Developments in Precision $b \to s\ell\ell$ Analyses

B. Capdevila ICCUB, University of Barcelona





Based on:

Capdevila, **Matias**, **Novoa-Brunet**, **Patel**, **Smith**, arXiv:2504.00949
Alvarez-Cartelle, **Capdevila**, **Lunghi**, **Matias**, arXiv:2605.06567

The $b \to s\ell\ell$ anomalies

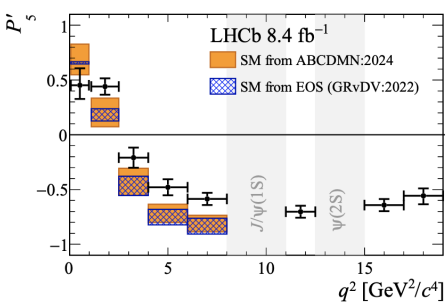
$b \to s\,\mu^+\mu^-$ anomalies

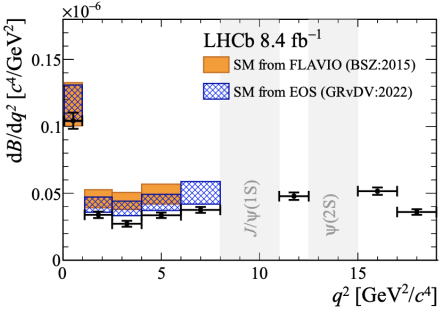
Several LHCb measurements deviate from SM predictions by 2-3σ:

- Angular observables in $B^{(0,+)} \to K^{*(0,+)}\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$ ($\ell = e,\ \mu$)
  - ⇒ CMS $B^0 \to K^{*0}\mu^+\mu^-$ angular analysis: largely compatible with LHCb analyses
  - ⇒ LHCb $B^0 \to K^{*0}\mu^+\mu^-$ angular analysis: different fit choices, q^2 binnings, **amplitude analyses** (more later), etc, (see also M. Birch's talk at Implications 2026)

LHCb, arXiv:2003.04831, arXiv:2012.13241
CMS, arXiv:2411.11820; LHCb, arXiv:2504.06346
LHCb, arXiv:2512.18053; see T. Mombächer's talk

- Branching ratios of $B \to K\ell^+\ell^-$, $B \to K^*\ell^+\ell^-$, and $B_s \to \phi\ell^+\ell^-$ LHCb, arXiv:1403.8044, arXiv:1506.08777
  - ⇒ LHCb $B_s \to \phi\ell^+\ell^-$ for $\ell = e,\ \mu$ branching ratio measurement LHCb, arXiv:1606.04731, arXiv:2105.14007





LFU violation in $b \to s\,\ell^+\ell^-$ decays

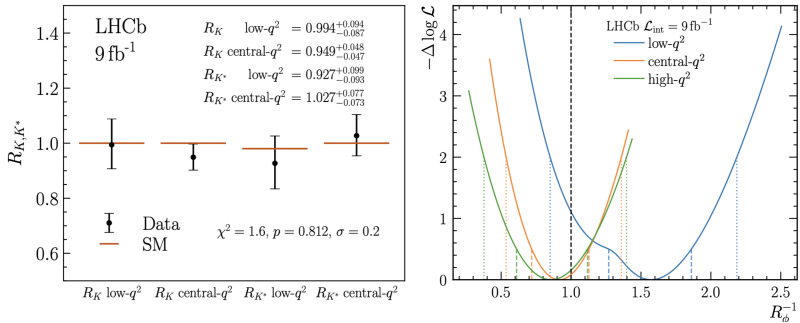
$$R_H = \frac{\mathcal{B}(B \to H\mu^+\mu^-)}{\mathcal{B}(B \to He^+e^-)}$$

- Combined measurement of the LFU ratios $R_K^{[0.1,1.1]}$, $R_K^{[1.1,6]}$, $R_{K*}^{[0.1,1.1]}$, $R_{K*}^{[1.1,6]}$

  LHCb, arXiv:2212.09152, arXiv:2212.09153
- Recent measurement of $R_\phi^{[0.1,1.1]}$, $R_\phi^{[1.1,6]}$, $R_\phi^{[15,19]}$

  LHCb, arXiv:2410.13748
- LFU ratio R_{pK}, CMS measurement of R_K

  LHCb, arXiv:1912.08139
  CMS, arXiv:2401.07090



Leptonic modes $B_{s,d} \to \mu^+\mu^-$

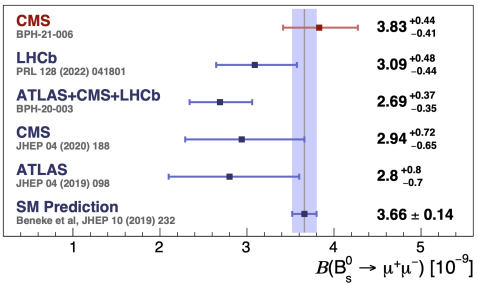
- Measurements of $\mathcal{B}(B_{s,d} \to \mu^+\mu^-)$ by LHCb, CMS, and ATLAS show deviations of only about $\sim 1\sigma$ with respect to SM predictions*

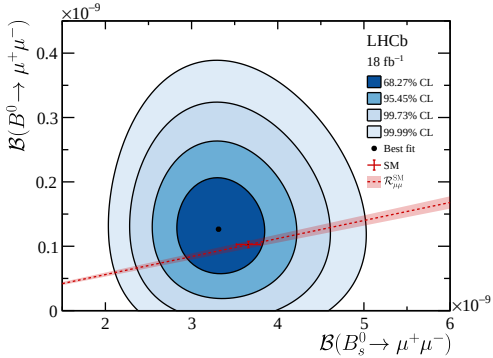
ATLAS, arXiv:1812.03017
CMS, arXiv:1910.12127, 2212.10311
LHCb, arXiv:1703.05747, 2108.09283

- New LHCb $\mathcal{B}(B_s \to \mu^+\mu^-)$ measurement (data from runs 1&2 and partially 3)

$$\mathcal{B}(B_s \to \mu^+\mu^-) = \left(3.31\,^{+\,0.31\,+\,0.15}_{-\,0.30\,-\,0.13}\right) \times 10^{-9}, \qquad \mathcal{B}(B_d \to \mu^+\mu^-) = \left(1.26\,^{+\,0.51\,+\,0.06}_{-\,0.47\,-\,0.05}\right) \times 10^{-10}$$

