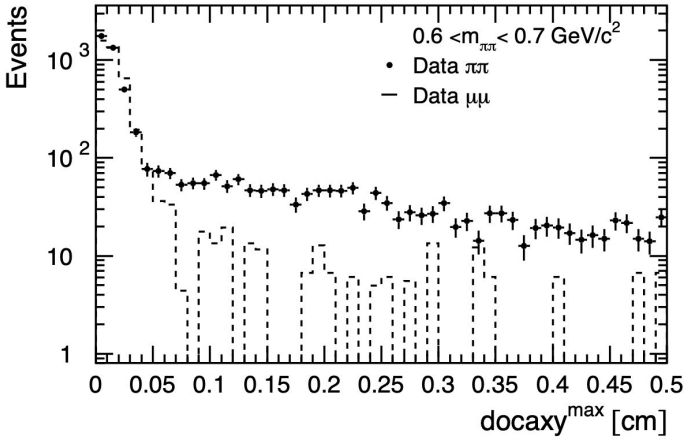
Effect of fake photons on χ^2 selection efficiency





Effect of pion interactions on χ^2 selection efficiency



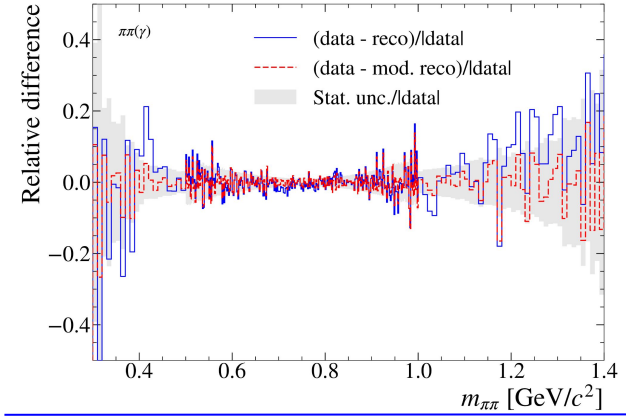


Unfolding and uncertainties

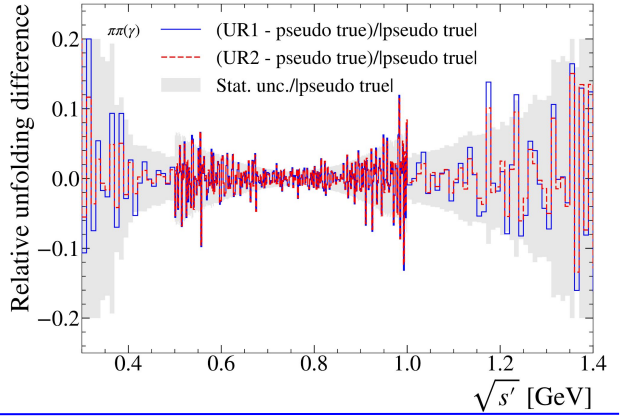
- One truth-MC reweighting removes data-MC shape differences at reconstructed level
- Data-driven closure test to evaluate potential residual uncertainty:

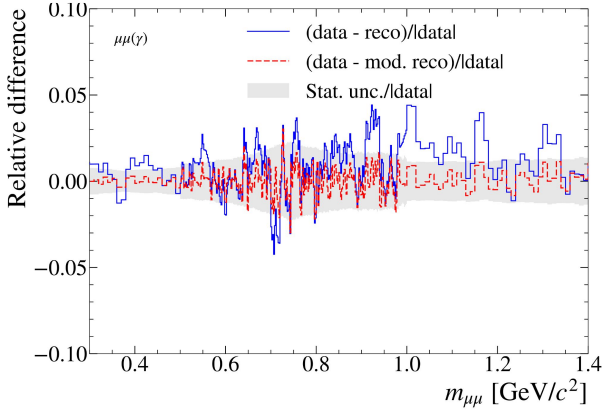
  → Build reweighed MC sample that looks very much like data on reconstructed level (with known truth shape)

  → *negligible methodological bias*

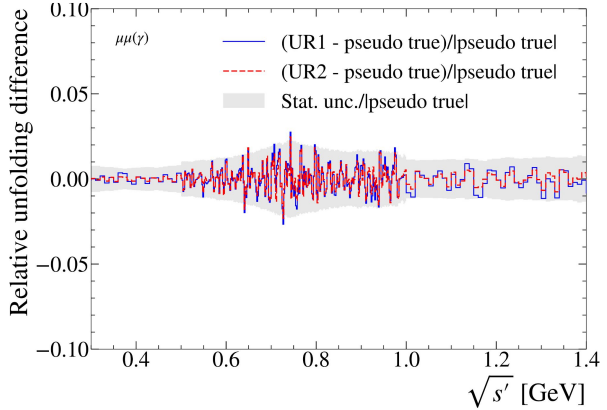


ππ(γ)





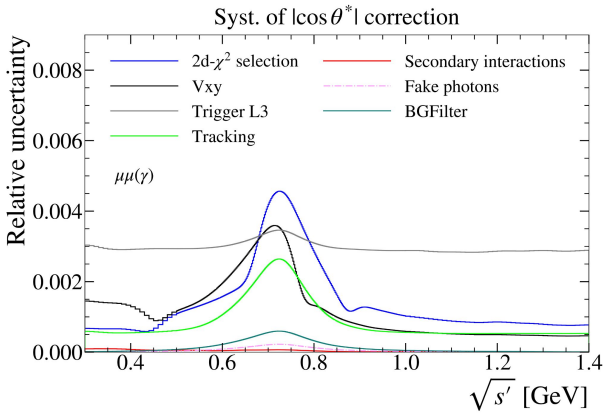
μμ(γ)

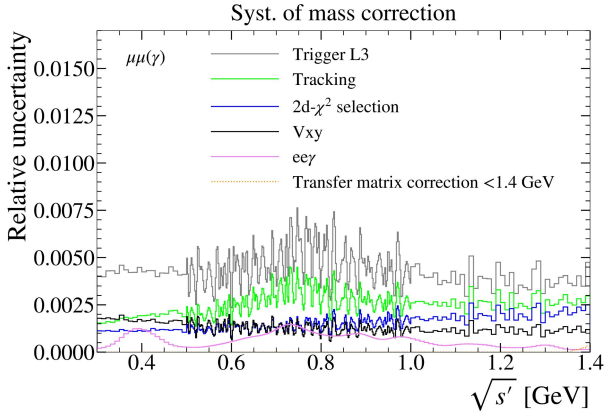


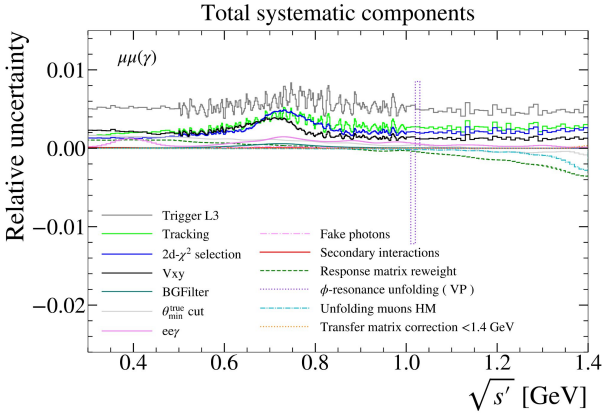
Back

Unfolding and uncertainties

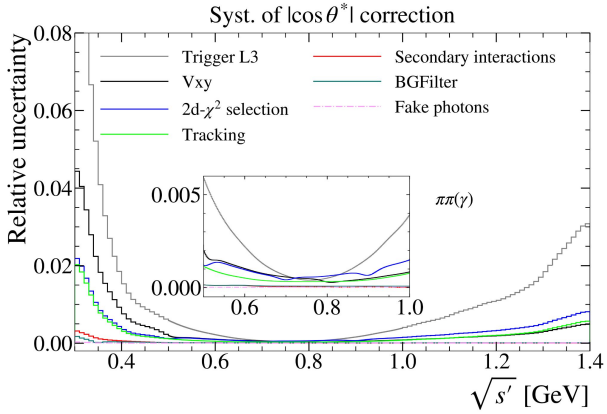
μμ(γ)

