B anomalies, subleading corrections to inclusive penguin decays and refactorisation

Discrete 2026, 13.-17.11.2026, Bratislava



PRISMA+

TOBIAS HURTH

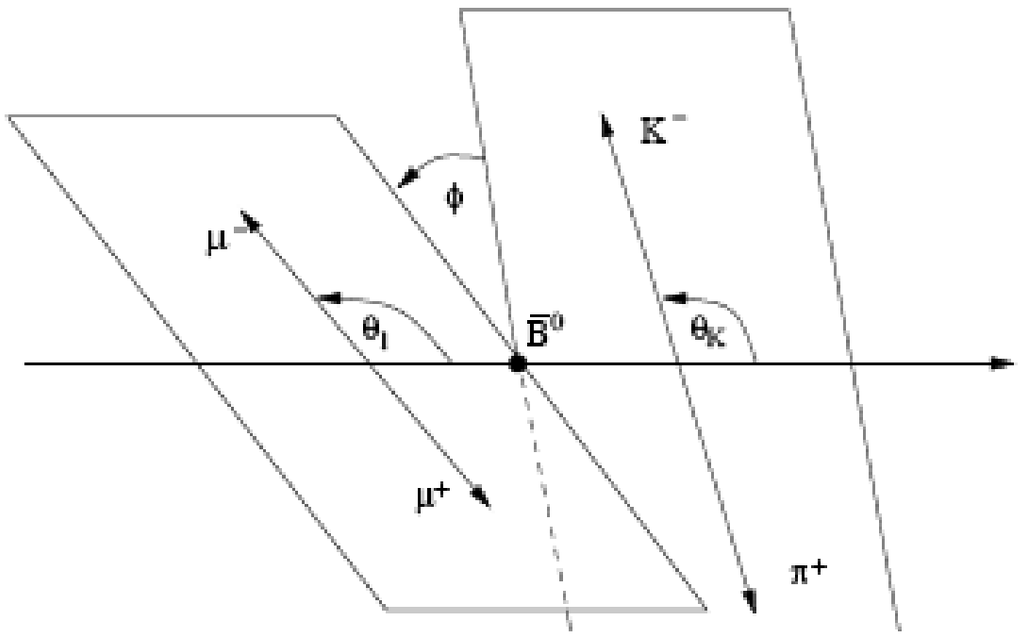
JG|U JOHANNES GUTENBERG UNIVERSITÄT MAINZ

The B anomalies

Differential decay rate of $B \to K^* \ell\ell$

Assuming the $\bar{K}^*$ to be on the mass shell, the decay $\bar{B}^0 \to \bar{K}^{*0}(\to K^-\pi^+)\ell^+\ell^-$ described by the lepton-pair invariant mass, s, and the three angles θ_l, θ_{K^*}, ϕ.

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_l\, d\cos\theta_K\, d\phi} = \frac{9}{32\pi} J(q^2, \theta_l, \theta_K, \phi)$$



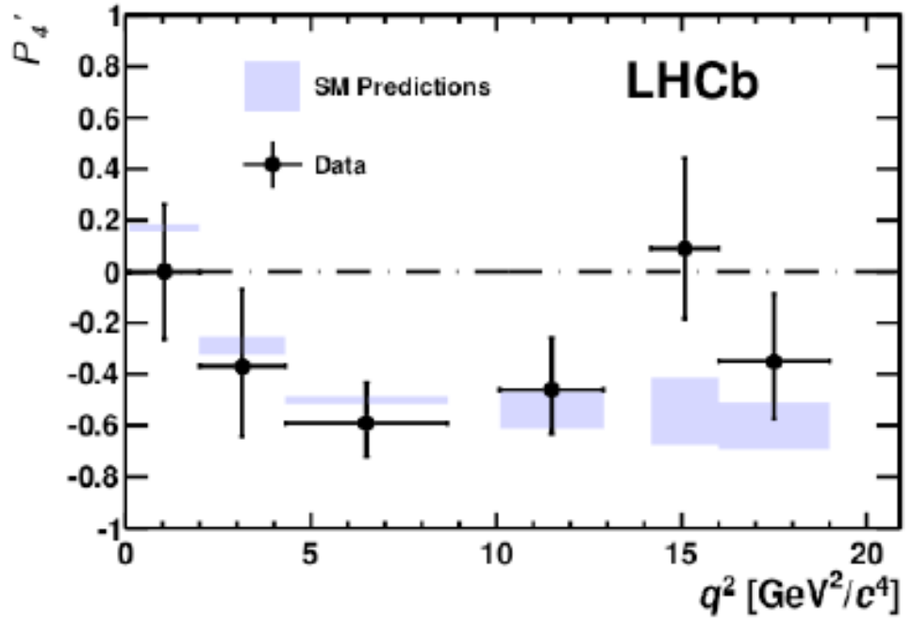
$$J(q^2, \theta_l, \theta_K, \phi) =$$

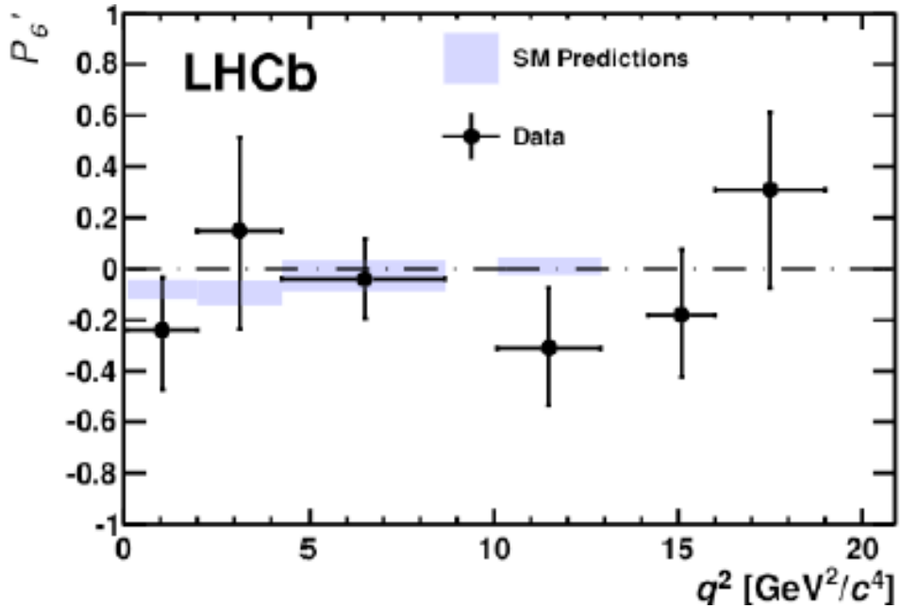
$$= J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_l + J_3 \sin^2\theta_K \sin^2\theta_l \cos 2\phi$$
$$+ J_4 \sin 2\theta_K \sin 2\theta_l \cos\phi + J_5 \sin 2\theta_K \sin\theta_l \cos\phi + (J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_l$$
$$+ J_7 \sin 2\theta_K \sin\theta_l \sin\phi + J_8 \sin 2\theta_K \sin 2\theta_l \sin\phi + J_9 \sin^2\theta_K \sin^2\theta_l \sin 2\phi$$

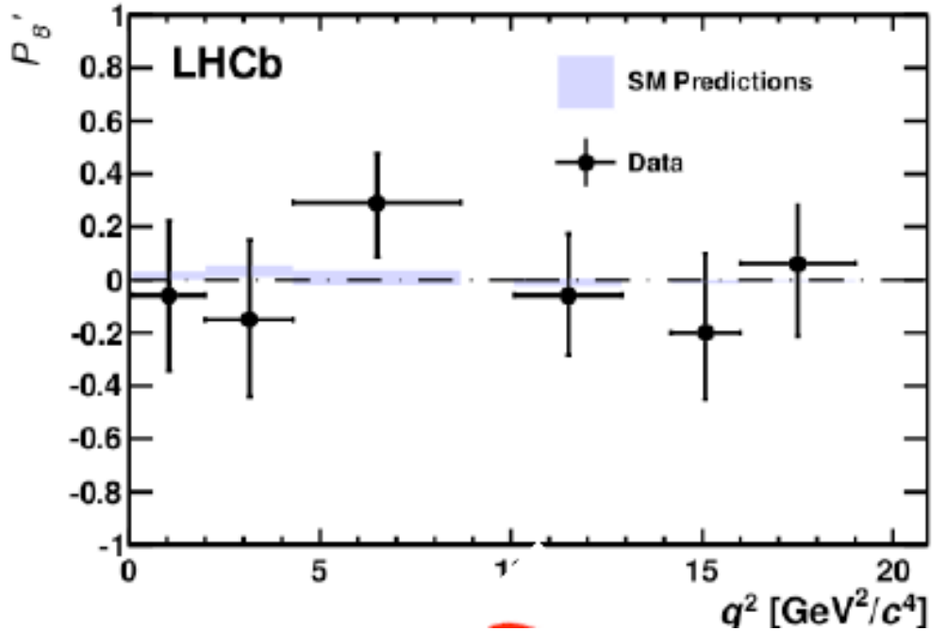
Large number of angular observables

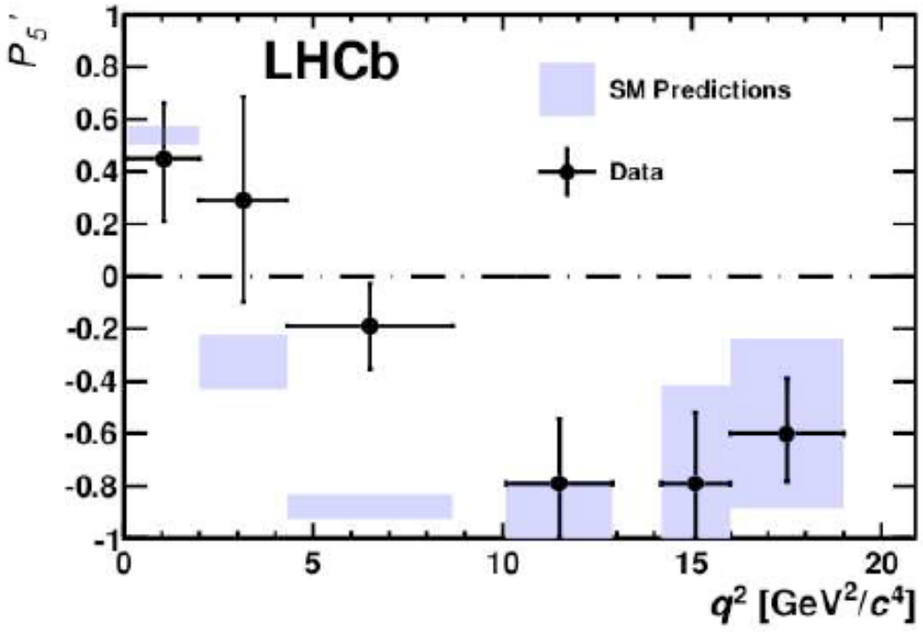
First measurements of new angular observables LHCb arXiv:1308.1707