LHCb, arXiv:2611.00230





Bobeth, Buras, arXiv:2104.09521

*: depends on parameters like V_{cb}

Theoretical framework: exclusive decays

Effective theory for $b \to s\ell\ell$

- Effective Hamiltonian at scale m_b: $\mathcal{H}_{\text{eff}} = \mathcal{H}_{\text{eff, sl}} + \mathcal{H}_{\text{eff, had}}$
- **Semileptonic operators**: ($\mathcal{N} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \approx (34\text{ TeV})^{-2}$)

$$\mathcal{H}_{\text{eff, sl}} = -\mathcal{N}\left(\mathcal{C}_7 O_7 + \mathcal{C}_7' O_7' + \sum_\ell \sum_{i=9,10,P,S} \left(\mathcal{C}_i^\ell O_i^\ell + \mathcal{C}_i'^\ell O_i'^\ell\right)\right) + \text{h.c.}$$

$$O_7^{(\prime)} = \frac{m_b}{e}(\bar{s}\sigma_{\mu\nu} P_{R(L)} b)\, F^{\mu\nu}\,, \quad O_9^{(\prime)\ell} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \ell)\,, \quad O_{10}^{(\prime)\ell} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

$$\mathcal{C}_7^{\text{SM}} \simeq -0.3\,, \qquad \mathcal{C}_9^{\text{SM}} \simeq 4\,, \qquad \mathcal{C}_{10}^{\text{SM}} \simeq -4$$

Not considered here: (pseudo)scalar $O_{P,S}$ vanish in SM, could appear at dim. 6 in SMEFT (and tensor O_T only at dim. 8 in SMEFT)

- **Hadronic operators**:

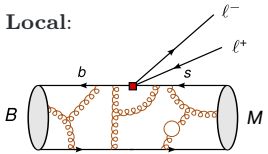
$$\mathcal{H}_{\text{eff, had}} = -\mathcal{N}\frac{16\pi^2}{e^2}\left(\mathcal{C}_8 O_8 + \mathcal{C}_8' O_8' + \sum_{i=1,\ldots,6} \mathcal{C}_i O_i\right) + \text{h.c.}$$

$$\text{e.g.} \quad O_1 = (\bar{s}\gamma_\mu P_L T^a c)(\bar{c}\gamma^\mu P_L T^a b)\,, \quad O_2 = (\bar{s}\gamma_\mu P_L c)(\bar{c}\gamma^\mu P_L b)$$

$B \to M\ell\ell$ decays: hadronic matrix elements ($M = K, K^*, \phi$)

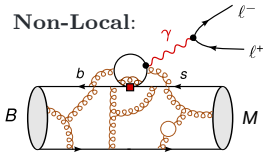
$$\mathcal{M}(B \to M\ell\ell) = \langle M\ell\ell|\mathcal{H}_{\rm eff}|B\rangle = \mathcal{N}\Big[\,(\mathcal{A}_V^\mu + \mathcal{H}^\mu)\;\bar u_\ell\gamma_\mu v_\ell + \mathcal{A}_A^\mu\;\bar u_\ell\gamma_\mu\gamma_5 v_\ell + \mathcal{A}_S\;\bar u_\ell v_\ell + \mathcal{A}_P\;\bar u_\ell\gamma_5 v_\ell\Big]$$

Local:



$$\mathcal{A}_V^\mu = -\tfrac{2im_b}{q^2}\,\mathcal{C}_7\langle M|\bar s\,\sigma^{\mu\nu}q_\nu\,P_R\,b|B\rangle + \mathcal{C}_9\langle M|\bar s\,\gamma^\mu\,P_L\,b|B\rangle + (P_L \leftrightarrow P_R, \mathcal{C}_i \to \mathcal{C}_i')$$
$$\mathcal{A}_A^\mu = \mathcal{C}_{10}\langle M|\bar s\,\gamma^\mu\,P_L\,b|B\rangle + (P_L \leftrightarrow P_R, \mathcal{C}_i \to \mathcal{C}_i')$$
$$\mathcal{A}_{S,P} = \mathcal{C}_{S,P}\langle M|\bar s\,P_R\,b|B\rangle + (P_L \leftrightarrow P_R, \mathcal{C}_i \to \mathcal{C}_i')$$

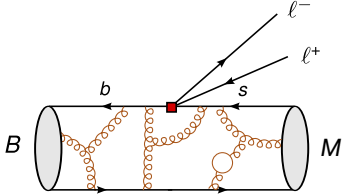
Non-Local:



$$\mathcal{H}^\mu = \tfrac{-16i\pi^2}{q^2}\sum_{i=1,\dots,6,8}\mathcal{C}_i\int d^4x\,e^{iq\cdot x}\langle M|T\{j^\mu_{\rm em}(x), O_i(0)\}|B\rangle\,, \qquad j^\mu_{\rm em} = \sum_q Q_q\,\bar q\gamma^\mu q$$

- **Wilson coefficients** $\mathcal{C}_i = \mathcal{C}_i^{\rm SM} + \mathcal{C}_i^{\rm NP}$:
  perturbative, short-distance physics (q^2 independent), well-known in SM, parameterise heavy NP
- **local** and **non-local** **hadronic matrix elements**:
  non-perturbative, long-distance physics (q^2 dependent), **main source of uncertainty**

Local matrix elements



$$\mathcal{A}_V^\mu = -\frac{2im_b}{q^2}\,\mathcal{C}_7\langle M|\bar{s}\,\sigma^{\mu\nu}q_\nu\,P_R\,b|B\rangle + \mathcal{C}_9\langle M|\bar{s}\,\gamma^\mu\,P_L\,b|B\rangle$$
$$+ (P_L \leftrightarrow P_R, \mathcal{C}_i \to \mathcal{C}_i')$$
$$\mathcal{A}_A^\mu = \mathcal{C}_{10}\langle M|\bar{s}\,\gamma^\mu\,P_L\,b|B\rangle + (P_L \leftrightarrow P_R, \mathcal{C}_i \to \mathcal{C}_i')$$
$$\mathcal{A}_{S,P} = \mathcal{C}_{S,P}\langle M|\bar{s}\,P_R\,b|B\rangle + (P_L \leftrightarrow P_R, \mathcal{C}_i \to \mathcal{C}_i')$$