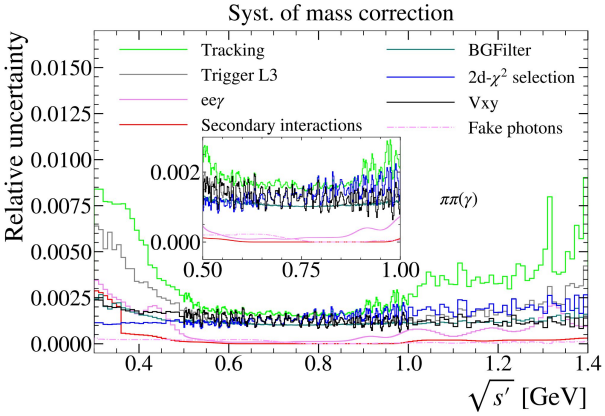


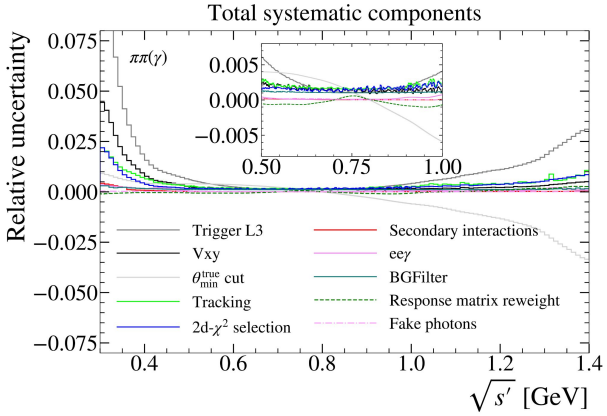




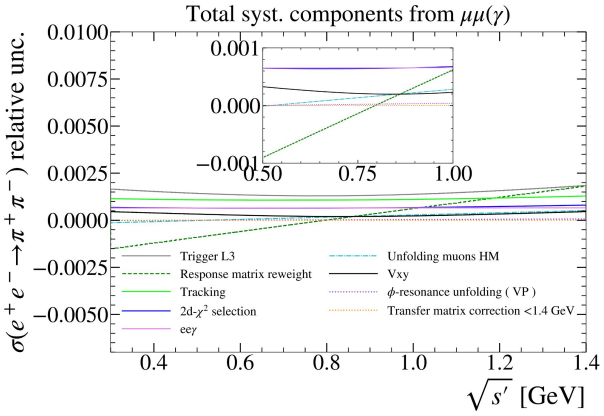
ππ(γ)

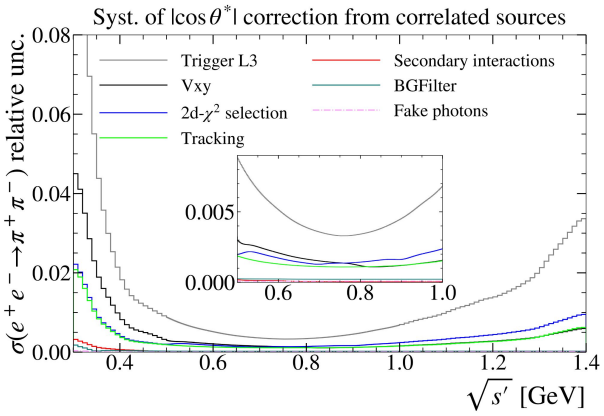


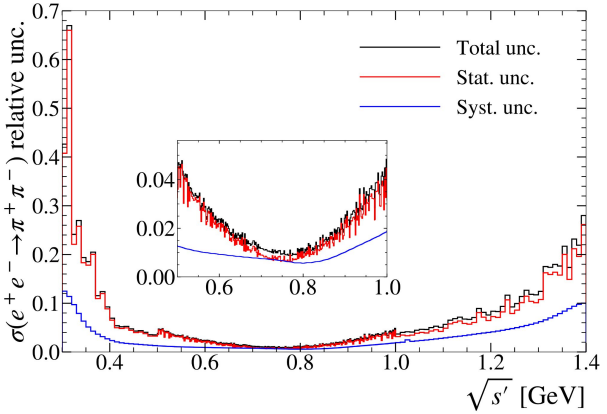


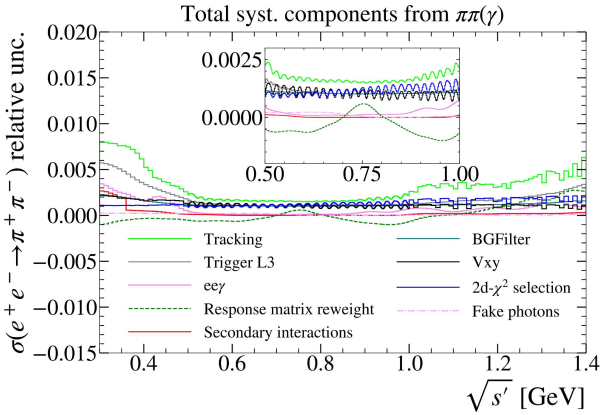


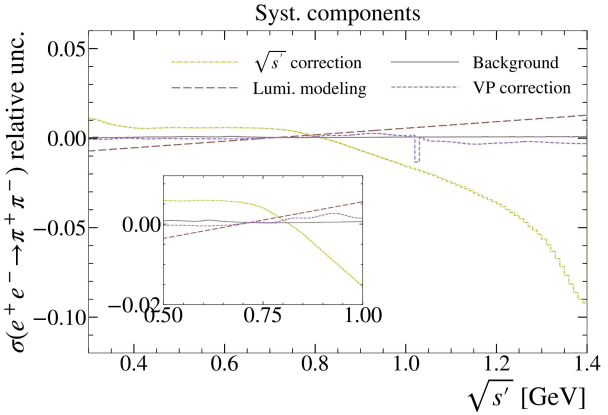
Cross-section uncertainties











Back

Statistical correlations among the BaBar 09 & BaBar 26 measurements

→ Two analyses with differences in: methodology (PID / cos θ* fits), luminosity, selection (~ x8 statistics)
These differences strongly reduce the correlations among the two results.