"SM predictions" Descotes-Genon, Hurth, Matias, Virto arXiv:1303.5794









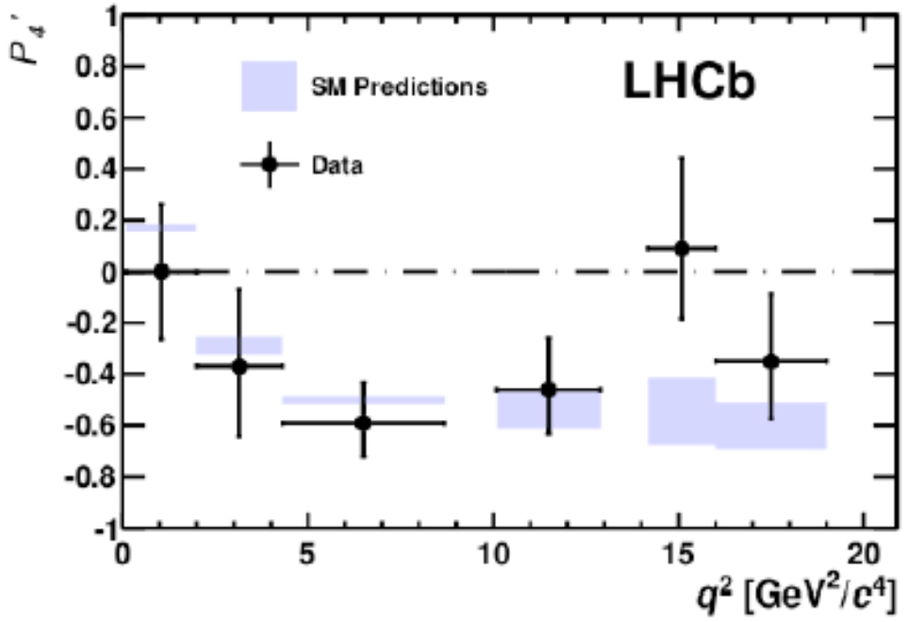
Good agreement with SM in P_4', P_6' and P_8',

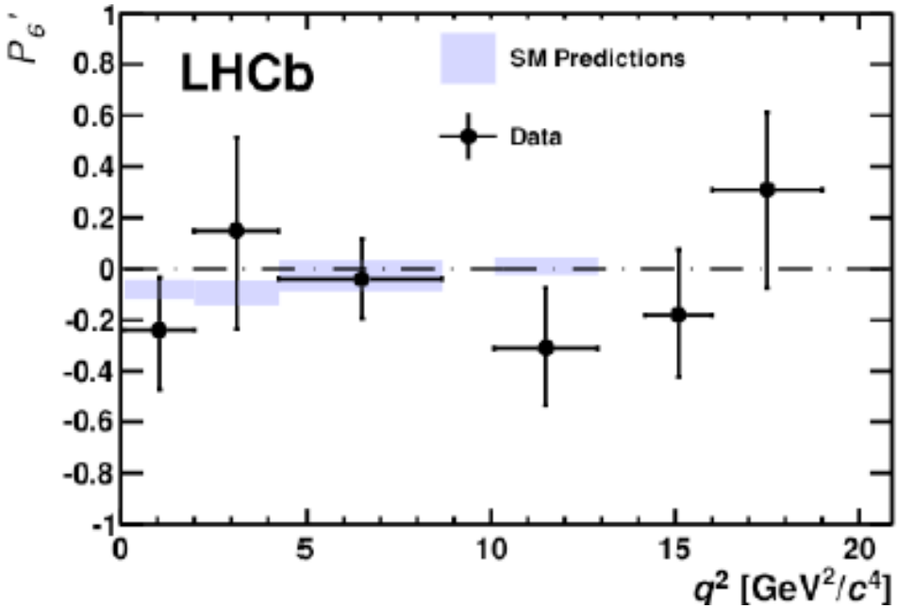
but a 3.7σ deviation in the third bin in P_5'

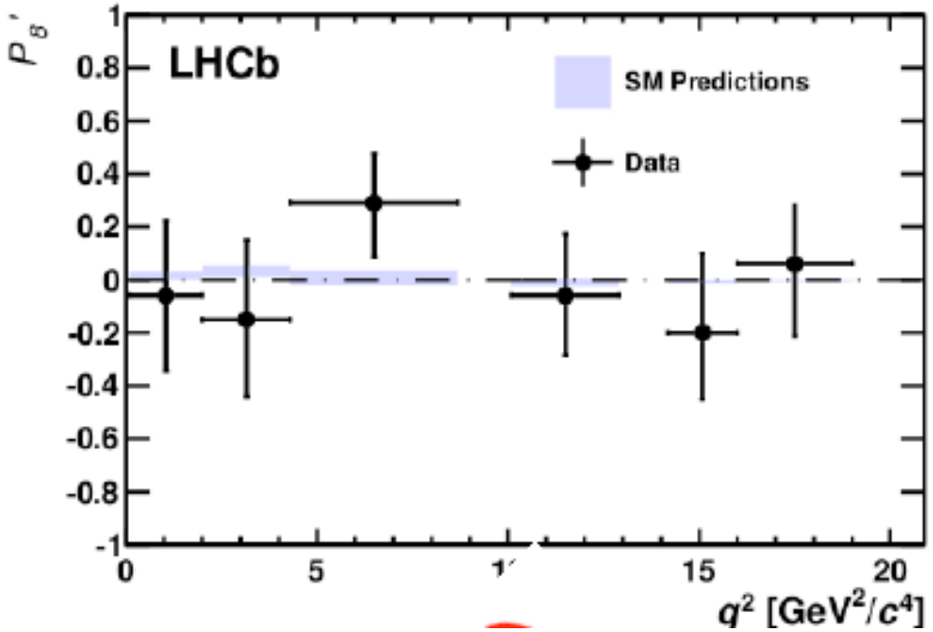
Based on 10% guesstimate of power corrections!

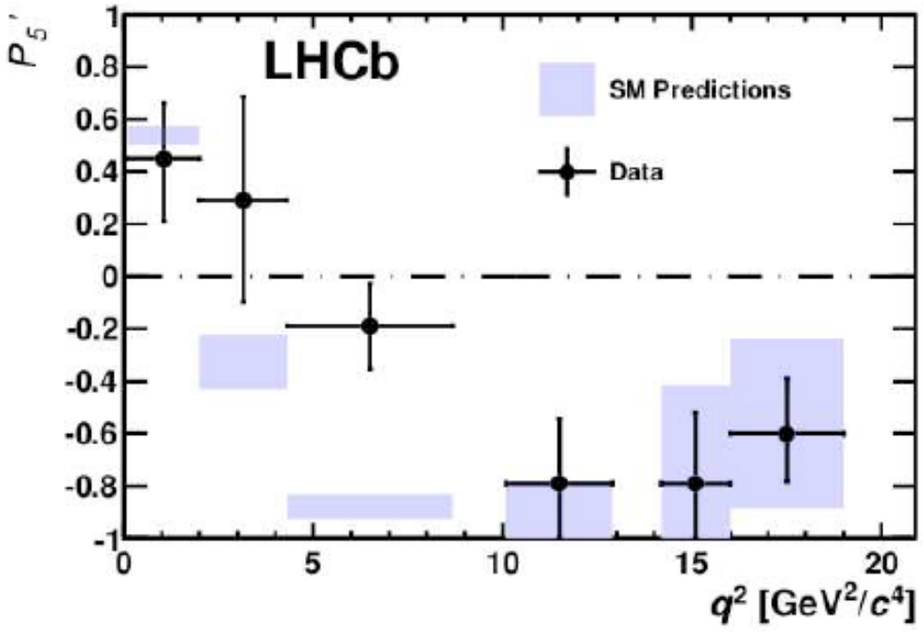
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New Physics or underestimated hadronic uncertainties

(form factors, power corrections) ?

Exclusive modes $B \to K^{(*)}\ell\ell$

Soft-collinear effective theory

$$\mathcal{T}_a^{(i)} = C_a^{(i)}\xi_a + \phi_B \otimes T_a^{(i)} \otimes \phi_{a,K^*} + O(\Lambda/m_b)$$

- Separation of perturbative hard kernels from process-independent nonperturbative functions like form factors

- Relations between formfactors in large-energy limit

- Limitation: insufficient information on power-suppressed Λ/m_b terms (breakdown of factorization: 'endpoint divergences')

The significance of the anomalies depends on the assumptions made for the unknown power corrections!

Problem of nonfactorizable power corrections

- Ongoing efforts: Estimate of power corrections based on analyticity

  van Dyk et al.: arXiv:2011.09813, 2206.03797

- In the long run: Solution with refactorization techniques *

  New developments in the SCET community

  Neubert et al., arXiv:2009.06779

- Crosscheck of the anomalies via inclusive modes

*Caveat:
Many nonperturbative functions in exclusive modes at subleading order.