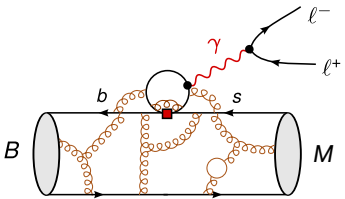
- $\langle M|\bar{s}\,\Gamma_i\,b|B\rangle$ matrix elements are parameterised by:
  - ⇒ **3 form factors** for each **spin zero** final state $M = \boldsymbol{K}$
  - ⇒ **7 form factors** for each **spin one** final state $M = \boldsymbol{K^*}, \boldsymbol{\phi}$
- Determination of form factors
  - ⇒ high q^2: **Lattice QCD**

    HPQCD, arXiv:1306.2384,2207.12468
    Fermilab, MILC, arXiv:1509.06235
    Horgan, Liu, Meinel, Wingate, arXiv:1310.3722, arXiv:1501.00367
  - ⇒ low q^2: **Continuum methods**
    e.g. Light-cone sum rules (LCSR)

    Ball, Zwicky, arXiv:hep-ph/0406232
    Khodjamirian, Mannel, Pivovarov, Wang, arXiv:1006.4945
    Bharucha, Straub, Zwicky, arXiv:1503.05534
    Gubernari, Kokulu, van Dyk, arXiv:1811.00983
  - ⇒ low + high q^2: Combined fit to **continuum methods** + **lattice** / **lattice**

    Altmannshofer, Straub, arXiv:1411.3161
    Bharucha, Straub, Zwicky, arXiv:1503.05534
    Gubernari, Kokulu, van Dyk, arXiv:1811.00983

Non-local matrix elements



$$\mathcal{H}^\mu = \frac{-16i\pi^2}{q^2} \sum_{i=1,\ldots,6,8} \mathcal{C}_i \int d^4x\, e^{iq\cdot x} \langle M|T\{j^\mu_{\rm em}(x), O_i(0)\}|B\rangle$$

$$j^\mu_{\rm em} = \sum_q Q_q\, \bar{q}\gamma^\mu q$$

- **Main source of uncertainty**
- Non-local contributions can mimic New Physics in $\mathcal{C}_9$
- Several approaches to estimate beyond-QCDF contributions at low q^2
  - ⇒ Fit of sum of resonances to data — Blake, Egede, Owen, Pomery, Petridis, arXiv:1709.03921
  - ⇒ Direct fit to angular data — Ciuchini, Fedele, Franco, Mishima, Paul, Silvestrini, Valli, arXiv:1512.07157
  - ⇒ Light-Cone Sum Rules estimates — Khodjamirian, Mannel, Pivovarov, Wang, arXiv:1006.4945; Gubernari, van Dyk, Virto, arXiv:2011.09813; Bobeth, Chrzaszcz, van Dyk, Virto, arXiv:1707.07305; Gubernari, van Dyk, Virto, arXiv:2011.09813; Gubernari, Reboud, van Dyk, Virto, arXiv: 2206.03797; Gubernari, Reboud, van Dyk, Virto, arXiv:2305.06301
  - ⇒ Analyticity + experimental data on $b \to sc\bar{c}$
  - ⇒ Proof-of-principle work on lattice computations — Frezzotti *et al*, arXiv:2508.03655

Unbinned analyses: theoretical perspective

Experimental analyses: from amplitudes to observables

- Theoretical $B \to K^* \ell^+ \ell^-$ P-wave transversity amplitudes ($\lambda = \perp, \parallel, 0$):

$$A_\lambda^{L,R}(q^2) = \mathcal{N}_\lambda \left[(\mathcal{C}_9 \mp \mathcal{C}_{10})\, \mathcal{F}_\lambda(q^2) + \frac{2 m_b m_B}{q^2} \left(\mathcal{C}_7 \mathcal{F}_\lambda^{\mathcal{T}}(q^2) - 16\pi^2 \frac{m_B}{m_b} \mathcal{H}_\lambda(q^2) \right) \right]$$

  - $\Rightarrow$ Local form factors and non-local amplitudes: their q^2 dependence is central to disentangling short and long distances
  - $\Rightarrow$ Angular coefficients $J_i(q^2)$ are bilinear combinations of transversity amplitudes:

$$J_i(q^2) = J_i\left(A_\lambda^{L,R}(q^2) \right), \qquad \frac{d^4\Gamma}{dq^2\, d\Omega} \propto \sum_i f_i(\Omega)\, J_i(q^2)$$

- **Binned analyses**: extract angular coefficients J_i in separate q^2 bins
- **Unbinned analyses**: extract the q^2 dependence of $J_i(q^2)$ or $A_\lambda^{L,R}(q^2)$ directly from data

Unbinned experimental analyses: different strategies

- **Unbinned angular-coefficient extraction**:
  - ⇒ **Moments** + k**NN**: extract $J_i(q^2)$ without imposing an amplitude parametrisation (pseudo-experiment proof of principle)

    Capdevila, Matias, Novoa-Brunet, Patel, Smith, arXiv:2504.00949

- **Unbinned amplitude analyses**:
  - ⇒ **Dispersive fit**: fit local/non-local form-factor parameters, with or without $q^2 < 0$ theory constraints

    LHCb, arXiv:2312.09102
  - ⇒ **Resonance-based amplitude fit**: include one- and two-particle intermediate states over the full q^2 spectrum

    LHCb, arXiv:2405.17347
  - ⇒ **Legendre amplitudes**: flexible polynomial basis in q^2, with amplitude symmetry fixed by convention, $1.1 < q^2 < 8\,\mathrm{GeV}^2$

    LHCb, arXiv:2608.12215

Non-trivial observable relations and geometrical bounds

- In the absence of scalar amplitudes, the J_i are **not independent**: $J_{2c} = f(J_{2s}, J_3, \ldots, J_9)$

  $\Rightarrow$ In the approximation $P_i^{\rm CP}, P_3, P'_{6,8} \to 0$ (no large weak or strong phases):

$$P_2 \simeq \frac{1}{2}\left[P'_4 P'_5 + \frac{1}{\beta_\ell}\sqrt{(-1+P_1+P'^2_4)(-1-P_1+\beta_\ell^2 P'^2_5)}\right]$$

Egede, Hurth, Matias, Ramon, Reece, arXiv:1005.0571
Matias, Serra, arXiv:1402.6855