→ Employing coherent Poisson bootstrap replicas to account for statistical correlations among:

- mass spectra of 09 and 26 measurements
- mass spectrum of 09 and Xsec of the 26 measurement

(unable to run the full analysis chain for the 09 measurement - expected to further reduce correlations)

→ Correlations above propagated to the covariance among 09 & 26 cross-sections

→ Statistical correlations among 09 & 26 cross-sections have little impact on their comparison

Potential systematic correlations among BaBar 09 & BaBar 26 measurements

→ Complication from the fact that '09 uncertainties were provided at the Xsec level, with merged contributions from the ππ and μμ channels (for the uncertainty components where both channels contribute).

When this induces ambiguities for the 09-26 correlations (i.e. *uncertainties on correlations*), tested configuration correlating a given '09 uncertainty with the corresponding dominant component for '26.

→ Further *uncertainties on correlations* from methodological differences among the two analyses

Back

Potential systematic correlations among BaBar 09 & BaBar 26 measurements

Independent uncertainties:

→ PID uncertainties specific to '09

→ Unfolding uncertainty corresponding to (small for) '09 are negligible for '26;
Other (un)folding uncertainties or the effective $\sqrt{s'}$ definition are specific to the '26 approach.

→ Luminosity smoothing / fit method different for '09 / '26

Potentially correlated components:

→ Trigger/filter '09 & L3 $\cos\theta^*_\pi$ ($\mu\mu\gamma$) '26 → Dominant trigger uncertainty for '26

→ Tracking '09 & π tracking (mass) '26 → Larger than μ tracking (mass) for '26

→ Background '09 & Background '26

→ Kinematic fit (χ^2) '09 &
π 2d-χ^2 (mass) OR μ 2d-χ^2 (mass) '26 → Similar amplitudes for these two components for '26
Actually, not obvious if one should correlate '09 with '26, given the observed differences in χ^2 distributions and differences of selection cuts.
Correlating Kinematic fit (χ^2) '09 & π 2d-χ^2 (mass) '26 (conservative)

→ Because of *uncertainties on the systematic correlations* among the two analyses, performed comparisons and combinations, both with & without correlations

Methodology for comparison/combination of cross section data

→ Goal: combine experimental spectra with arbitrary point spacing / binning → *Here simpler use-case with common binning*

→ Requirements:

- Properly propagate uncertainties and correlations
  - *Between data bins of a given measurements*
    (covariance matrices and/or detailed split of uncertainties in sub-components)
  - *Between measurements*
    based on detailed information from split of uncertainties into sub-components
  - *Between different channels* – motivated by understanding of the meaning of systematic uncertainties and identifying the common ones
- Minimize biases
- Optimize average/integrals uncertainty
  (*without overestimating the precision with which the uncertainties of the measurements are known*)

Back

Combination procedure implemented in HVPTools software

For each final bin:

→ Compute an average value for each measurement and its uncertainty

→ Compute correlation matrix between measurements

→ Minimize χ^2 and get average coefficients (weights)

→ Compute average between measurements and its uncertainty

Evaluation of integrals and propagation of uncertainties:

→ Integral(s) evaluated for nominal result and for each set of toy pseudo-experiments; uncertainty of integrals from RMS of results for all toys

→ The pseudo-experiments also used to derive (statistical & systematic) covariance matrices of combined cross sections → Integral evaluation

→ Uncertainties also propagated through $\pm 1\sigma$ shifts of each uncertainty

→ *Checked consistency between the different approaches*

Combination procedure: weights of various measurements

For each final bin:

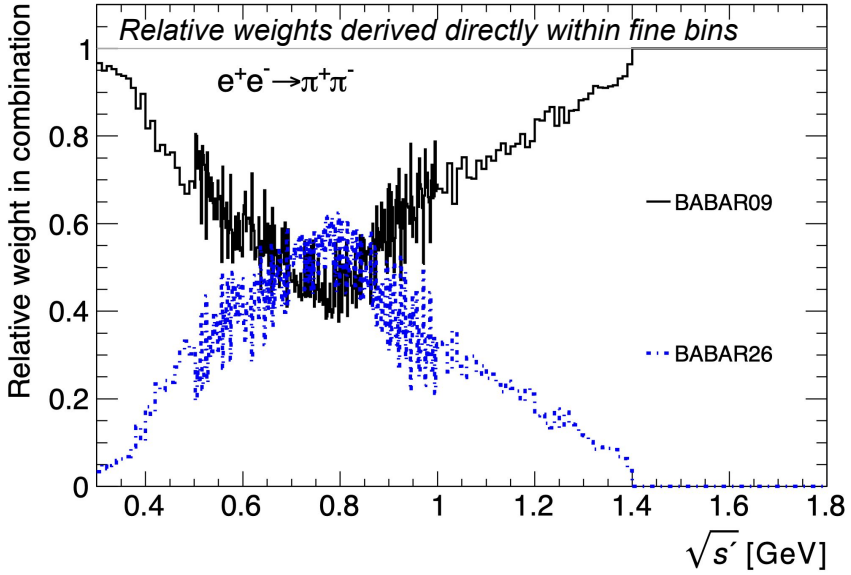
→ Minimize χ^2 and get average coefficients

Note: average weights designed account for bin sizes / point spacing of measurements
(do not over-estimate the weight of experiments with large bins)

→ Weights in fine bins evaluated using a common (large) binning for measurements + interpolation

→ Compare the precisions on the same footing

→ Here this procedure allows to also explore the sensitivity of weights, χ^2/ndf, integrals, uncertainties, to possible choices for treatment of correlations in intermediate mass ranges of ~100 MeV: *little impact*



→ Relative weights reflect the precision of the two measurements in different mass regions

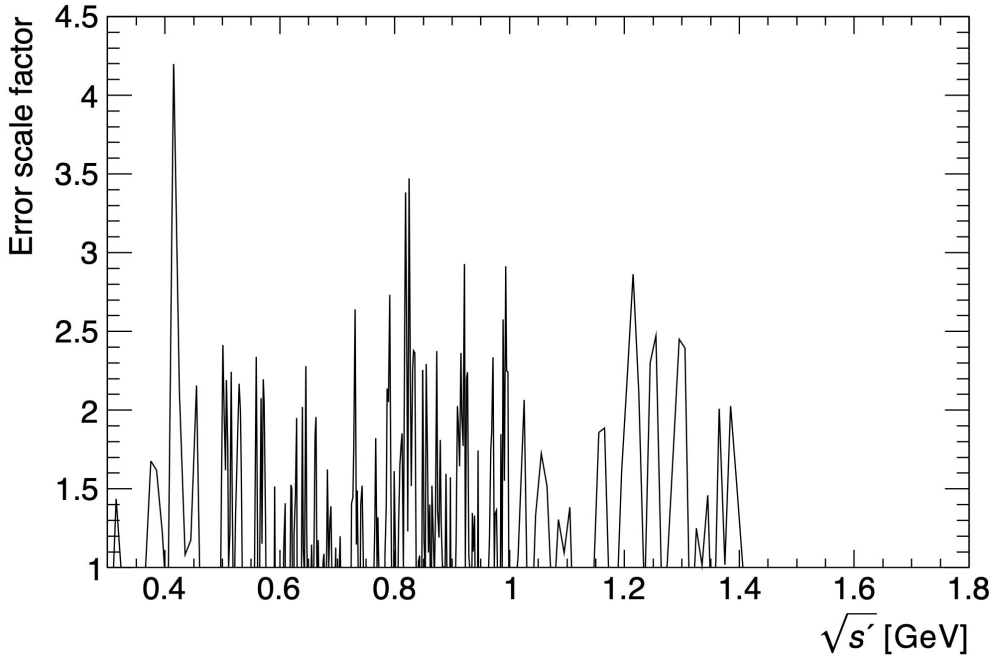
→ BaBar09 covering full range

Combination procedure: compatibility between measurements

For each final bin:

→ χ^2 /ndof: test locally the level of agreement between input measurements, *taking into account correlations*

→ Scale uncertainties in bins with χ^2 /ndof > 1 (PDG): *locally* conservative

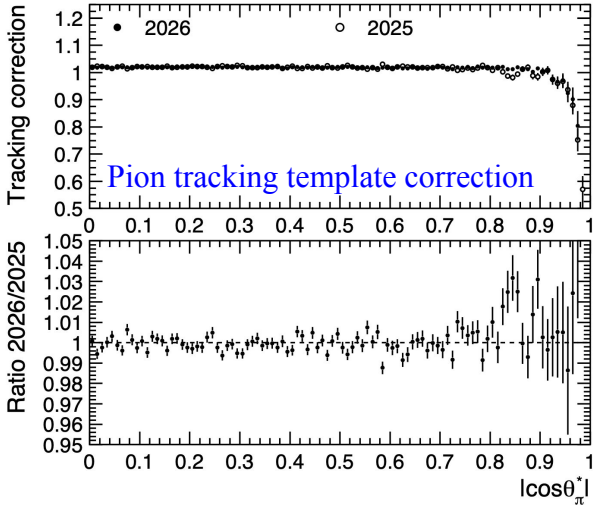


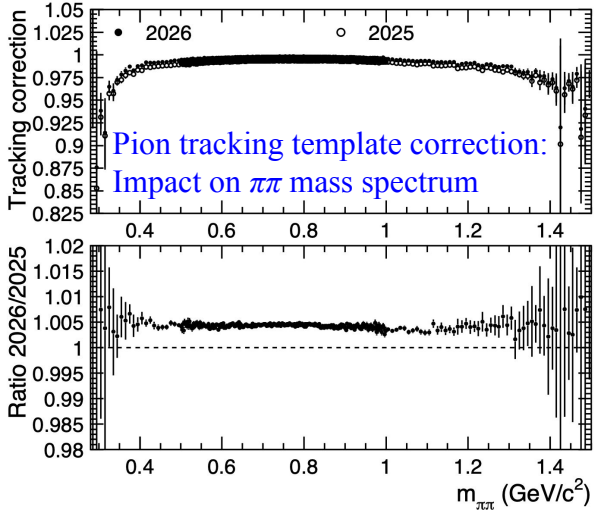
Main changes w.r.t. preliminary ’25 result

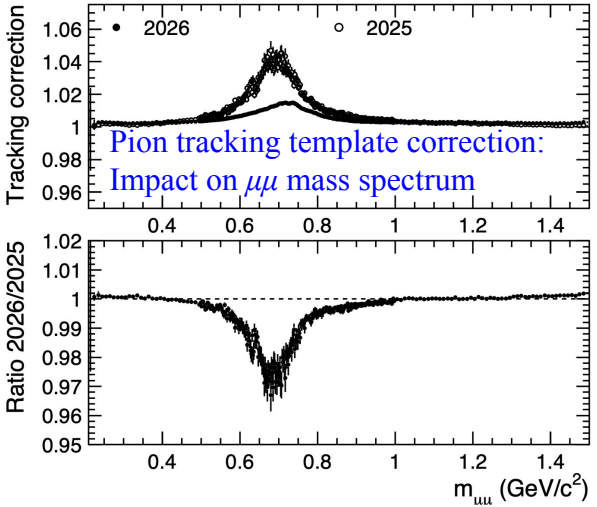
- Dressed pion form factor in MC changed to bare one
- Pion form factor parametrization in MC → negligibly small uncertainty with new fitting strategy; would have been over 2% with the 2025 muon fitting strategy
- Changed muon fitting strategy
- Pion tracking efficiency corrections → largest technical fix (bookkeeping fix)
- Pion BGFilter efficiencies → improved to correct for a bias due to the correlated loss
- Improved unfolding procedure → new three-step procedure to also correct for some boundary event loss at the high energy tail (in addition to detector resolution and FSR, which were already accounted for)
- Scaled kaon background prediction → data/MC normalisation from template fit (performed below 0.4 GeV) propagated to normalise MC at higher masses
- Improved systematic error propagation → improved fit stability + smoothing to mitigate statistical noise