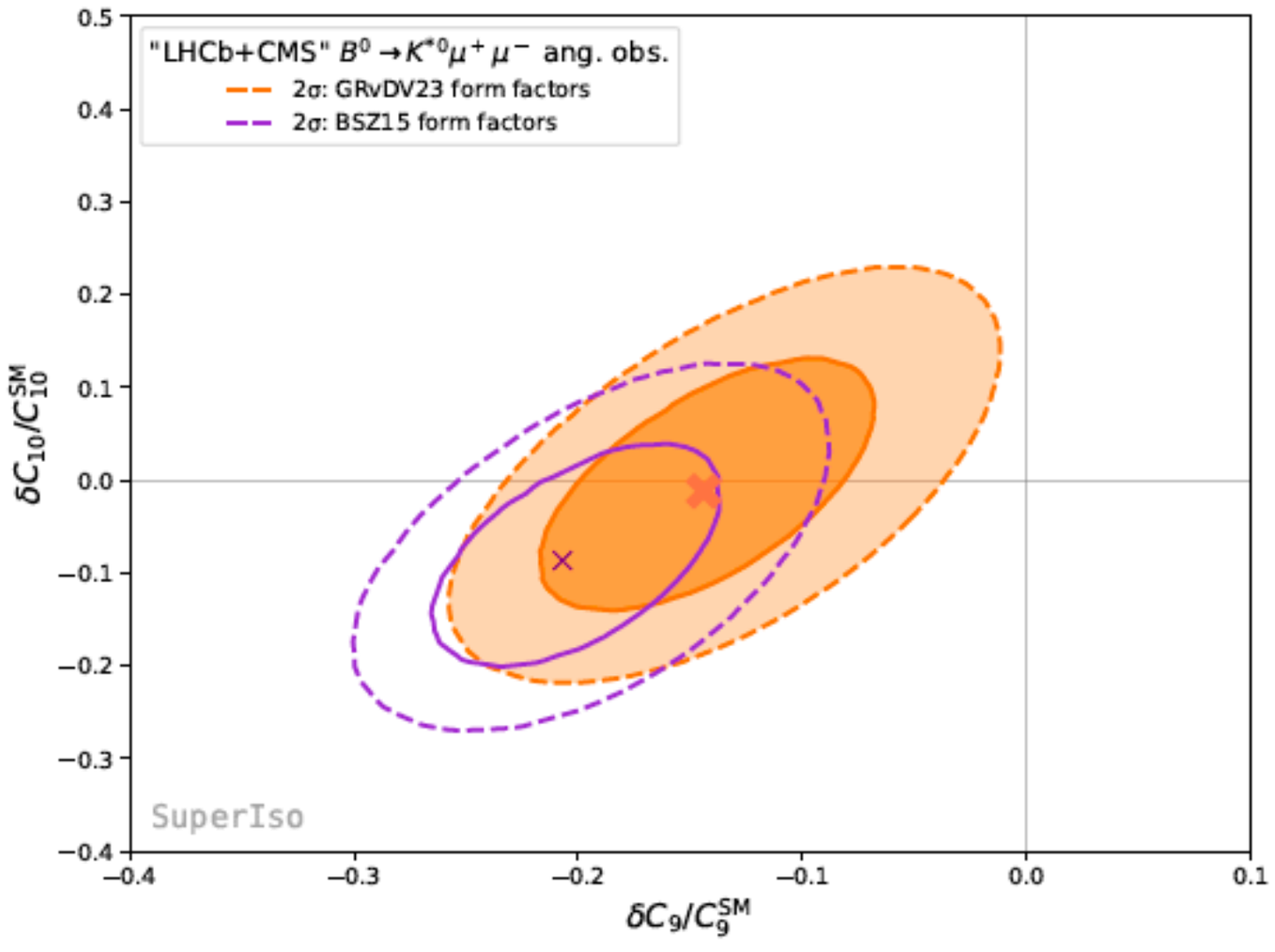
Side remarks on the $b \to s$ anomalies

Hurth,Mahmoudi,Monceaux,Neshatpour arXiv:2508.09986

Large ambiguity of the significance due to the local formfactors

The 1 and 2σ C.L. of the $\{C_9, C_{10}\}$, **without** $[6, 8]$ and $[6., 8.68]$ GeV^2 bins

Combined LHCb and CMS results, impact of the choice of the form factors



Orange: GRvDV23-FF (pull_{SM}: 2.8)

Purple contours : BSZ15-FF (pull_{SM}: 4.0)

We need latttice calculations....

Based on 10% guesstimate of power corrections!

Side remarks on the $b \to s$ anomalies

Data-driven approach

Hurth,Mahmoudi,Monceaux,Neshatpour arXiv:2508.09986
see also arXiv:1702.02234

Instead of making assumptions on the size of the power corrections (h_λ) they can be parameterized by a general ansatz (compatible with the analycity structure):

$$h_\pm(q^2) = h_\pm^{(0)} + \frac{q^2}{1\ \text{GeV}^2} h_\pm^{(1)} + \frac{q^4}{1\ \text{GeV}^4} h_\pm^{(2)}, \quad h_0(q^2) \quad = \sqrt{q^2} \times \left(h_0^{(0)} + \frac{q^2}{1\ \text{GeV}^2} h_0^{(1)} + \frac{q^4}{1\ \text{GeV}^4} h_0^{(2)} \right)$$

Jäger,Camalich, arXiv:1212.2263,1412.3183

NP ansatz with δC_9 and δC_7 can be embedded in hadronic ansatz

$$\delta C_7 \sim h_\pm^{(0)} \qquad \delta C_9 \sim h_\pm^{(1)}$$

Wilk's test

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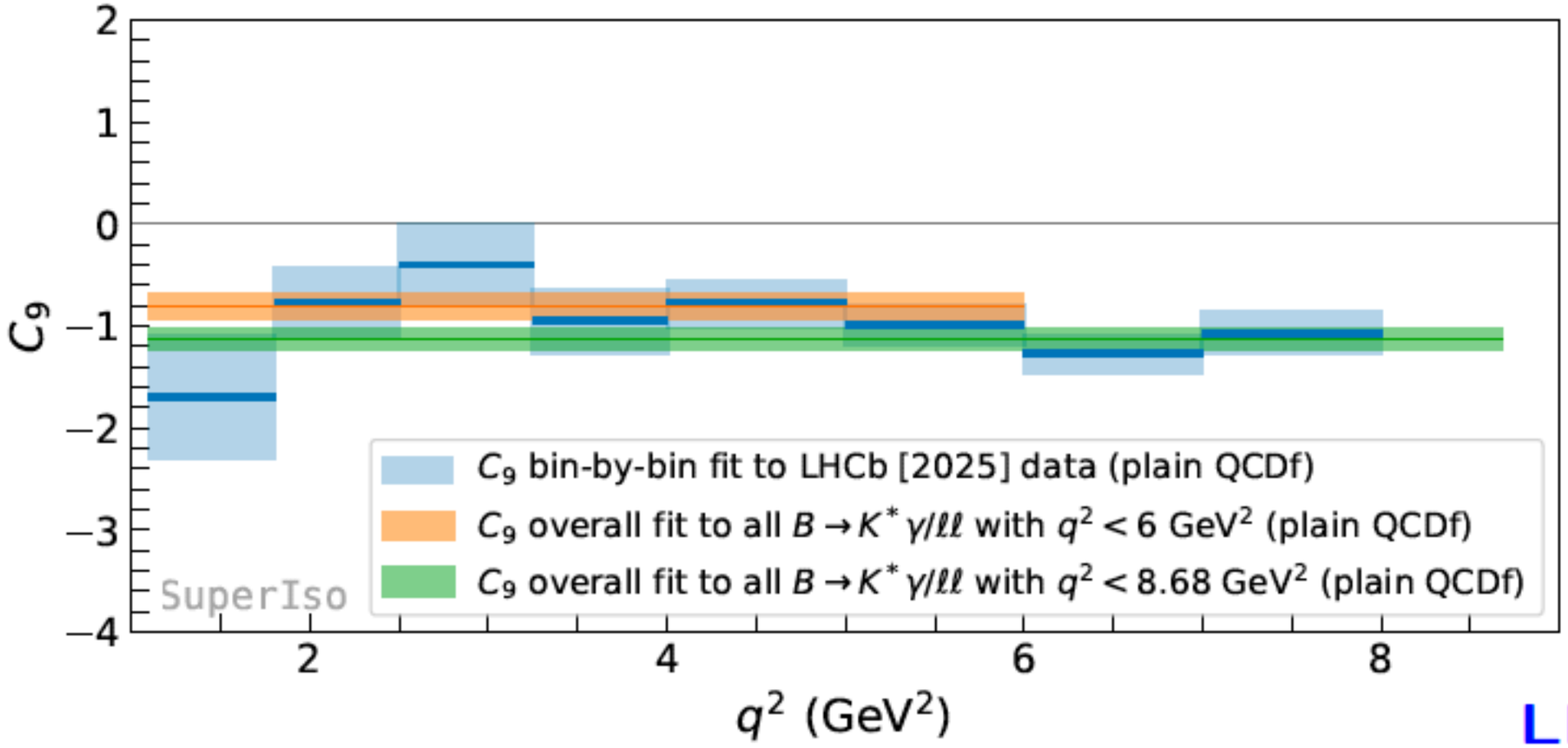
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Wilk's test

- **Crosscheck helicity and q^2 independence of NP coefficients**



LHCb Run3, ...

Inclusive modes $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$

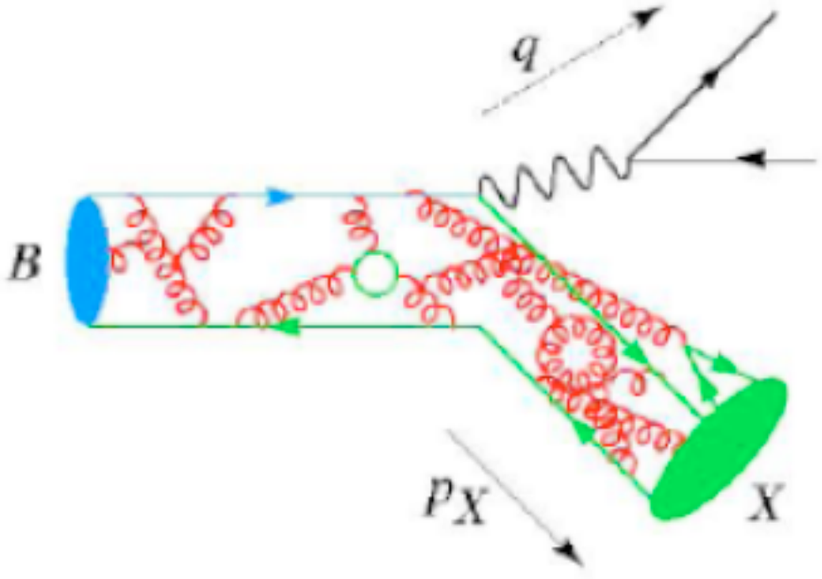
How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?

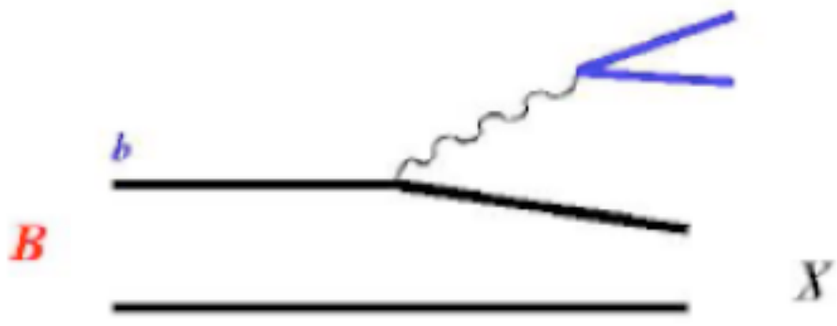
- Heavy mass expansion for inclusive modes:

$$\Gamma(\bar{B} \to X_s\gamma) \overset{m_b\to\infty}{\longrightarrow} \Gamma(b \to X_s^{parton}\gamma)\,, \quad \Delta^{nonpert.} \sim \Lambda^2_{QCD}/m_b^2$$

No linear term Λ_{QCD}/m_b (perturbative contributions dominant)

Chay,Georgi,Grinstein 1990





Inclusive modes $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$

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Old story:

– If one goes beyond the leading operator ($\mathcal{O}_7$, $\mathcal{O}_9$):
breakdown of local expansion

Dedicated analysis:

naive estimate of non-local matrix elements leads to 5% uncertainty.