- **Geometrical bounds** follow from Cauchy–Schwarz and the transversity vector structure:

$$|P_2| \leq \frac{1}{2\beta_\ell}\sqrt{1-4P_3^2}, \qquad |P'_4| \leq \sqrt{1-P_1-\beta_\ell^2 P'^2_6},$$

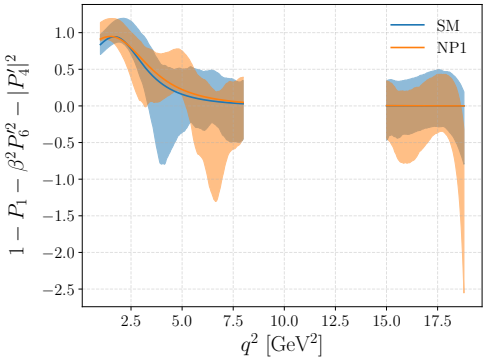
$$|P'_5| \leq \frac{1}{\beta_\ell}\sqrt{1+P_1-P'^2_8}, \quad |P'_6| \leq \frac{1}{\beta_\ell}\sqrt{1-P_1-P'^2_4}$$

  $\Rightarrow$ An unbinned q^2 measurement permits direct **self-consistency tests**

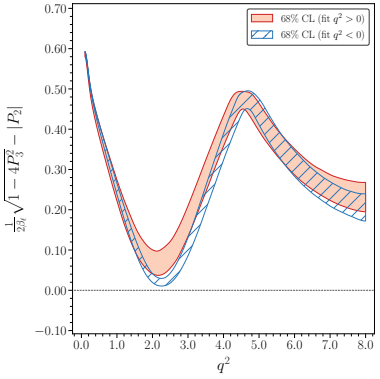
Capdevila, Matias, Novoa-Brunet, Patel, Smith, arXiv:2504.00949

Geometrical bounds: moments pseudo-experiment versus LHCb fit

Moments pseudo-experiment



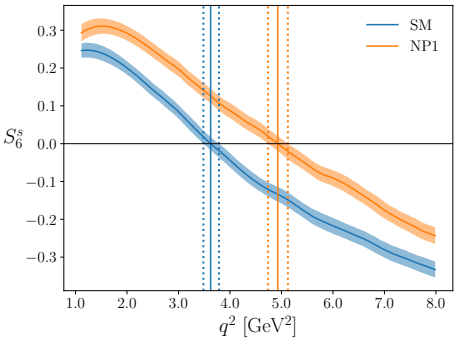
LHCb dispersive amplitude fit



- Illustrative P_2 bound
  - ⇒ LHCb analysis of arXiv:2312.09102 determined the amplitudes: geometric bounds **satisfied by construction**
  - ⇒ **Consistency test**: the **reconstructed observables from moments** need not satisfy

Capdevila, Matias, Novoa-Brunet, Patel, Smith, arXiv:2504.00949

Zero-crossings from the continuous q^2 dependence



- Leading-order illustration:

$$q_0^2(P_2) \simeq -2 m_b m_B \frac{\mathcal{C}_7^{\text{eff}}}{\text{Re}\,\mathcal{C}_9^{\text{eff}}(q_0^2)}$$

- Full NLO predictions have non-negligible form-factor and non-local uncertainties
  - ⇒ Within the SM

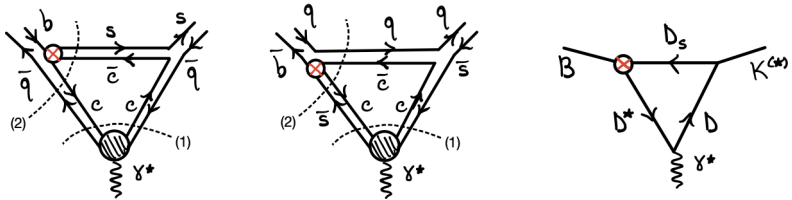
$$q_0^2(P_2) = 3.60 \pm 0.40\,\text{GeV}^2,$$
$$q_0^2(P_5') = 2.16 \pm 0.29\,\text{GeV}^2.$$

- Unbinned analyses yield continuous observables:
  - ⇒ We can experimentally determine zeroes and extrema
  - ⇒ These contain information on the Wilson coefficients
  - ⇒ Ensembles of pseudo-experiments are needed to calibrate **zero-position uncertainties** and correlations

Capdevila, Matias, Novoa-Brunet, Patel, Smith, arXiv:2504.00949

Hadronic rescattering: anomalous thresholds

Charm rescattering: anomalous thresholds



- Rescattering triangles generate **anomalous branch cuts** in q^2, also present in the NLO partonic matching functions
- Their contributions to non-local form factors **require explicit treatment** in dispersion relations, beyond parametrisations of the ordinary cut

M Ciuchini, M Fedele, E Franco, L Silvestrini, A Paul, M Valli, arXiv:2212.10516
G Isidori, Z Polonsky, A Tinari, arXiv:2405.1755
S Mutke, M Hoferichter, B Kubis, arXiv:2406.14608
G Isidori, Z Polonsky, A Tinari, arXiv:2507.17824
M Hoferichter, B Kubis, S Mutke, arXiv:2604.01284

Could an unknown hadronic contribution imitate $\mathcal{C}_{9\mu}$?

- In exclusive $B \to K^{(*)}\mu\mu$, consider the schematic decomposition

$$\mathcal{C}_{9\mu}^{\text{eff}}(q^2; j, M) = \mathcal{C}_9^{\text{SM}} + Y(q^2) + \mathcal{C}_{9,j}^{\text{non-pert},M}(q^2) + \mathcal{C}_{9,\text{extra}} + \mathcal{C}_{9\mu}^{\text{NP}}$$

  - $\Rightarrow$ $j = \perp$, $\parallel$, 0 and $M = K, K^*$
  - $\Rightarrow$ $Y(q^2)$ denotes perturbative contributions and $\mathcal{C}_{9,j}^{\text{non-pert},M}(q^2)$ the non-perturbative ones
  - $\Rightarrow$ $\mathcal{C}_{9,\text{extra}}$ is any non-perturbative contribution not included in $\mathcal{C}_{9,j}^{\text{non-pert},M}(q^2)$