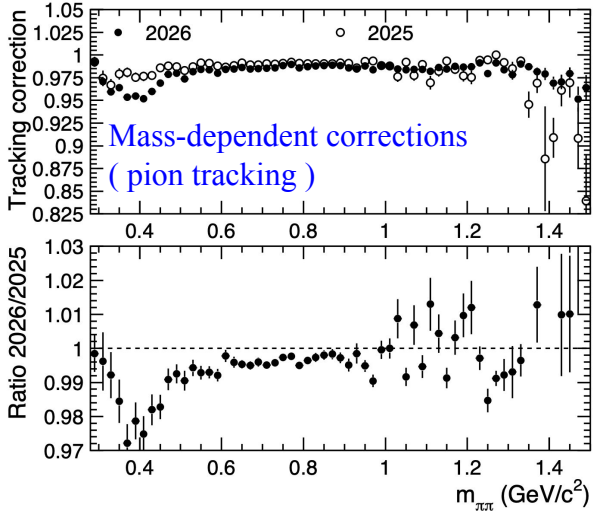
Back

Main changes w.r.t. preliminary ’25 result

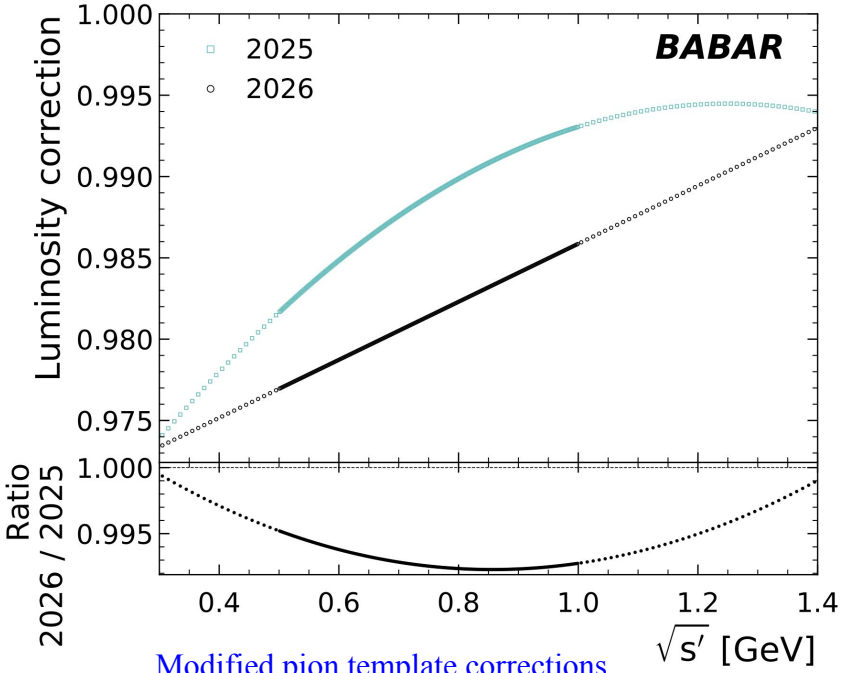




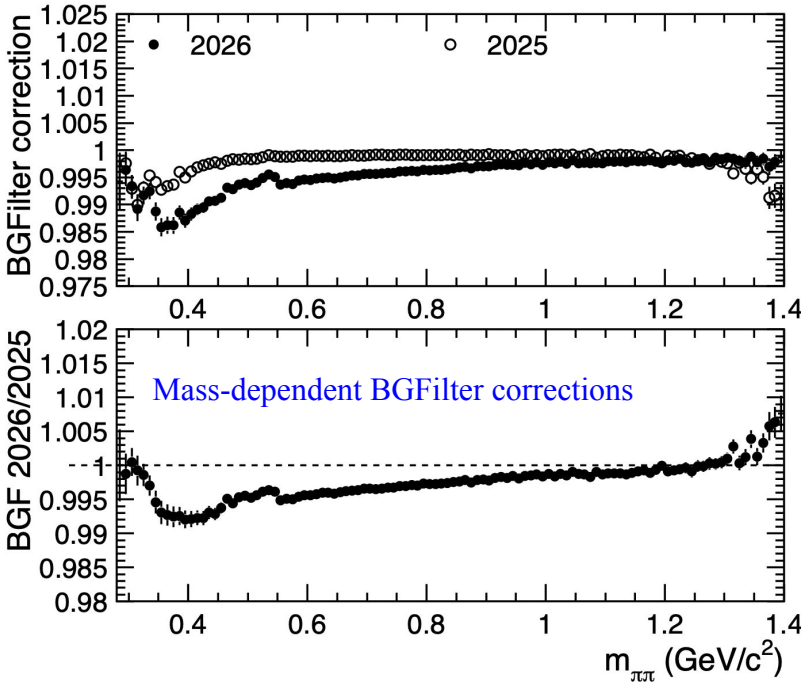




Main changes w.r.t. preliminary ’25 result

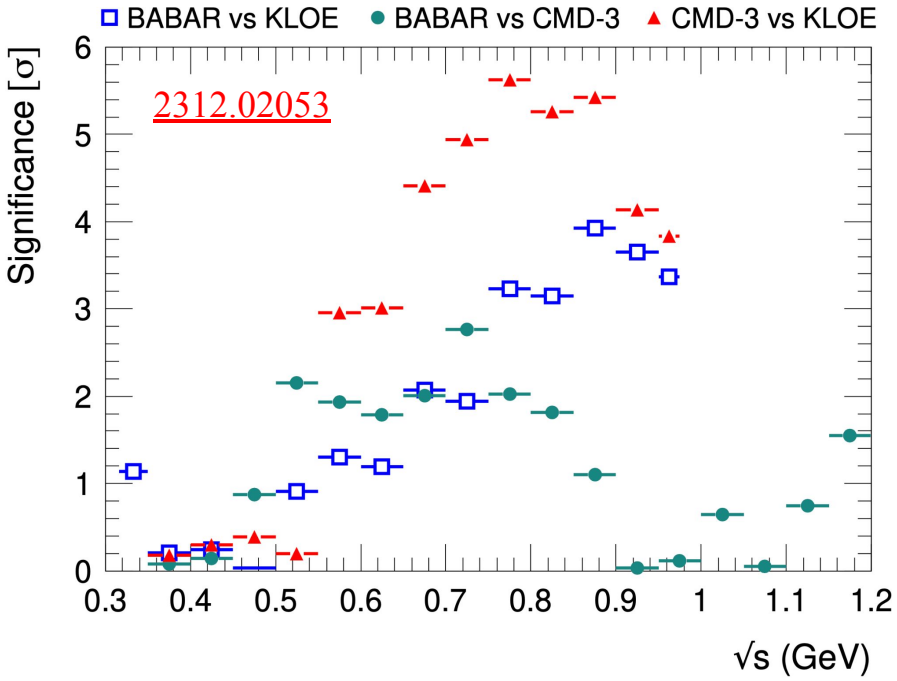


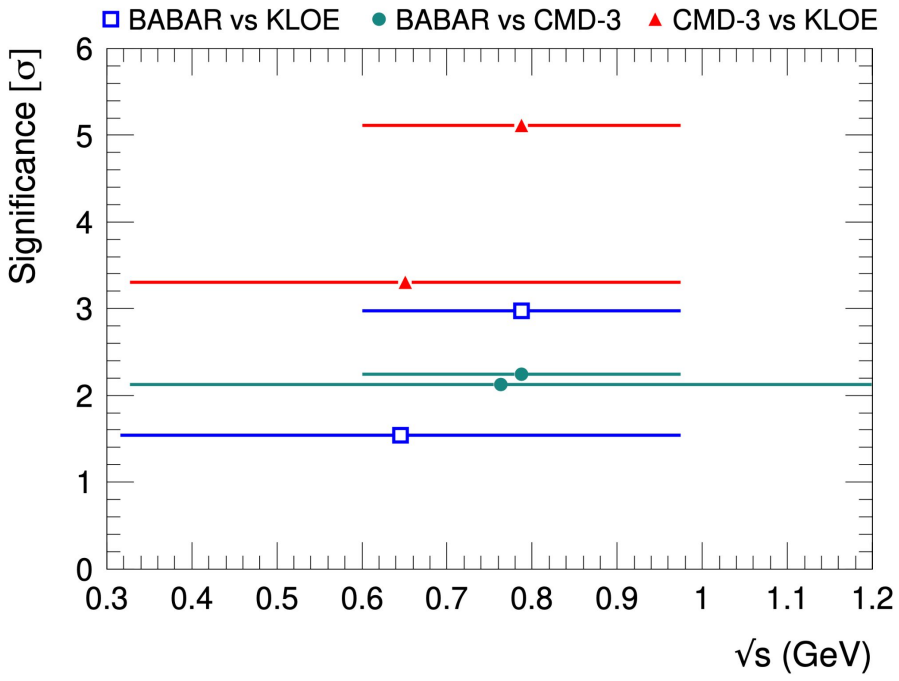
Modified pion template corrections
→ Choice of functional forms for data/Phokhara fit
→ Effective ISR luminosity



Quantitative comparisons for a_μ^{HVP}

→ Comparison of integrals computed in various restricted energy ranges, for individual e^+e^- experiments: significance of the difference between different experiments, taking into account correlations