$b \to s\gamma$: Benzke,Lee,Neubert,Paz,arXiv:1003.5012



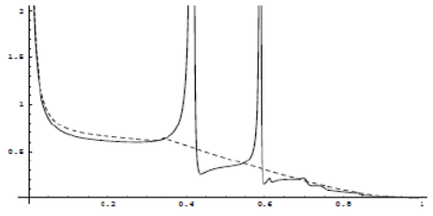
$b \to s\ell\ell$: Benzke,Hurth,Turczyk,arXiv:1705.10366

Nonlocal subleading contributions

Subleading power factorization in $B \to X_s \ell^+ \ell^-$

Benzke,Hurth,Turczyk,arXiv:1705.10366; Benzke,Hurth,arXiv:2006.00624

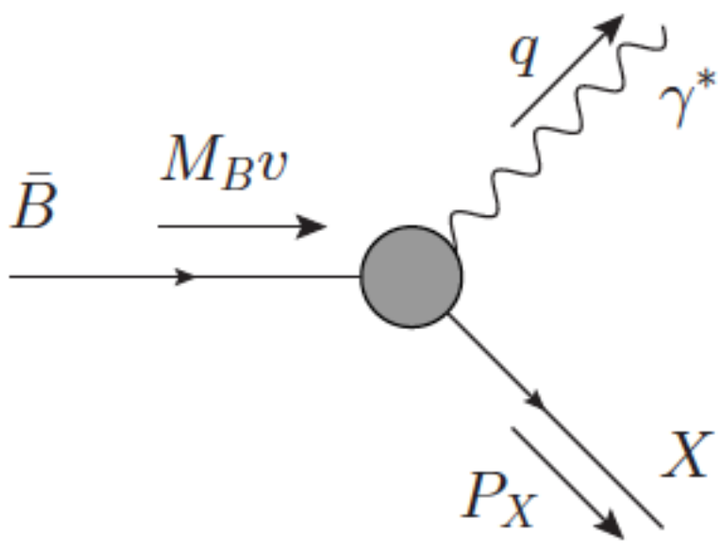
- Cuts in the dilepton mass spectrum necessary due to $c\bar{c}$ resonances



- Additional cut in the hadronic mass spectrum (X_s) needed for background suppression (i.e. $b \to c(\to s e^+ \nu) e^- \bar{\nu}$)

- Kinematics: X_s is jetlike and $m_x^2 \leq m_b \Lambda_{QCD}$ (shapefunction region)

- Multiscale problem $\Rightarrow$ SCET with scaling Λ_{QCD}/m_b



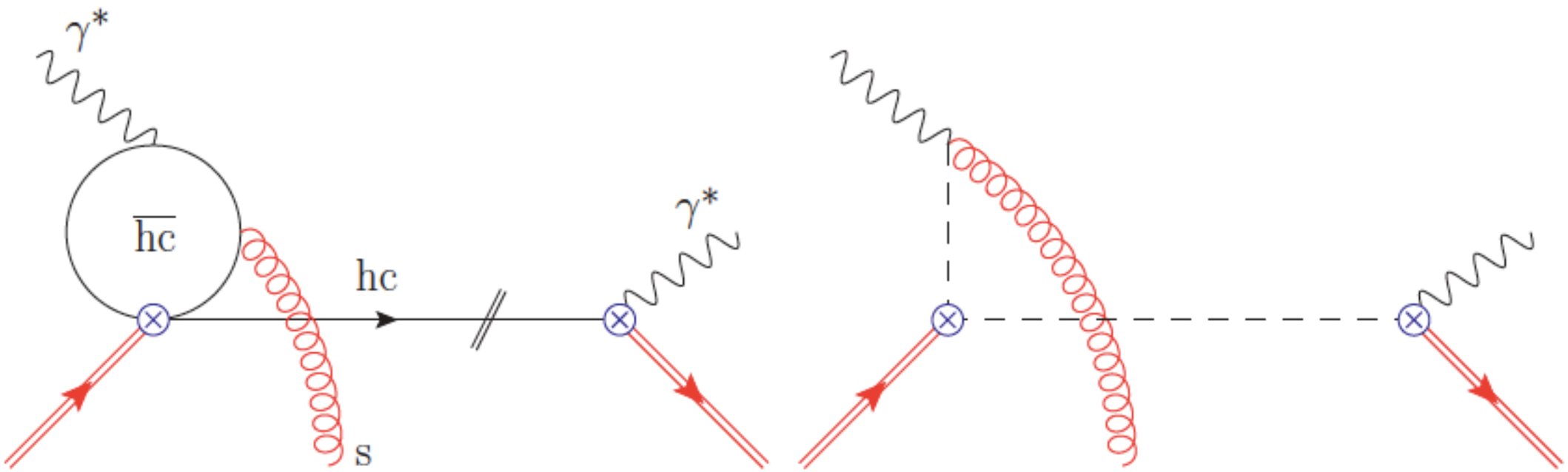
$$M_B^2 \sim m_b^2 \gg m_x^2 \sim m_b \Lambda_{QCD} \gg \Lambda_{QCD}^2$$

hard **hardcollinear** **soft** **modes**

Interference of Q_1 and Q_7

$$d\Gamma \sim H \cdot J \otimes s \otimes \bar{J}$$

In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.



$$\frac{d\Gamma^{\rm res}}{dn\cdot q\, d\bar{n}\cdot q} \sim \frac{1}{m_b}\int d\omega\, \delta(\omega + p_+) \int \frac{d\omega_1}{\omega_1 + i\varepsilon}$$
$$\frac{1}{\omega_1}\left[\bar{n}\cdot q\left(F\left(\frac{m_c^2}{n\cdot q\bar{n}\cdot q}\right)-1\right) - (\bar{n}\cdot q+\omega_1)\left(F\left(\frac{m_c^2}{n\cdot q(\bar{n}\cdot q+\omega_1)}\right)-1\right)\right.$$
$$\left. +\bar{n}\cdot q\left(G\left(\frac{m_c^2}{n\cdot q\bar{n}\cdot q}\right) - G\left(\frac{m_c^2}{n\cdot q(\bar{n}\cdot q+\omega_1)}\right)\right)\right] g_{17}(\omega,\omega_1)$$

$$g_{17}(\omega,\omega_1) = \int \frac{dr}{2\pi} e^{-i\omega_1 r}\int \frac{dt}{2\pi} e^{-i\omega t}\frac{1}{M_B}\langle \bar{B}|\bar{h}(\mathbf{tn})\ldots G_s^{\alpha\beta}(\mathbf{r\bar{n}})\ldots h(0)|\bar{B}\rangle$$

- Shape function is nonlocal in both light cone directions
- It survives $M_X \to 1$ limit (irreducible uncertainty)

Numerical evaluation of the resolved contributions

Strategy:

- Use explicit definition of shape function as HQET matrix element to derive properties
  - PT invariance implies that soft functions are real
  - Moments of shape functions are related to HQET parameters
  - Soft functions have no significant structure outside the hadronic range
  - Values of soft functions are within the hadronic range
- Perform convolution integrals with model functions

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New input:

$$\int_{-\infty}^{\infty} d\omega_1 \, {\omega_1}^0 \, h_{17}(\omega_1, \mu) = 0.237 \pm 0.040 \, \mathrm{GeV}^2$$

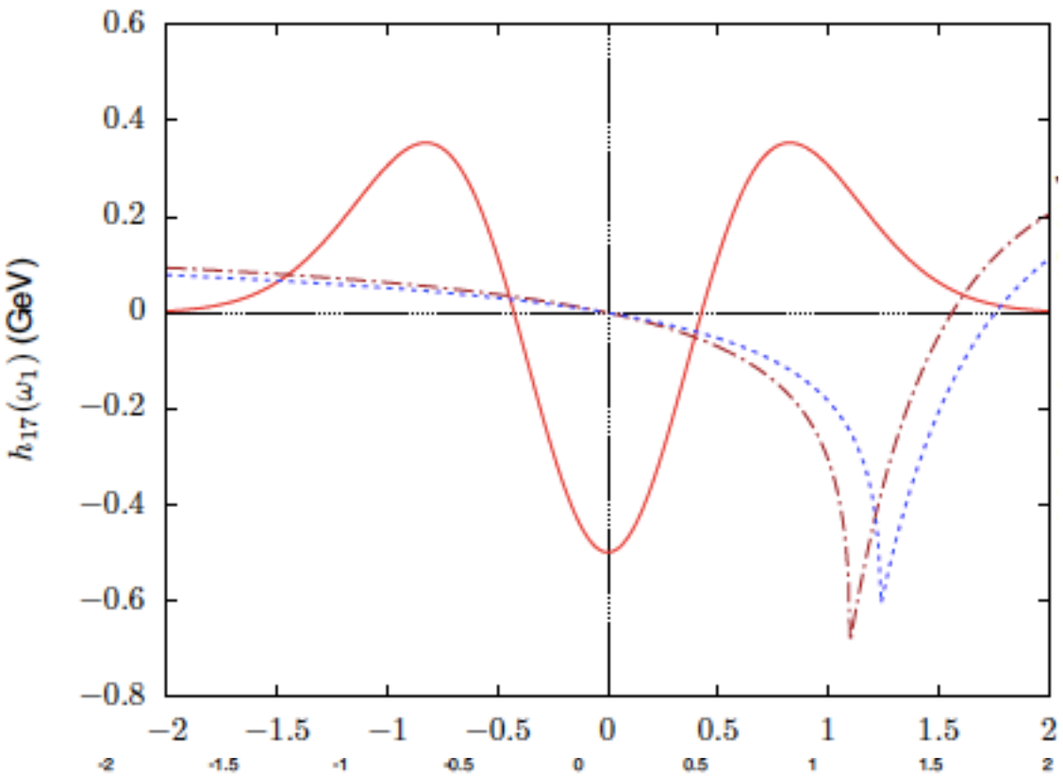
$$\int_{-\infty}^{\infty} d\omega_1 \, {\omega_1}^2 \, h_{17}(\omega_1, \mu) = 0.15 \pm 0.12 \, \mathrm{GeV}^4$$