- **Nightmare scenario:** $\mathcal{C}_{9,\text{extra}}$ is constant in q^2, universal across exclusive modes and polarisations, and can mimic a Wilson-coefficient shift
  - $\Rightarrow$ It is **not** a short-distance Wilson coefficient
  - $\Rightarrow$ It parameterises additional hadronic effects beyond those already included

- **Test:** inclusive $B \to X_s\ell\ell$ should depend on genuine $\mathcal{C}_9^{\text{NP}}$, but not on a large *exclusive-specific* residual hadronic term

Alvarez-Cartelle, Capdevila, Lunghi, Matias, arXiv:2605.06567

Theoretical framework: inclusive decays

Inclusive $B \to X_s \ell^+ \ell^-$: theoretical framework

- Inclusive decays can be calculated using the **Heavy Quark Expansion** (HQE) away from phase-space boundaries (up to quark–hadron duality violations)

  ⇒ The differential rate in the HQE:

$$\frac{d\Gamma(B \to X_s \ell\ell)}{dq^2} = \frac{d\Gamma(b \to s\ell\ell)}{dq^2} + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{m_b^2}, \frac{\Lambda^2_{\rm QCD}}{m_c^2}\right) + \Delta^{\rm LD}_{\rm fact} + \Delta^{\rm LD}_{\rm non\text{-}fact}$$

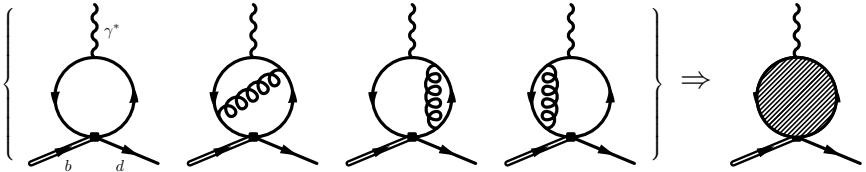
  ⇒ **Low q^2:** HQE, plus long-distance non-factorizable resolved-photon contributions in **SCET** (estimated uncertainty $\sim 5\%$)

  ⇒ **High q^2:** endpoint-enhanced power terms, weak annihilation at $1/m_b^3$, and high-q^2 OPE for long-distance non-factorizable soft-gluon contributions

  ⇒ Krüger–Sehgal mechanism for factorisable long-distance corrections at both low and high q^2

Benzke, Hurth, Turczyk, arXiv:1705.10366
Bartocci, Böer, Hurth, arXiv:2411.16634; see T. Hurth's talk
Huber, Hurth, Jenkins, Lunghi, Qin, Vos, arXiv:1908.07507
Huber, Hurth, Jenkins, Lunghi, Qin, Vos, arXiv:2007.04191

Factorisable corrections in $B \to X_s \ell^+ \ell^-$



- **Factorisable contributions** correspond to matrix elements of the form

$$\langle X_s \ell^+ \ell^- | \, T\{j_{\rm em}^\mu, \mathcal{O}_{1,2}\} \, |\bar{B}\rangle \;\propto\; \langle X_s | \bar{s} \gamma^\nu P_L b | \bar{B} \rangle \, \Pi_\nu{}^\mu(q)$$

- **Krüger–Sehgal:** replace the perturbative factorisable quark-loop functions by a dispersive determination from $e^+e^- \to$ hadrons:

$$(q^\mu q^\nu - q^2 g^{\mu\nu}) \, \Pi(q^2) = i \int d^4x \, e^{iq\cdot x} \langle 0 | T\{ j_{\rm em}^\mu(x) j_{\rm em}^\nu(0) \} | 0 \rangle, \qquad R_{\rm had}(s) = 12\pi \, \mathrm{Im}\, \Pi(s)$$

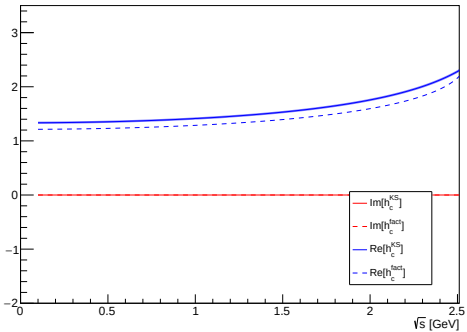
  $\Rightarrow$ **Optical theorem**: fixes $\mathrm{Im}\,\Pi$ with the spectral function $R(s)$

  $\Rightarrow$ A subtracted dispersion relation reconstructs $\mathrm{Re}\,\Pi$, with perturbative matching at spacelike q^2

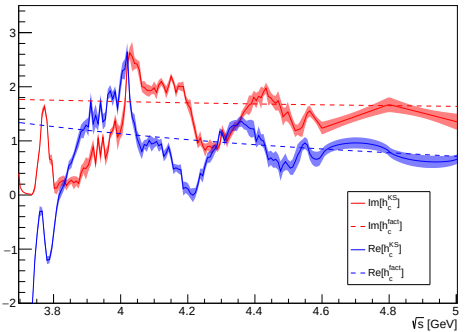
Krüger, Sehgal, hep-ph/9603237
Huber, Hurth, Jenkins, Lunghi, Qin, Vos, arXiv:1908.07507

Krüger–Sehgal mechanism at work

Low q^2



High q^2



⇒ Charm-resonance structure enters through the measured vacuum-polarisation spectral function, rather than a purely perturbative quark loop

Huber, Hurth, Jenkins, Lunghi, Qin, Vos, arXiv:1908.07507

High q^2: local OPE and semileptonic normalisation

- The HQE expansion parameter $\Lambda_{\rm QCD}/(m_b - \sqrt{q^2})$ grows at high q^2
  - ⇒ Enhanced dimension-six **Darwin-operator** and **weak-annihilation** (WA) effects
  - ⇒ Compute **the ratio of rare and charmless semileptonic rates with the same cut:**

$$\mathcal{R}(q^2_{\rm min}) = \frac{\displaystyle\int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \, \frac{d\Gamma(B \to X_s \ell^+ \ell^-)}{dq^2}}{\displaystyle\int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \, \frac{d\Gamma(B \to X_u \ell \nu)}{dq^2}}$$

$$\mathcal{B}(B \to X_s \ell^+ \ell^-)_{>15} = \mathcal{R}(15\,\text{GeV}^2)\, \mathcal{B}(B \to X_u \ell \nu)^{\rm exp}_{>15}$$