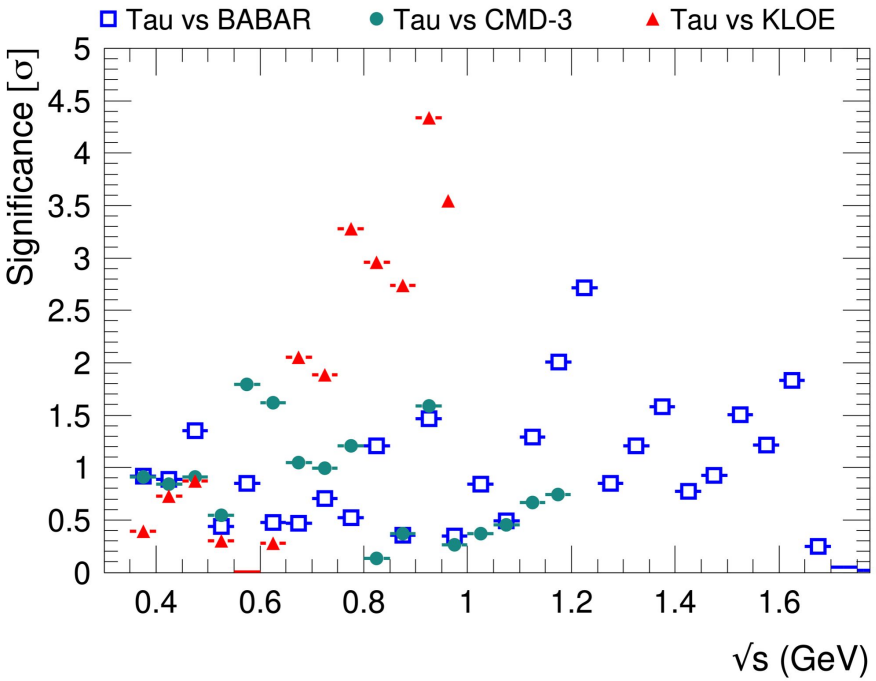


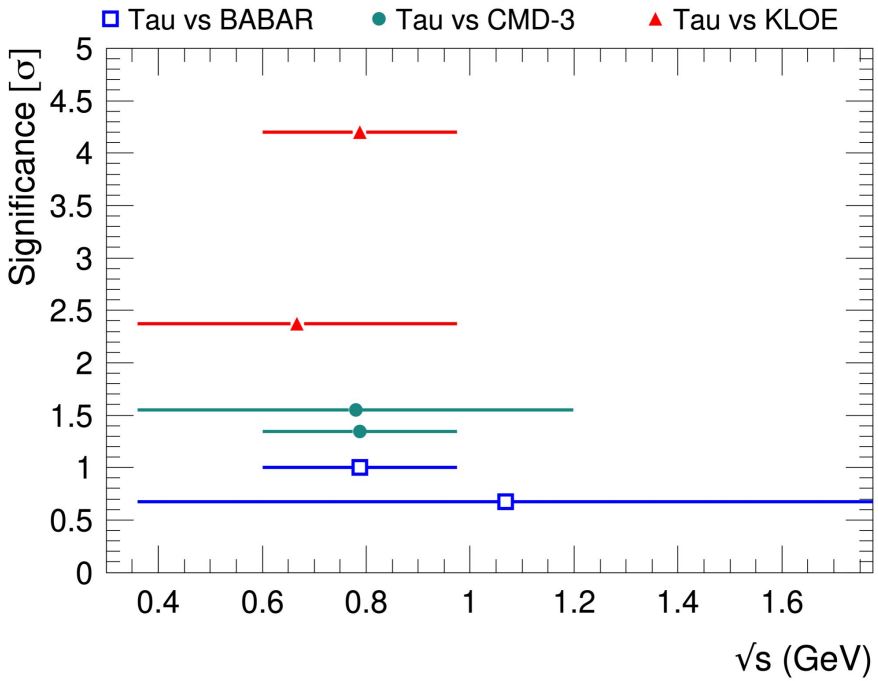


→ Largest tensions between CMD3 and KLOE

Quantitative comparisons for a_μ^{HVP}

→ Comparison of integrals computed in various restricted energy ranges, for τ / individual e^+e^- experiments: significance of the difference between different experiments, taking into account correlations





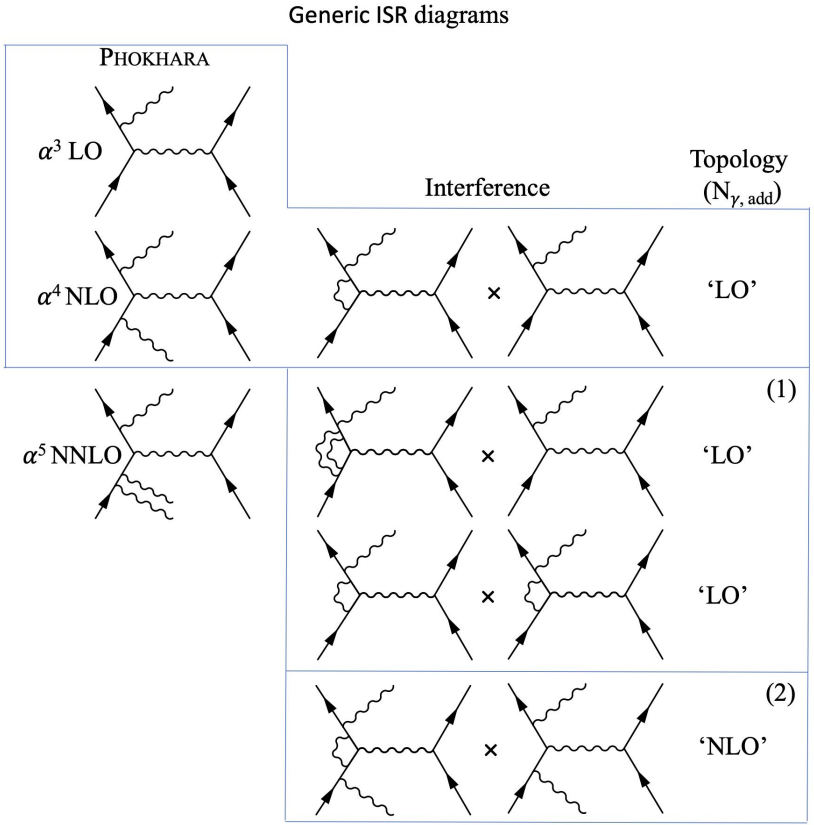
→ Largest tensions between Tau and KLOE

→ Good agreement among the Tau measurements

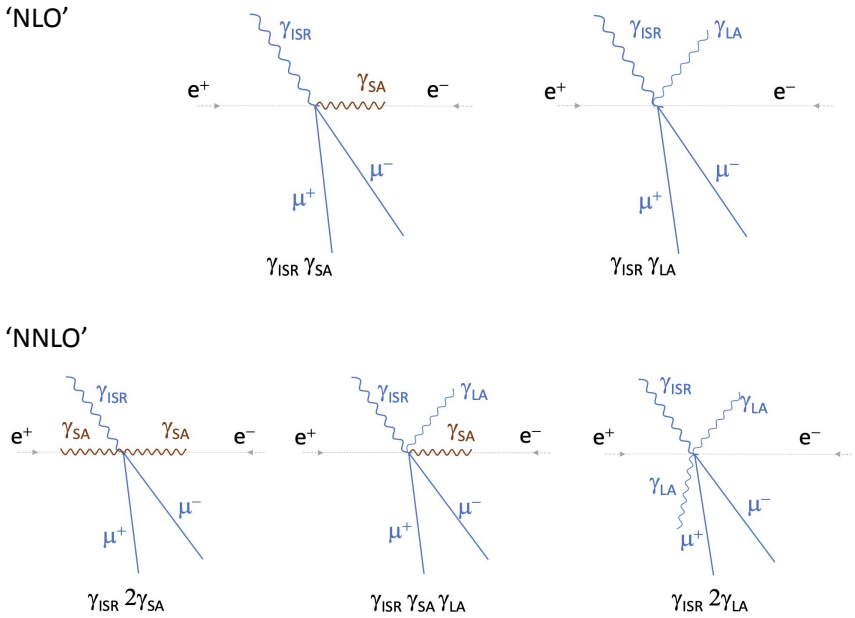
Impact of higher order photon emissions: *Unique '(N)NLO' BaBar study*