Paz et al. arXiv:1908.02812

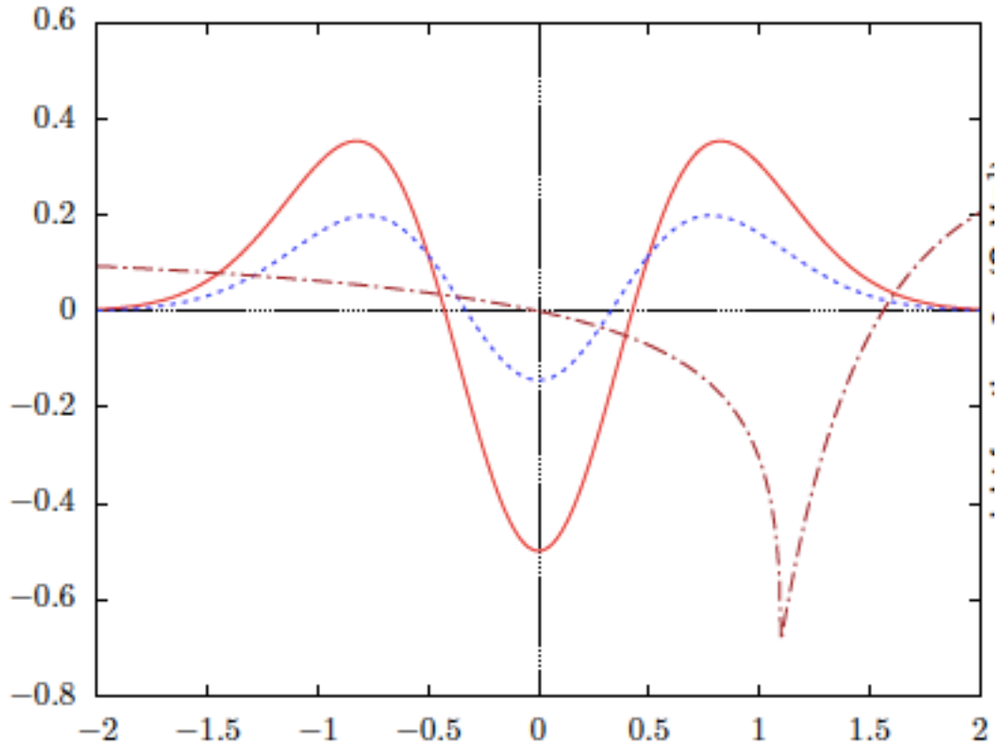
Updated result for $\bar{B} \to X_s\gamma$

Benzke,Hurth,arXiv:2006.00624

Charm dependence of jet function:



Constraint on shape function:



$$\mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-0.4\%,\, 4.7\%]$$

Uncertainties are not Gaussian !

Still: Largest uncertainty in the prediction of the decay rate of $\bar{B} \to X_s\gamma$

Intermezzo

Comparison with the numerical analysis in Paz et al. arXiv:1908.02812

$$\mathcal{F}^{17}_{b\to s\gamma} \in [-0.4\%, 1.9\%] \quad \text{versus} \quad \mathcal{F}^{17}_{b\to s\gamma} \in [-0.4\%, 4.7\%]$$

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- For charm dependence only the parametric uncertainty was used

  $$1.17\,\text{GeV} \le m_c \le 1.23\,\text{GeV}$$

  We use scale variation of the hard-collinear scale and get

  $$\mu_{hc} \sim \sqrt{m_b\,\Lambda_{QCD}} \quad \text{from} \quad 1.3\,\text{GeV to } 1.7\,\text{GeV}$$

  $$1.14\,\text{GeV} \le m_c \le 1.26\,\text{GeV}$$

- Numerically large $1/m_b^2$ term due to kinematic factors was dropped compared to the original analysis in 2010 where the large $1/m_b^2$ term was used as conservative estimate of all $1/m_b^2$ contributions to resolved $\mathcal{O}_{7\gamma} - \mathcal{O}_1$. Neubert et al.,arXiv:1003.5012 Benzke,Hurth,arXiv:2006.00624

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Underestimation of the uncertainty due to the resolved contribution.

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But used in recent $b \to s\gamma$ analysis. $\mathcal{B}^{SM}_{s\gamma} = (3.54 \pm 0.14) \times 10^{-4}$ NNLO

" since the remaining unknown $\mathcal{O}\left(\bar{\Lambda}^2/m_b^2\right)$ corrections may tend to cancel the currently known ones, we prefer to neglect all contributions of this order in the $Q_{1,2}$-Q_7 interference for the purpose of the present analysis "

Misiak et al.,arXiv:2608.29864

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- Great achievement based on 25 years of scientific efforts:

  $\mathcal{B}^{SM}_{s\gamma} = (3.60 \pm 0.30) \times 10^{-4}$ NLO Gambino,Misiak,arXiv:0104034

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  There is an ambiguity in the charm mass renormalisation at NLO

End of Intermezzo

Motivation for NLO Calculation

- There is a significant scale ambiguity of around 40% if one chooses the hard-collinear instead of the hard scale for the Wilson coefficient in LO result.

- A NLO analysis will diminish this scale ambiuguity and will establish a well-defined scale dependence of $O(\alpha_s^2)$ and will also reduce the dependence on the charm mass.

- Local Voloshin term of around 3% which is a natural part of the resolved contribution has to be added. Benzke,Garzelli,Hurth,arXiv:2512.08902

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- **Task 1** For NLL analysis we have to establish a factorisation theorem.

Hurth,Szafron,arXiv:2301.01739

$$d\Gamma \propto H \times J \otimes S \otimes \bar{J}$$

- **Task 2** Various steps of the NLL analysis

Bartocci,Böer,Brune,Hurth

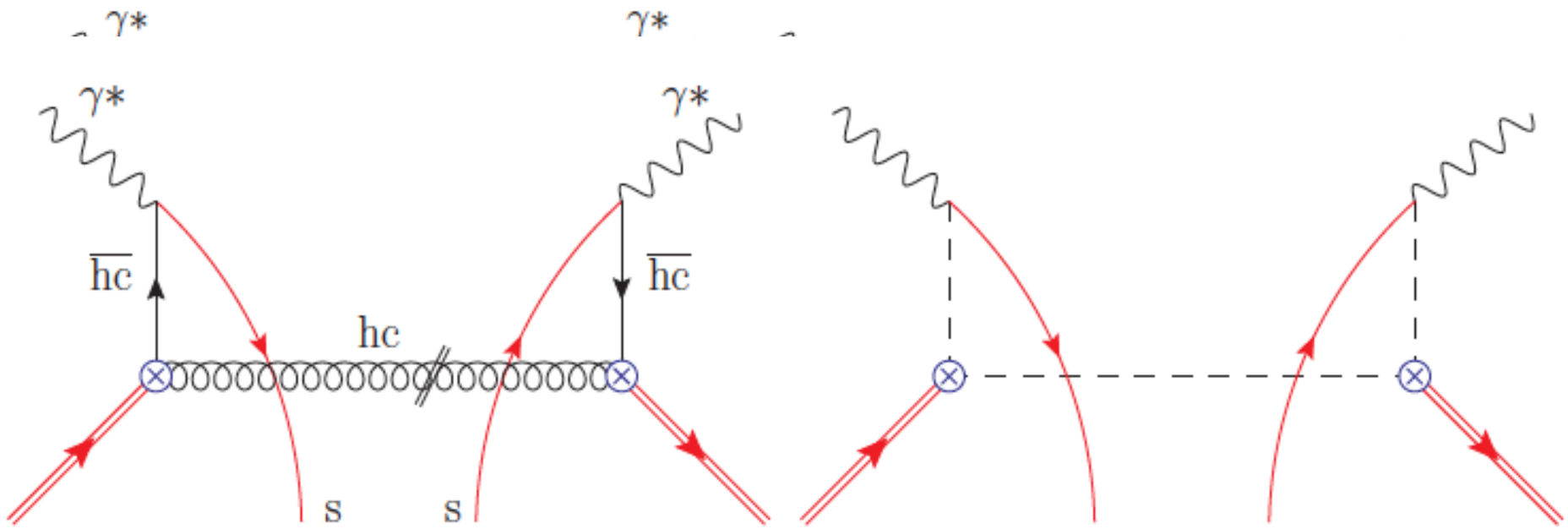
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- **Task 1** For **NLL analysis** we have to establish a factorisation theorem.

Interference of Q_8 and Q_8



$$\frac{d\Gamma^{\rm res}}{dn\cdot q\, d\bar n\cdot q} \sim \frac{e_s^2\alpha_s}{m_b}\int d\omega\,\delta(\omega+p_+)\int\frac{d\omega_1}{\omega_1+\bar n\cdot q+i\varepsilon}\int\frac{d\omega_2}{\omega_2+\bar n\cdot q-i\varepsilon}g_{88}(\omega,\omega_1,\omega_2)$$

$$g_{88}(\omega,\omega_1,\omega_2)=\frac{1}{M_B}\langle\bar B|\bar h(\mathbf{tn})\ldots s(\mathbf{tn}+\mathbf{u\bar n})\bar s(\mathbf{r\bar n})\ldots h(\mathbf{0})|\bar B\rangle_{\rm F.T.}$$

- Subtlety in the Q_8-Q_8 contribution: convolution integral is UV divergent
  - This implies that there is no complete proof of the factorization formula yet.

Benzke,Lee,Neubert,Paz,arXiv:1003.5012

- **Task 1** For **NLL analysis** we have to establish a factorisation theorem.