  - ⇒ Both channels share the dominant endpoint-sensitive HQE structures
  - ⇒ Normalisation reduces the Darwin/WA uncertainties, without eliminating all power corrections

Ligeti, Tackmann, arXiv:0707.1694
Huber, Hurth, Jenkins, Lunghi, Qin, Vos, arXiv:1908.07507
Alvarez-Cartelle, Capdevila, Lunghi, Matias, arXiv:2605.06567

- **Soft non-factorisable contributions at high** q^2: admit a local OPE, using $r = q^2/(4m_c^2)$ and expanding in $t = k \cdot q/(2m_c^2)$, with k the soft-gluon momentum

Buchalla, Isidori, Rey, hep-ph/9705253

Probing unknown non-perturbative effects

The four $\mathcal{F}$ observables

- Fractions of the inclusive rate accounted for by $B \to K\mu\mu$ and $B \to K^*\mu\mu$:

$$\mathcal{F}_{K^{(*)}}^{[1.1,6]} = \frac{\mathcal{B}(B \to K^{(*)}\mu\mu)_{[1.1,6]}}{\mathcal{B}(B \to X_s\ell\ell)_{[1,6]}}, \qquad \mathcal{F}_{K^{(*)}}^{>15} = \frac{\mathcal{B}(B \to K^{(*)}\mu\mu)_{>15}}{\mathcal{B}(B \to X_s\ell\ell)_{>15}}$$

- **Theory**: Under the assumption that there are no sizeable sources of missing hadronic uncertainties in $B \to X_s\ell^+\ell^-$,
  - $\Rightarrow$ **NP** $\mathcal{C}_9^{\rm NP}$ enters both numerator and denominator
  - $\Rightarrow$ An unknown **exclusive hadronic** $\mathcal{C}_{9,\rm extra}$ enters the numerator only
- **Experiment:** These ratios can, in principle, be measured **without external charmonium branching fractions**
  - $\Rightarrow$ Isospin averaging and fragmentation-fraction assumptions still require care

Alvarez-Cartelle, Capdevila, Lunghi, Matias, arXiv:2605.06567

First experimental estimates of the $\mathcal{F}$ ratios

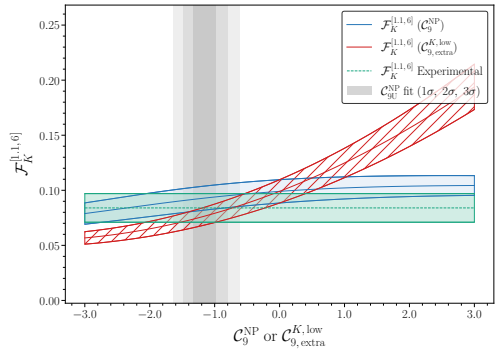
	$B \to K\mu\mu$ fraction	$B \to K^*\mu\mu$ fraction
$[1.1, 6]\,\mathrm{GeV}^2$	$\mathcal{F}_K^{[1.1,6]} = \mathbf{0.084 \pm 0.013}$	$\mathcal{F}_{K^*}^{[1.1,6]} = \mathbf{0.110 \pm 0.017}$
$q^2 > 15\,\mathrm{GeV}^2$	$\mathcal{F}_K^{>15} = \mathbf{0.309 \pm 0.015}$	$\mathcal{F}_{K^*}^{>15} = \mathbf{0.629 \pm 0.021}$

- Low q^2: denominator is the inclusive e/μ average, with a *different lower cut* (1.0 instead of $1.1\,\mathrm{GeV}^2$)
- High q^2: denominator estimated by summing K, K^*, $(K\pi)_S$ and $K\pi\pi$ modes, with common-normalisation correlations treated in the averages

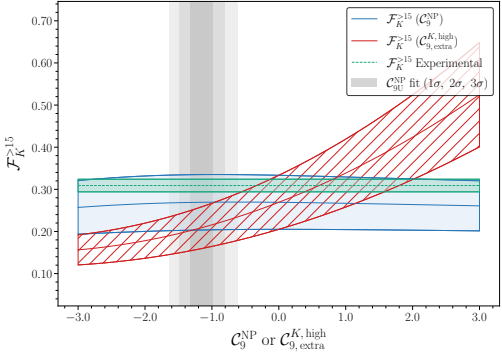
Alvarez-Cartelle, Capdevila, Lunghi, Matias, arXiv:2605.06567

$\mathcal{F}$ ratios: sensitivity to $\mathcal{C}_9^{\rm NP}$ and $\mathcal{C}_{9,\rm extra}$

Low q^2: $\mathcal{F}_K$



High q^2: $\mathcal{F}_K$



- The NP-only curve is relatively flat: numerator–denominator $\mathcal{C}_9^{\rm NP}$ dependence largely cancels
- A hypothetical exclusive-only $\mathcal{C}_{9,\rm extra}$ produces a more pronounced change in the ratios; current experimental bands still admit sizeable uncertainty

Alvarez-Cartelle, Capdevila, Lunghi, Matias, arXiv:2605.06567

High q^2: summing exclusive modes to reconstruct inclusive

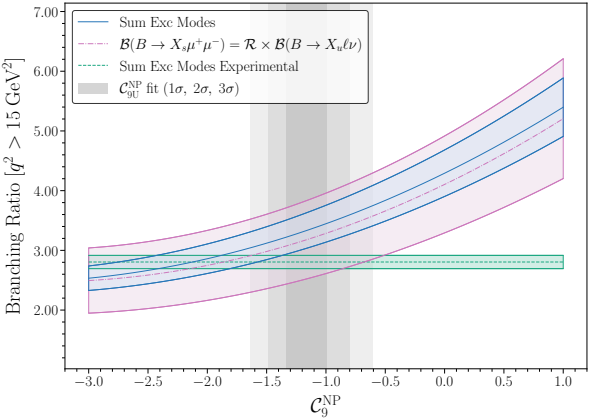
- At high q^2, $\mathcal{B}(B \to X_s \ell^+ \ell^-)^{>15}$ is nearly saturated by K, K^* and residual $(K\pi)_S$, $K\pi\pi$ modes

$$10^7 \times \mathcal{B}^{>15}_{\sum \text{exc}} \simeq 4.292 + 0.976\,(\mathcal{C}_9^{\text{NP}} + \mathcal{C}_{9,\text{extra}}) + 0.130\,(\mathcal{C}_9^{\text{NP}} + \mathcal{C}_{9,\text{extra}})^2,$$

$$10^7 \times \mathcal{B}^{>15}_{\text{inc}} \simeq 4.10 + 0.96\,\mathcal{C}_9^{\text{NP}} + 0.14\mathcal{C}_9^{\text{NP}\,2}$$