→ Studied in-situ in BaBar data, using kinematic fits: test the most frequently used Monte Carlo generators

- PHOKHARA: full NLO matrix element for ISR and FSR
- AFKQED: NLO and NNLO, with collinear approximation for additional ISR

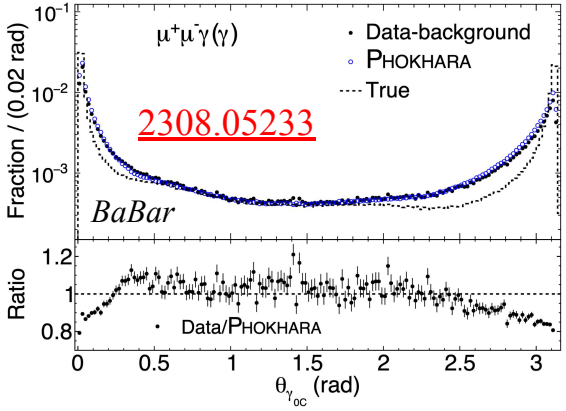


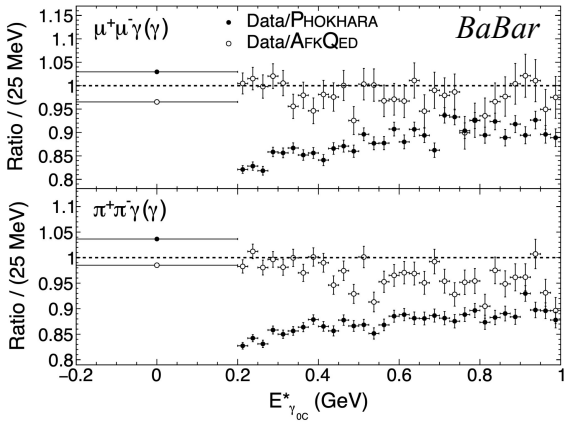
2308.05233

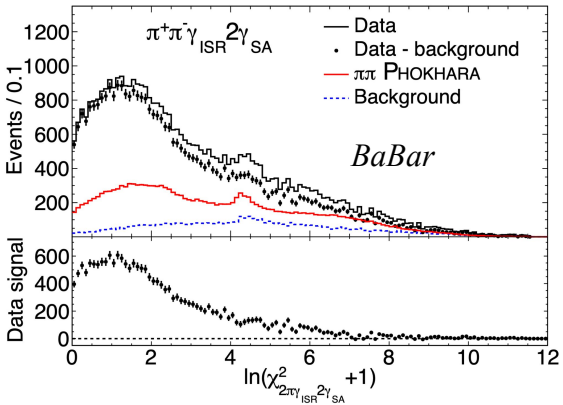


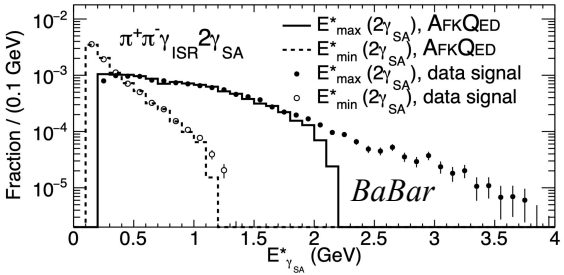
'(N)NLO': γ_{add} counting ($E_{\gamma\,add} > E_{min}$)

BaBar results on higher order photon emissions









→ NLO small-angle ISR in PHOKHARA higher than in data; large-angle ratios consistent with unity

→ *Independent Belle II confirmation of PHOKHARA problem*

→ AFKQED: reasonable description of rate and energy distributions for '(N)NLO' data

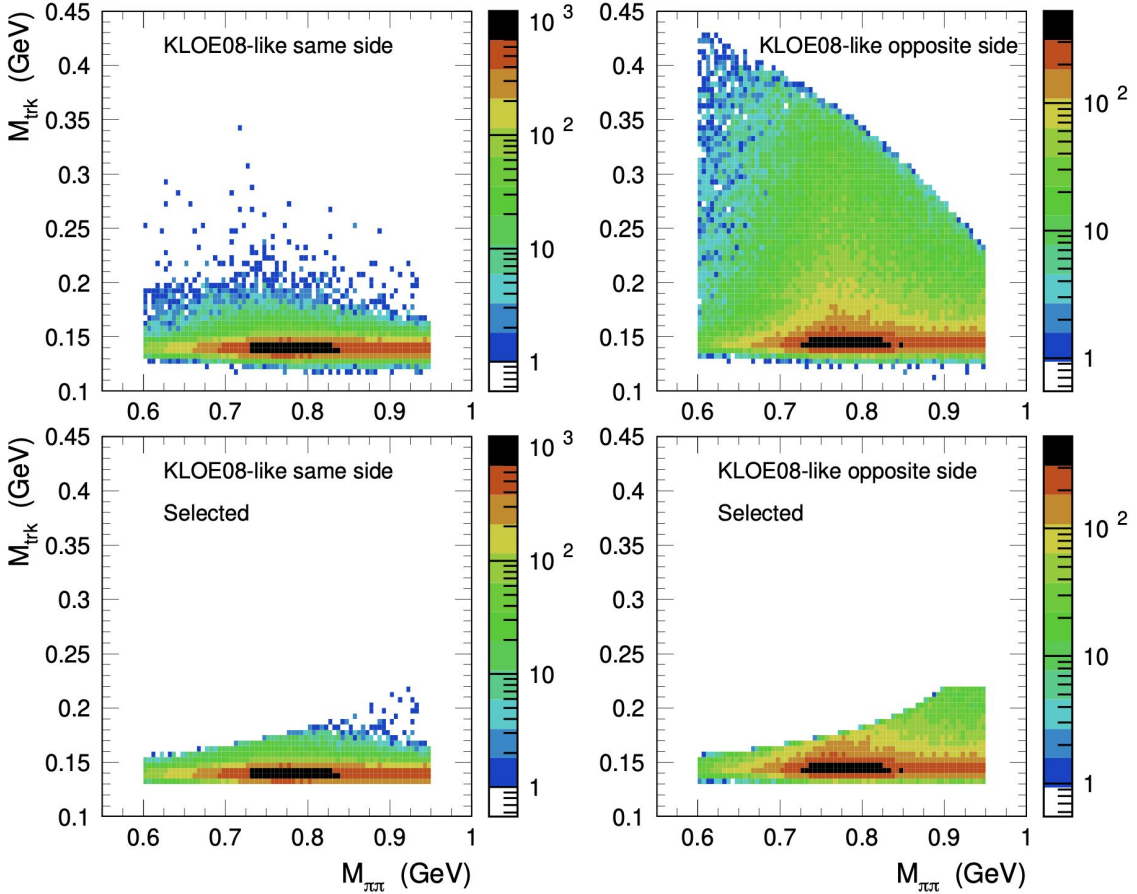
→ *NNLO contributions clearly observed in data*