Refactorisation in subleading $\bar{B} \to X_s\gamma$ Hurth,Szafron,arXiv:2301.01739

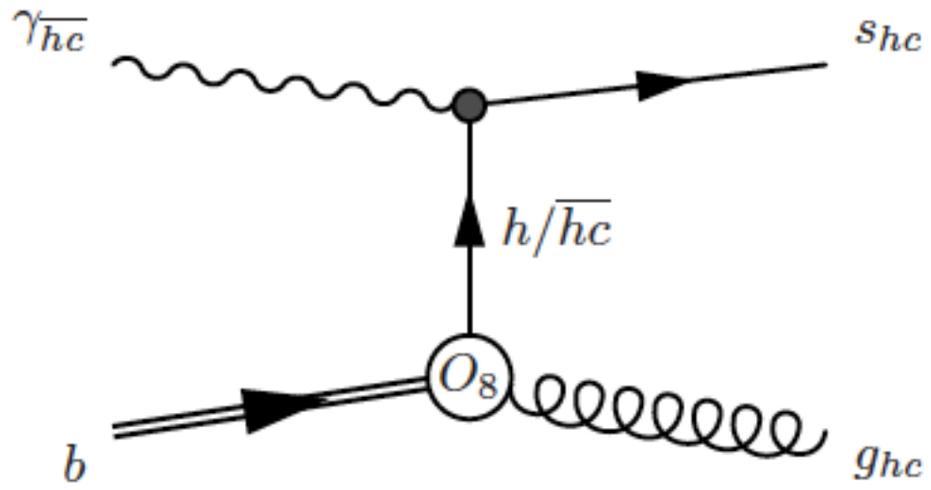
- Naive factorisation theorem with anti-hardcollinear Jet functions $\overline{J}$

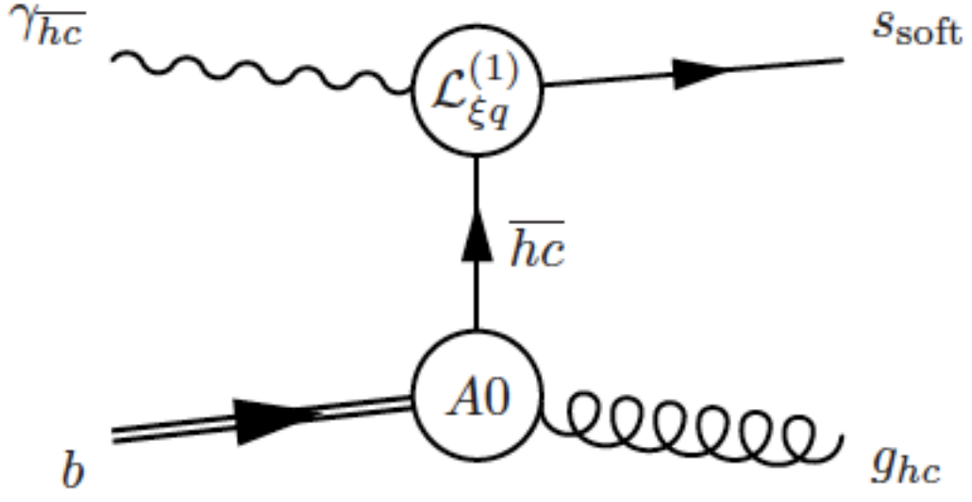
$$d\Gamma(\bar{B} \to X_s\gamma) = \sum_{n=0}^{\infty} \frac{1}{m_b^n} \sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)}$$
$$+ \sum_{n=1}^{\infty} \frac{1}{m_b^n} \left[\sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} + \sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} \otimes \bar{J}_i^{(n)} \right]$$

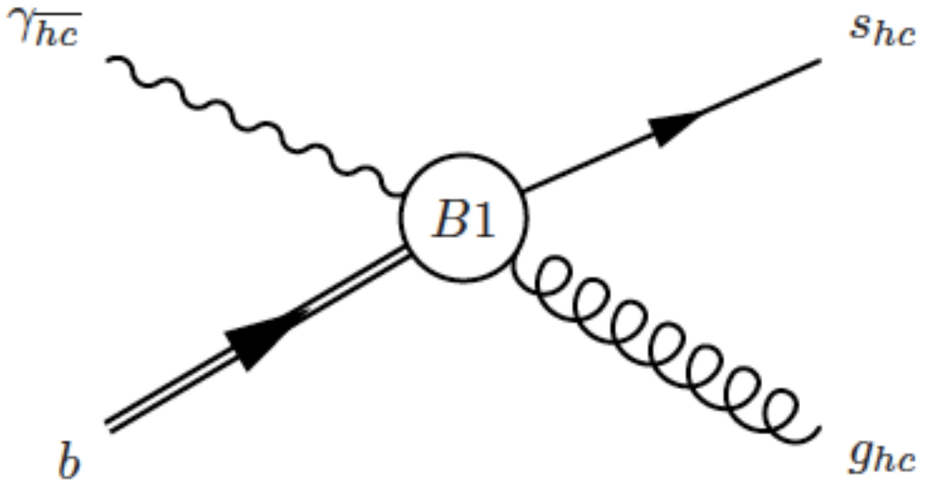
- One can identify divergences in resolved *and* direct contribution in SCET-I as endpoint-divergences
- One can use refactorisation techniques developed in collider examples Neubert et al.,arXiv:2009.06779
- First QCD application with nonperturbative objects in flavour physics

Degeneracy in EFT leads to endpoint divergences

Hurth,Szafron,arXiv:2301.01739







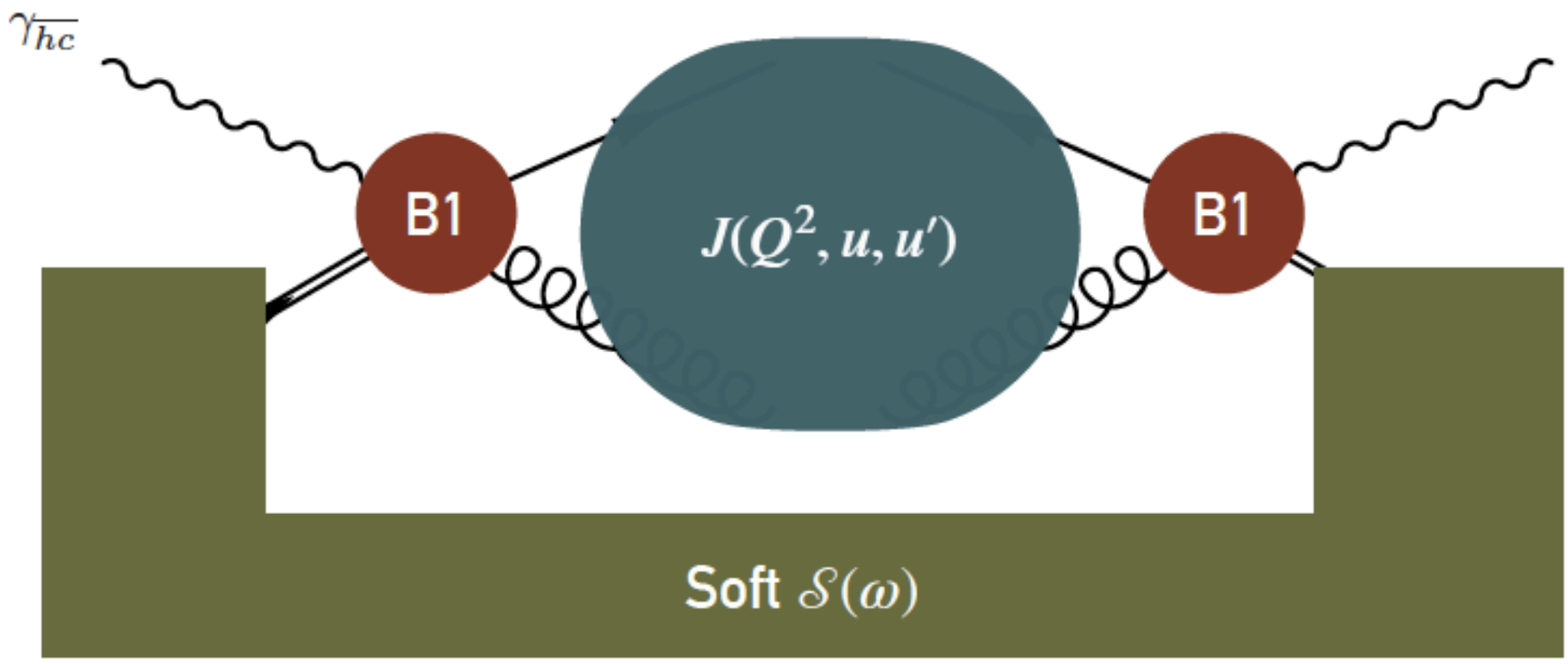
$$\mathcal{O}^{A0}_{8g}(0) = \overline{\chi}_{\overline{hc}}(0)\,\frac{\not{n}}{2}\gamma_{\mu\perp}\mathcal{A}^{\mu}{}_{hc\perp}(0)\,(1+\gamma_5)\,h(0)$$

$$\mathcal{O}^{B1}_{8g}(u) = \int \frac{dt}{2\pi} e^{-ium_b t}\overline{\chi}_{hc}(t\bar{n})\,\gamma_{\nu\perp}\,Q_s\,\mathcal{B}^{\nu}{}_{\overline{hc}\perp}(0)\,\gamma_{\mu\perp}\,\mathcal{A}^{\mu}{}_{hc\perp}(0)\,(1+\gamma_5)\,h(0)$$

Factorisation of direct contribution

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_B \int_0^1 du\, \mathbf{C^{B1}}\left(\mathbf{m_b}, \mathbf{u}\right) \int_0^1 du'\, \mathbf{C^{B1*}}\left(\mathbf{m_b}, \mathbf{u'}\right) \int_{-p_+}^{\overline{\Lambda}} d\omega\, \mathbf{J}\left(\mathbf{M_B}\left(\mathbf{p_+} + \omega\right), \mathbf{u}, \mathbf{u'}\right) \mathcal{S}\left(\omega\right)$$

$$\mathcal{S}\left(\omega\right) = \frac{1}{2m_B} \int \frac{dt}{2\pi} e^{-i\omega t} \left\langle B\right| h\left(tn\right) S_n\left(tn\right) S_n^\dagger\left(0\right) h\left(0\right) \left|B\right\rangle$$



Endpoint divergence in direct contribution at leading order

Hard matching coefficients

$$\mathbf{C_{LO}^{B1}\left(m_b, u\right)} = (-1)\frac{\overline{u}}{u}\frac{m_b^2}{4\pi^2}\frac{G_F}{\sqrt{2}}\lambda_t\, C_{8g} = (-1)\frac{\overline{u}}{u}C_{LO}^{A0}\left(m_b\right)$$

convoluted with jet function

$$\mathbf{J\left(p^2, u, u'\right)} = C_F\frac{\alpha_s}{4\pi\, m_b}\theta(p^2)\, A(\epsilon)\, \delta(u-u')u^{1-\epsilon}(1-u)^{-\epsilon}\left(\frac{p^2}{\mu^2}\right)^{-\epsilon}$$