⇒ Use estimates for $\mathcal{B}(B \to (K\pi)_{S-\text{wave}}\mu^+\mu^-)$ from HHChPT

Isidori, Polonsky, Tinari, arXiv:2305.03076



⇒ Theoretical inclusive and exclusive-sum predictions agree closely in their SM values and NP dependence

⇒ Experimental summed rate:

$$10^7 \times \mathcal{B}^{\text{exp}}_{\sum \text{exc}} = (2.80 \pm 0.12)$$

Testing a universal $\mathcal{C}_{9,\text{extra}}$ at high q^2

- Impose the agreement of **two theoretical descriptions** of the inclusive high-q^2 branching fraction, assuming $\mathcal{C}_9^{\text{NP}} = 0$:

$$\mathcal{B}^{\text{th}}_{\sum \text{exc}}(\mathcal{C}_{9,\text{extra}}) = \mathcal{B}^{\text{th}}_{\text{inc}}(\mathcal{C}_9^{\text{NP}} = 0).$$

$$\mathcal{C}_{9,\text{extra}} = -0.20^{+0.90}_{-1.10} \quad (68\%\ \text{CL})$$

- Compatible with zero
  - ⇒ Considerably less negative than $\mathcal{C}_{9,\text{extra}} \sim -1.16$ required to reproduce the exclusive deficits in the pure-hadronic scenario
- The likelihood is **asymmetric and non-parabolic**
  - ⇒ A second far-negative solution exists
  - ⇒ Precision is dominated by the inclusive high-q^2 rate

Alvarez-Cartelle, Capdevila, Lunghi, Matias, arXiv:2605.06567

Conclusions

Conclusions

- **Amplitude analyses** offer complementary routes: theory-constrained fits, flexible amplitude extraction, and unbinned angular moments
- **Consistency conditions, observable relations and geometrical bounds** test the amplitude hypothesis when $J_i(q^2)$ are measured independently of it; continuous observables also expose zero-crossings sensitive to $\mathcal{C}_9$
- **Exclusive–inclusive comparisons** probe a hypothetical constant, universal *hadronic* contribution to $\mathcal{C}_9$ that could imitate short-distance NP
- At high q^2, the required inclusive–exclusive consistency yields $\mathcal{C}_{9,\text{extra}} = -0.20^{+0.90}_{-1.10}$ (68% CL; $\mathcal{C}_9^{\text{NP}} = 0$), **compatible with zero**; precision improvements will sharpen the test

Thank you!

Backup slides

Counting independent amplitude degrees of freedom

$$n_{\text{dof}} = 2n_A - n_{\text{sym}} = n_c - n_{\text{rel}}$$

P-wave hypothesis	n_A	n_{sym}	n_c	n_{rel}	n_{dof}	conditions
$m_\ell = 0$, no scalars	6	4	11	3	**8**	8
$m_\ell \neq 0$, no scalars	7	4	11	1	**10**	10
$m_\ell \neq 0$, with scalar	8	4	12	0	**12**	12

- Six complex amplitudes $A^{L,R}_{\perp,\parallel,0}$
  - $\Rightarrow$ Massive case adds A_t
  - $\Rightarrow$ Scalar case adds A_S
- Four continuous reparametrisation symmetries of the angular distribution
  - $\Rightarrow$ Only symmetry-invariant combinations are measurable
- In the massless case, two relations are kinematic and **one is non-trivial** (the latter survives for $m_\ell \neq 0$ without scalars)

Algueró *et al.*, arXiv:2107.05301
Capdevila, Matias, Novoa-Brunet, Patel, Smith, arXiv:2504.00949

Consistency conditions: amplitudes versus angular coefficients

- For P-wave amplitudes, define the L/R doublets:

$$n_\parallel = \begin{pmatrix} A_\parallel^L \\ A_\parallel^{R*} \end{pmatrix}, \quad n_\perp = \begin{pmatrix} A_\perp^L \\ -A_\perp^{R*} \end{pmatrix}, \quad n_0 = \begin{pmatrix} A_0^L \\ A_0^{R*} \end{pmatrix}$$

These give eight independent real combinations:

$$|n_\perp|^2 = \frac{2J_{2s} + J_3}{\beta_\ell^2}, \qquad |n_\parallel|^2 = \frac{2J_{2s} - J_3}{\beta_\ell^2},$$

$$n_\parallel^\dagger n_\perp = \frac{J_{6s}}{2\beta_\ell} + i\frac{J_9}{\beta_\ell^2}, \qquad n_\parallel^\dagger n_0 = \frac{\sqrt{2}}{\beta_\ell^2} J_4 + i\frac{J_7}{\sqrt{2}\beta_\ell}, \qquad n_\perp^\dagger n_0 = \frac{J_5}{\sqrt{2}\beta_\ell} + i\frac{\sqrt{2}}{\beta_\ell^2} J_8.$$

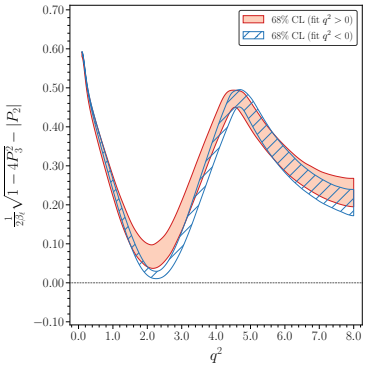
with $\beta_\ell = \sqrt{1 - 4m_\ell^2/q^2}$

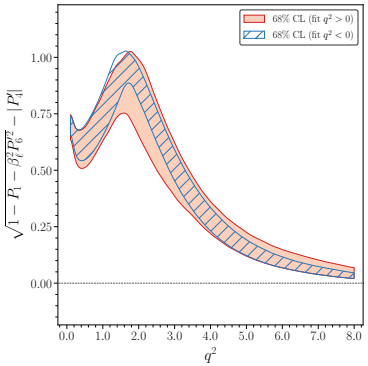
⇒ **Unbinned $J_i(q^2)$ measurement**: test whether the angular coefficients admit the assumed amplitude structure and whether a theoretical amplitude description reproduces them