→ BaBar measurements: *loose selection incorporates NLO and HO radiation, minimising MC-dependence*

→ Other ISR measurements *select 'LO' topology and rely on PHOKHARA for hard NLO (but with no NNLO)*

→ Aspects further studied with fast simulation, questioning the KLOE systematic uncertainties (2312.02053)

Impact of higher order photon emissions studied with fast simulation

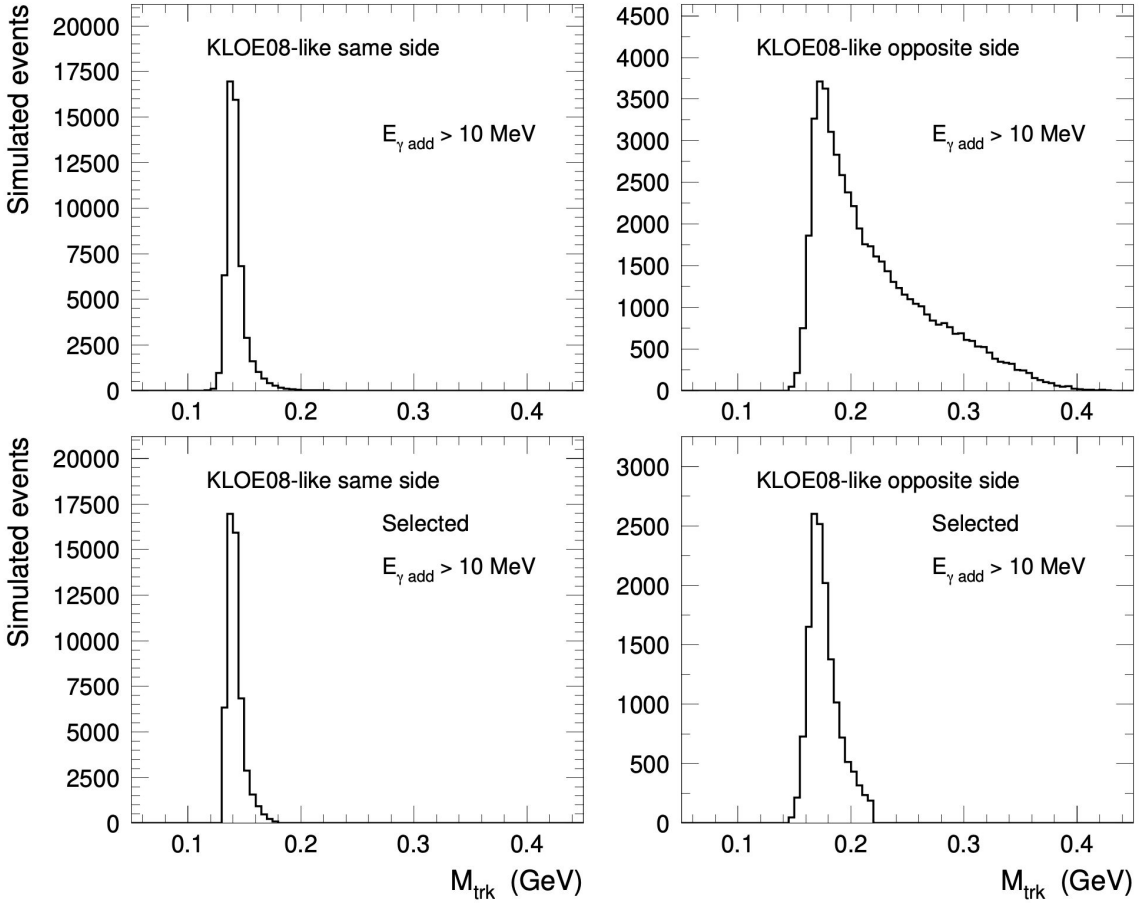


2312.02053

M_{trk} : common track mass of the two charged particles, computed under the LO assumption

→ Sensitivity of the KLOE selection to additional 'NLO' photon emissions (E > 5 MeV) in the same/opposite hemispheres w.r.t. the hard ISR photon

Impact of higher order photon emissions studied with fast simulation



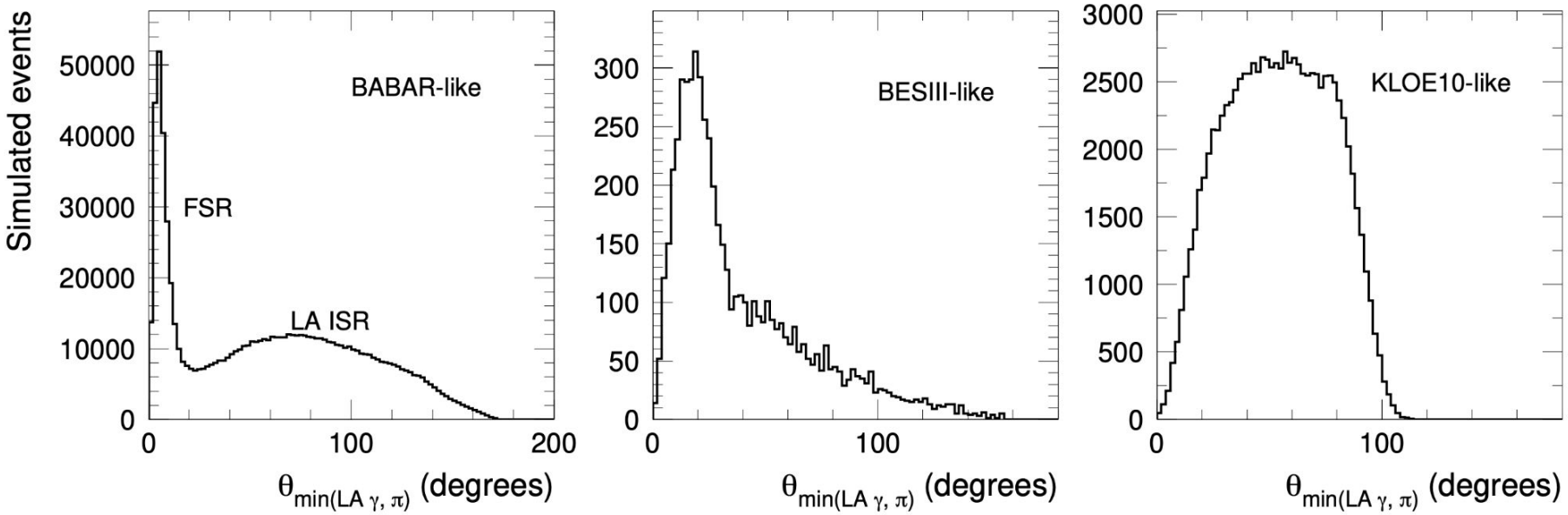
2312.02053

M_{trk} : common track mass of the two charged particles, computed under the LO assumption

→ Sensitivity of the KLOE selection to additional 'NLO' photon emissions (E > 10 MeV) in the same/opposite hemispheres w.r.t. the hard ISR photon

Impact of higher order photon emissions studied with fast simulation

2312.02053



→ The angular separation between FSR and LA ISR events is pronounced at high CM energy (BABAR), still visible at intermediate CM energy (BESIII), and vanishes at low CM energy (KLOE)