lead to endpoint divergence in the $u \to 0$ limit

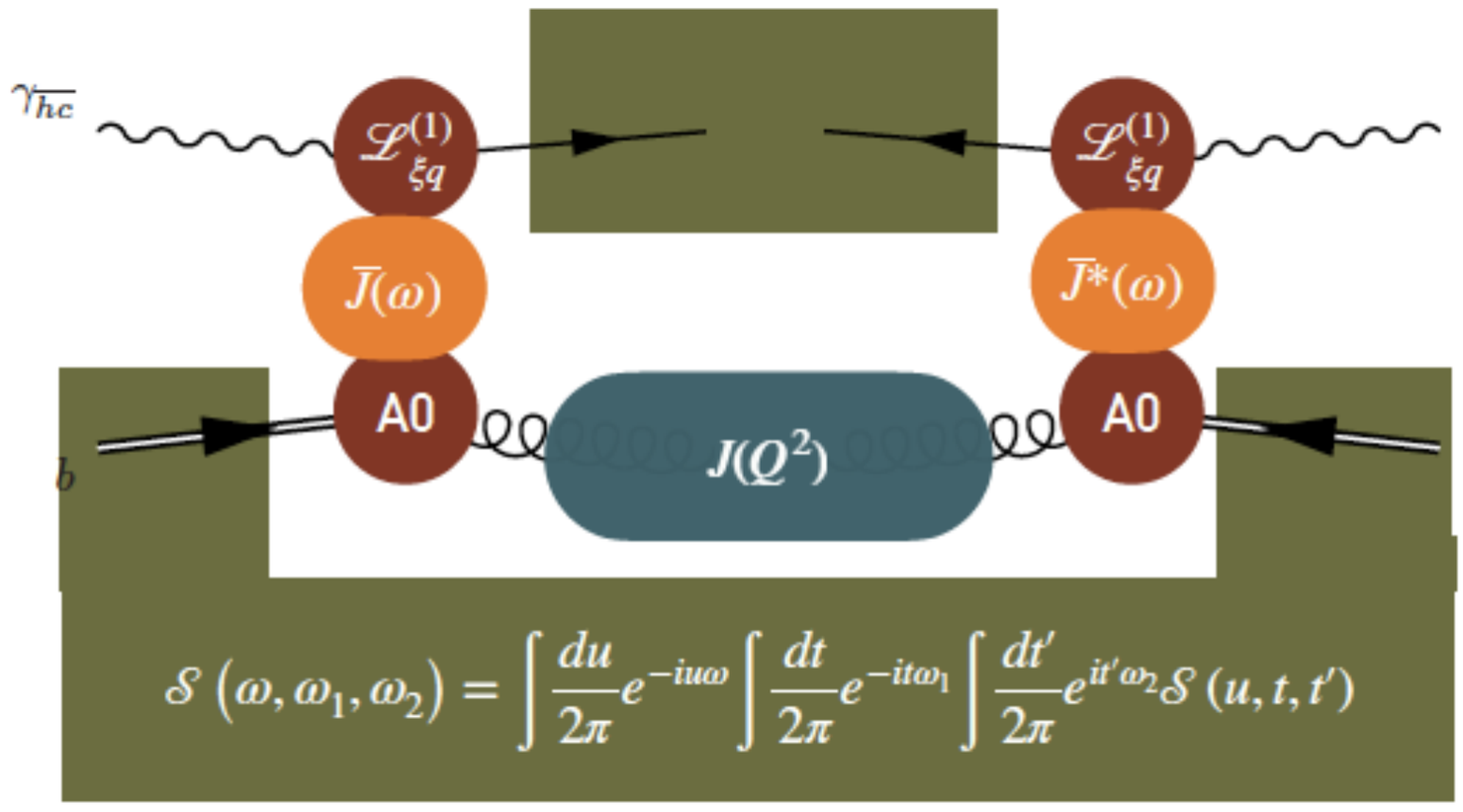
$$\int_0^1 du\frac{1}{u}\int_u^1 du'\frac{1}{u'}u^{1-\epsilon}\delta(u-u') \sim \int_0^1 du\frac{1}{u^1+\epsilon}$$

Factorisation of resolved contribution

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left|\mathbf{C^{A0}}\left(\mathbf{m_b}\right)\right|^2 \int_{-p_+}^{\overline{\Lambda}} d\omega\, \mathbf{J_g}\left(\mathbf{m_b}\left(\mathbf{p_+}+\omega\right)\right) \int d\omega_1 \int d\omega_2\, \overline{\mathbf{J}}\left(\omega_1\right)\, \overline{\mathbf{J}}^*\left(\omega_2\right) \mathcal{S}\left(\omega,\omega_1,\omega_2\right)$$

Operatorial definition of the soft function in position space $\mathcal{S}\left(\mathbf{u},\mathbf{t},\mathbf{t}'\right)$

$$\mathcal{S}\left(\mathbf{u},\mathbf{t},\mathbf{t}'\right) = (d-2)^2 g_s^2 \left\langle B\right| \overline{h}\left(un\right)\left(1-\gamma_5\right)\left[S_n\left(un\right) t^a S_n^\dagger\left(un\right)\right] S_{\bar{n}}\left(un\right) S_{\bar{n}}^\dagger\left(t'\bar{n}+un\right)$$
$$\frac{\not{n}\not{\bar{n}}}{4} q_s\left(t'\bar{n}+un\right) \overline{q}_s\left(t\bar{n}\right) \frac{\not{\bar{n}}\not{n}}{4} S_{\bar{n}}\left(t\bar{n}\right) S_{\bar{n}}^\dagger\left(0\right)\left[S_n\left(0\right) t^a S_n^\dagger\left(0\right)\right]\left(1+\gamma_5\right) h\left(0\right)\left|B\right\rangle \,/\,(2m_B)$$



Endpoint divergence in resolved contribution at leading order

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left|\mathbf{C^{A0}}\left(\mathbf{m_b}\right)\right|^2 \int_{-p_+}^{\overline{\Lambda}} d\omega\, \mathbf{J_g}\left(\mathbf{m_b}\left(\mathbf{p_+}+\omega\right)\right) \int d\omega_1 \int d\omega_2\, \overline{\mathbf{J}}\left(\omega_1\right)\, \overline{\mathbf{J}}^*\left(\omega_2\right) \mathcal{S}\left(\omega,\omega_1,\omega_2\right)$$

- Endpoint divergence occurs only for asymptotic $\omega_1 \sim \omega_2 \gg \omega$
- For $\omega_1 \sim \omega_2 \gg \omega$ light quarks become "hard-collinear" and can be decoupled from the soft gluons
- As a consequence the structure of the soft function corresponds to the leading power shape function $\mathcal{S}(\omega)$

$\omega_{1,2} \to \infty$ corresponds to $t, t' \to 0$ and $q_s(un) \to S_n(un) q_{hc}(un)$, $\bar{q}_s(0) \to q_{hc} S_n^+(0)$

$$\mathcal{S}\left(u,t,t'\right) = (d-2)^2 g_s^2 \left\langle B\right| \overline{h}\left(un\right)\left(1-\gamma_5\right)\left[S_n\left(un\right) t^a S_n^\dagger\left(un\right)\right] S_{\bar{n}}\left(un\right) S_{\bar{n}}^\dagger\left(t'\bar{n}+un\right)$$
$$\frac{\not{n}\not{\bar{n}}}{4} q_s\left(t'\bar{n}+un\right) \bar{q}_s\left(t\bar{n}\right) \frac{\not{\bar{n}}\not{n}}{4} S_{\bar{n}}\left(t\bar{n}\right) S_{\bar{n}}^\dagger\left(0\right)\left[S_n\left(0\right) t^a S_n^\dagger\left(0\right)\right]\left(1+\gamma_5\right) h\left(0\right)\left|B\right\rangle \,/\left(2m_B\right)$$

$$\mathcal{S}\left(u\right) = \left\langle B\right| \overline{h}\left(un\right) S_n\left(un\right)\, S_n^\dagger\left(0\right) h\left(0\right)\left|B\right\rangle \,/\left(2m_B\right)$$

Refactorisation at leading order

$$\frac{d\Gamma}{dE_\gamma}\Big|_B^{u,u'\to 0} = -\mathcal{N}\,\left|C_{LO}^{A0}\left(m_b\right)\right|^2\,\frac{\alpha_s C_F}{(4\pi)\,m_b}\,\frac{1}{\epsilon}\,A(\epsilon)\int_{-p_+}^{\overline{\Lambda}} d\omega\,\mathcal{S}_{LO}(\omega)\left(\frac{m_b(\omega+p_+)}{\mu^2}\right)^{-\epsilon}$$

$$\frac{d\Gamma}{dE_\gamma}\Big|_A^{\text{asy}} = \mathcal{N}\left|C_{LO}^{A0}\left(m_b\right)\right|^2\,\frac{\alpha_s C_F}{(4\pi)\,m_b}\,\frac{1}{\epsilon}A(\epsilon)\int_{-p_+}^{\overline{\Lambda}} d\omega\,\mathcal{S}_{LO}(\omega')\left(\frac{m_b(\omega+p_+)}{\mu^2}\right)^{-\epsilon}$$

One verifies that

$$\frac{d\Gamma}{dE_\gamma}\Big|_A^{\text{asy}} = (-1)\,\frac{d\Gamma}{dE_\gamma}\Big|_B^{u,u'\to 0}$$

Refactorisation conditions can be formulated on the operator level

Express the fact that in the limits $u \sim u' \ll 1$ and $\omega_1 \sim \omega_2 \gg \omega$ the two terms of the subleading $\mathcal{O}_8 - \mathcal{O}_8$ contribution have the same structure.

- $[\![C^{B1}(m_b, u)]\!] = (-1) C^{A0}(m_b)\, m_b \overline{J}(u m_b)$
  ($[\![g(u)]\!]$ only denotes the leading term of a function $g(u)$ in the limit $u \to 0$)
- $\widetilde{\mathcal{S}}(\omega, \omega_1, \omega_2)$ corresponds to $\mathcal{S}(\omega, \omega_1, \omega_2)$ in the limit $\omega_1 \sim \omega_2 \gg \omega$
  (In this limit: $q_s \to q_{sc}$ and higher power corrections in $\omega/\omega_{1,2}$ are neglected)
- $\int_{-p_+}^{\overline{\Lambda}} d\omega\, [\![J(m_b(p_+ + \omega), u, u')\, \mathcal{S}(\omega)]\!] = \int_{-p_+}^{\overline{\Lambda}} d\omega J_g(m_b(p_+ + \omega)) \widetilde{\mathcal{S}}(\omega, m_b u, m_b u')$
  (In this limit $\chi_{hc} \to q_{sc}$, brackets indicate again that the $u \to 0$ *and* $u' \to 0$ limits)

The refactorisation relations are operatorial relations that guarantee the cancellation of endpoint divergences between the two terms to all orders in α_s.

Finally we show that refactorisation and renormalisation commute.