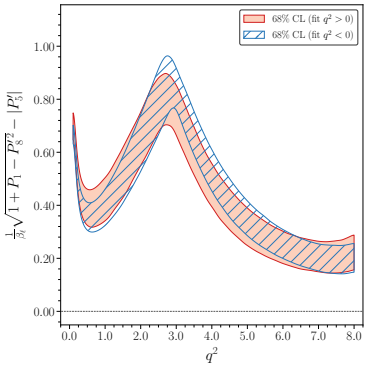
⇒ **Unbinned amplitudes extracted directly**: compare $A_\lambda^{L,R}(q^2)$ with theoretical amplitudes, after matching phase, angular and symmetry-fixing conventions

Capdevila, Matias, Novoa-Brunet, Patel, Smith, arXiv:2504.00949
LHCb, arXiv:2608.12215

Additional geometrical bounds

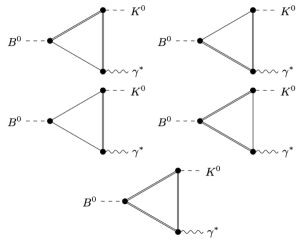


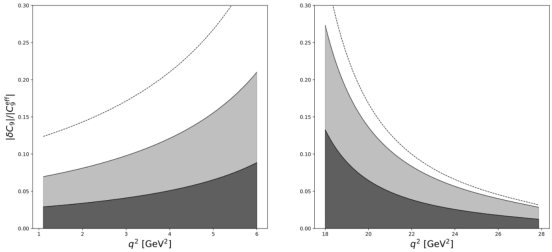




Capdevila, Matias, Novoa-Brunet, Patel, Smith, arXiv:2504.00949

Model estimates for charm rescattering





- Model estimates based on **Heavy–Hadron Chiral Perturbation Theory** (HHChPT)
  - ⇒ Estimates for $B^0 \to K^0 \ell^+ \ell^-$
  - ⇒ HHChPT valid near $q^2 \simeq m_B^2$ (soft–kaon region)
- Typical impact of 5% to 20% on the amplitude, depending on interference and multiplicity assumptions

  G Isidori, Z Polonsky, A Tinari, arXiv:2405.1755
  G Isidori, Z Polonsky, A Tinari, arXiv:2507.17824
- Recent dispersive analysis of anomalous thresholds in the **light–quark loop** finds anomalous branch cuts suppressed on the $\rho(770)$ but potentially $\mathcal{O}(10\%)$ off–peak, paving the way for similar studies of the charm loop

  S Mutke, M Hoferichter, B Kubis, arXiv:2406.14608

Inclusive $B \to X_s \ell^+ \ell^-$: differential distribution

- The triple-differential distribution can be decomposed into three angular observables:

$$\frac{d^3\Gamma}{dq_0\, dq^2\, dz} = \frac{3}{8}\left[(1+z^2)\frac{d^2H_T}{dq_0\, dq^2} + 2(1-z^2)\frac{d^2H_L}{dq_0\, dq^2} + 2z\frac{d^2H_A}{dq_0\, dq^2}\right], \qquad q_0 = v\cdot q, \quad z = \cos\theta_\ell$$

- After integration over q_0, the angular functions have the schematic form:

$$H_I(q^2) = \mathcal{N}(q^2)\sum_{i,j}\mathrm{Re}\Big[\mathcal{C}_i(\mu)\mathcal{C}_j^*(\mu)\,\widehat{K}_{ij}^I(q^2,\mu)\Big] + \Delta H_I^{\mathrm{KS}} + \Delta H_I^{\text{soft nonfact}} + \Delta H_I^{\mathrm{HQE}} + \Delta H_I^{\mathrm{EM}}$$

  - $\Rightarrow$ $\widehat{K}_{ij}^I$: perturbative kernels, including factorisable and hard non-factorisable QCD effects
  - $\Rightarrow$ ΔH_I^{KS}: dispersive correction to the perturbative factorisable loops (no double counting)
  - $\Rightarrow$ $\Delta H_I^{\text{soft nonfact}}$: non-local soft contributions, including resolved effects
  - $\Rightarrow$ $\Delta H_I^{\mathrm{HQE}}$: local heavy-quark expansion (HQE) power corrections to the partonic rate
  - $\Rightarrow$ ΔH_I^{EM}: QED effects (relevant for Belle II, already included in experimental treatment at LHCb)

Benzke, Hurth, Turczyk, arXiv:1705.10366
Huber, Hurth, Jenkins, Lunghi, Qin, Vos, arXiv:1908.07507
Huber, Hurth, Jenkins, Lunghi, Qin, Vos, arXiv:2007.04191

Soft non-factorisable corrections

- Non-factorisable soft effects are not included in the factorisable Krüger–Sehgal approach
- **Low q^2:** resolved-photon contributions are described in SCET, with $1 \lesssim q^2 \lesssim 6\,\text{GeV}^2$ and a cut $m_{X_s}^2 \sim m_b \Lambda_{\text{QCD}}$

$$\frac{d\Gamma}{dq^2} \sim \underbrace{\sum_n \frac{1}{m_b^n}\, H_n\, J_n \otimes S_n}_{\text{Direct}} + \underbrace{\sum_{n\geq 1} \frac{1}{m_b^n}\, H_n\, J_n \otimes S_n \otimes \bar{J}_n}_{\text{Resolved}}$$

  - ⇒ **Direct terms:** photon emission from the short-distance weak vertex
  - ⇒ **Resolved terms:** photon emission from light partons, involving non-local soft functions
- SCET factorisation:
  - ⇒ Hard functions H_n and jet functions $J_n, \bar{J}_n$ are perturbatively calculable
  - ⇒ Shape functions S_n are nonperturbative and need to be modelled
- Resolved terms:
  - ⇒ Contain contributions from $\mathcal{O}_1^c$–$\mathcal{O}_{7\gamma}$, $\mathcal{O}_{7\gamma}$–$\mathcal{O}_{8g}$ and $\mathcal{O}_{8g}$–$\mathcal{O}_{8g}$ interferences
  - ⇒ Poorly known subleading shape functions yield an estimated $\sim 5\%$ uncertainty from resolved contributions

Benzke, Hurth, Turczyk, arXiv:1705.10366
Huber, Hurth, Jenkins, Lunghi, Qin, Vos, arXiv:1908.07507