- **Task 2** Various steps of the NLL analysis

Bartocci,Böer,Hurth,arXiv:2411.16634,arXiv:2512.08902

Bartocci,Hurth,arXiv:2610.14211

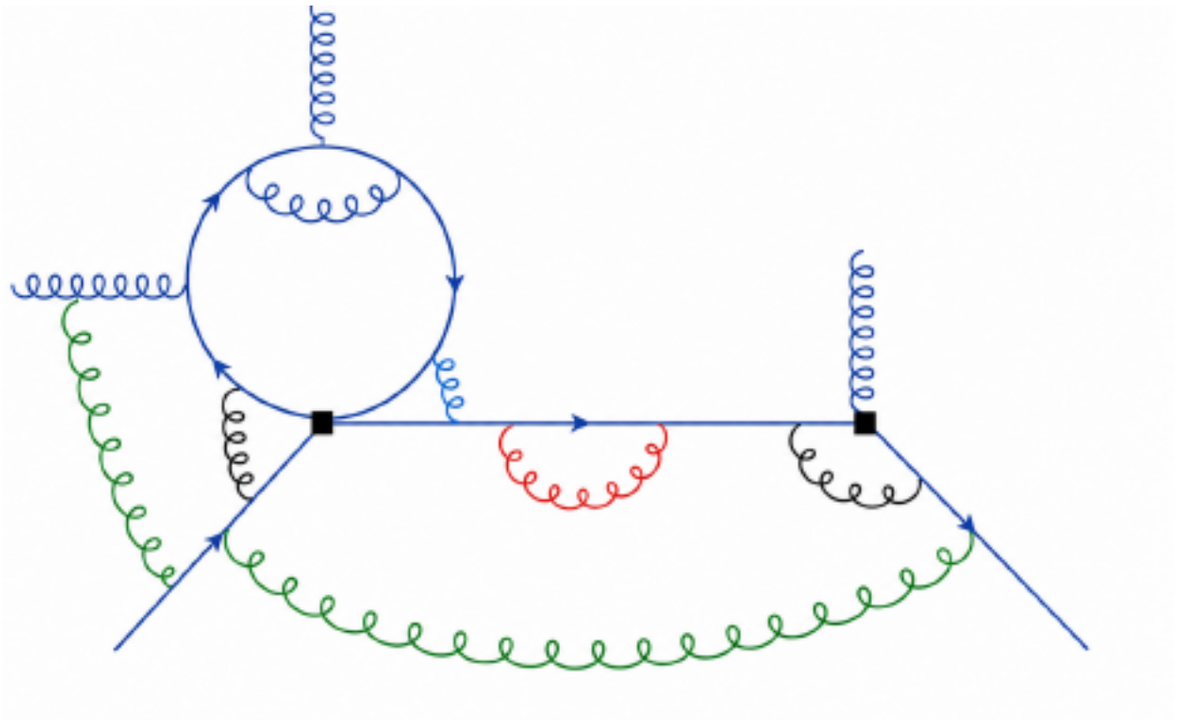
Bartocci,Böer,Brune,Hurth,arXiv:2611.14318

- **Task 2** Various steps of the NLL analysis

  – factorisation theorem beyond the LO has to be established

$$d\Gamma(\bar{B} \to X_s\gamma) \sim H \cdot J \otimes g_{17} \otimes \bar{J}$$

1. Hard function ($\mu^2 \sim m_b^2$) (perturbative)
2. Jet function ($\mu^2 \sim m_b\Lambda_{QCD}$) (perturbative)
3. Subleading shape function ($\mu^2 \sim \Lambda_{QCD}^2$) (non-perturbative)
4. "Penguin"-jet function ($\mu^2 \sim m_b\Lambda_{QCD}$) (perturbative)



In SCET, we can compute gauge invariant pieces separately.

- **Task 2** Various steps of the NLL analysis

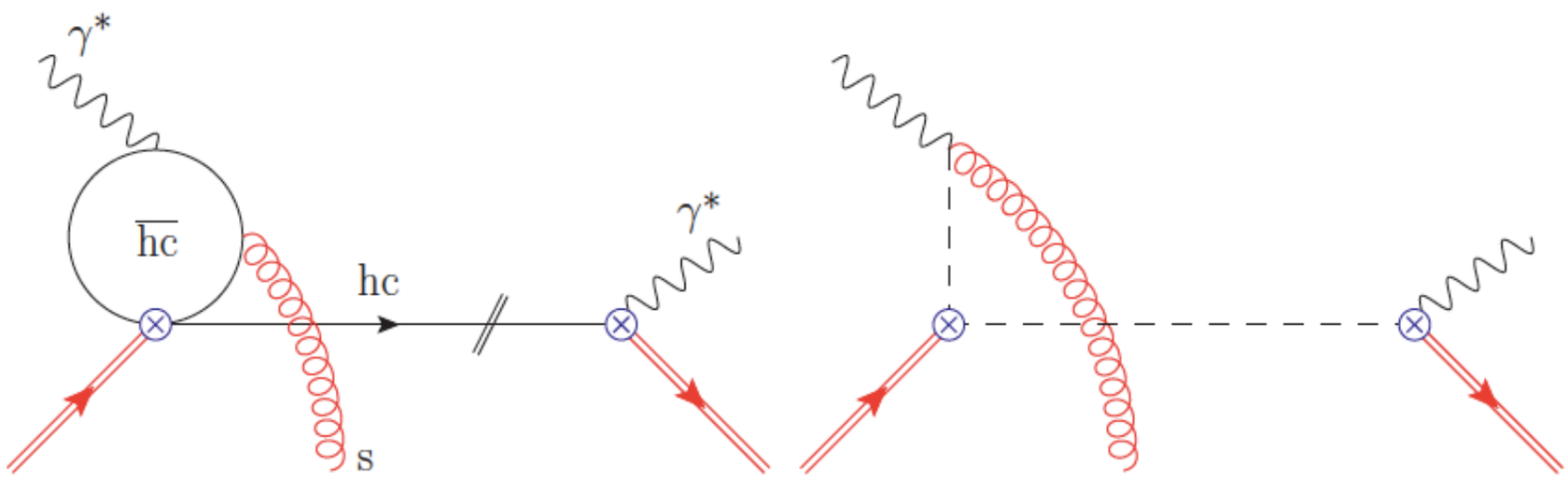
  – analysis of renormalisation properties of the soft function

$$\mathcal{S}(\omega,\omega_1;\mu)) = \frac{1}{2M_B}\int\frac{dr}{2\pi}\,e^{-i\omega_1 r}\int\frac{dt}{2\pi}\,e^{-i\omega t}\,\langle\bar{B}_v|\mathcal{O}_{17}(t,r)|\bar{B}_v\rangle$$

Related to $\bar{n}$ Related to n

Shape function is nonlocal in both light cone directions

$$\mathcal{S}_{ren}(\omega,\omega_1) = \int_{-\infty}^{\Lambda} d\omega'\, Z_{\mathcal{S}}(\omega,\omega',\omega_1,\omega_1')\,\mathcal{S}_{bare}(\omega',\omega_1').$$



Bartocci,Böer,Hurth, arXiv:2411.16634

- **Task 2** Various steps of the NLL analysis

  Factorisation indicates that the dependence on the two light-cone directions should decouple:

$$Z^{(1)}(\omega,\omega_1,\omega',\omega_1';\mu) = \delta(\omega_1-\omega_1')Z_n^{(1)}(\omega,\omega';\mu) + \delta(\omega-\omega')Z_{\bar n}^{(1)}(\omega_1,\omega_1';\mu)$$

  However, calculation of the anomalous dimensions of the operator $\mathcal{O}_{1,7}(r,s)$ leads also to mixing terms.

a) Decay rate is defined via restricted cuts:

$$d\Gamma(\bar B\to X_s\gamma) \propto \mathrm{Disc}_{\underline{\mathrm{restr.}}}\left[i\int d^4x\,\langle\bar B|\mathcal{H}_{\mathrm{eff}}^\dagger(x)\,\mathcal{H}_{\mathrm{eff}}(0)|\bar B\rangle\right]$$

   Cutting rules (Keldish formalism) show that mixing terms do not contribute!

b) Mixing terms also vanish when convoluted with the physical jet functions.

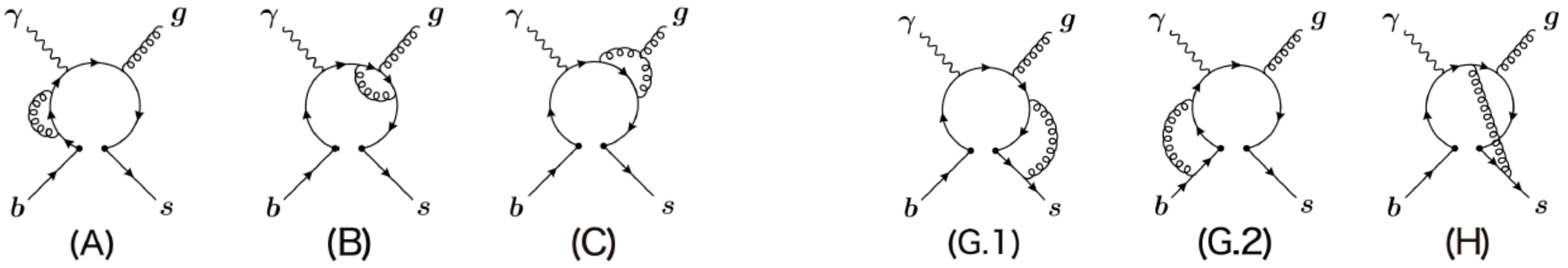
   Highly relevant observation also for exclusive modes (where the soft function is defined on the amplitude level) : RG simplifies heavily.

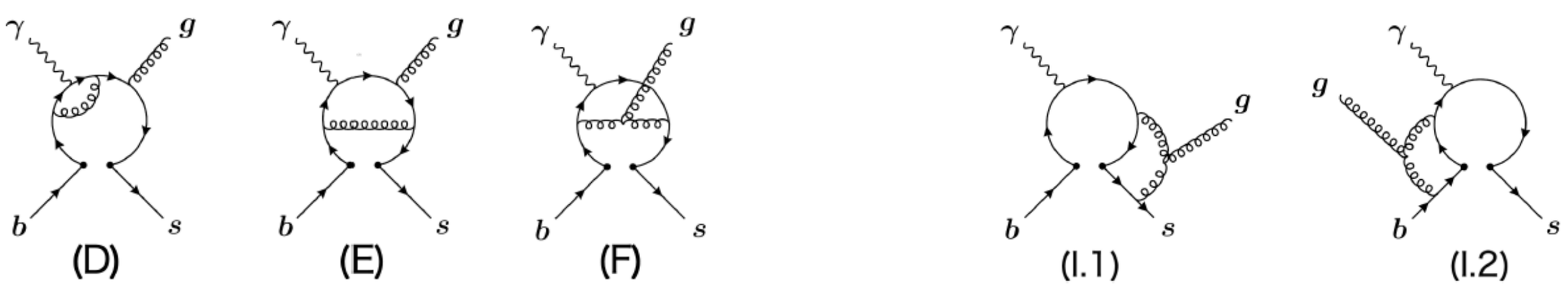
Bartocci,Böer,Hurth, arXiv:2411.16634

- **Task 2** Various steps of the NLL analysis

  – calculation of the anti-hard-collinear jet function at NLO

  Extracted from the anti-hard-collinear region of $b \to s\gamma g$



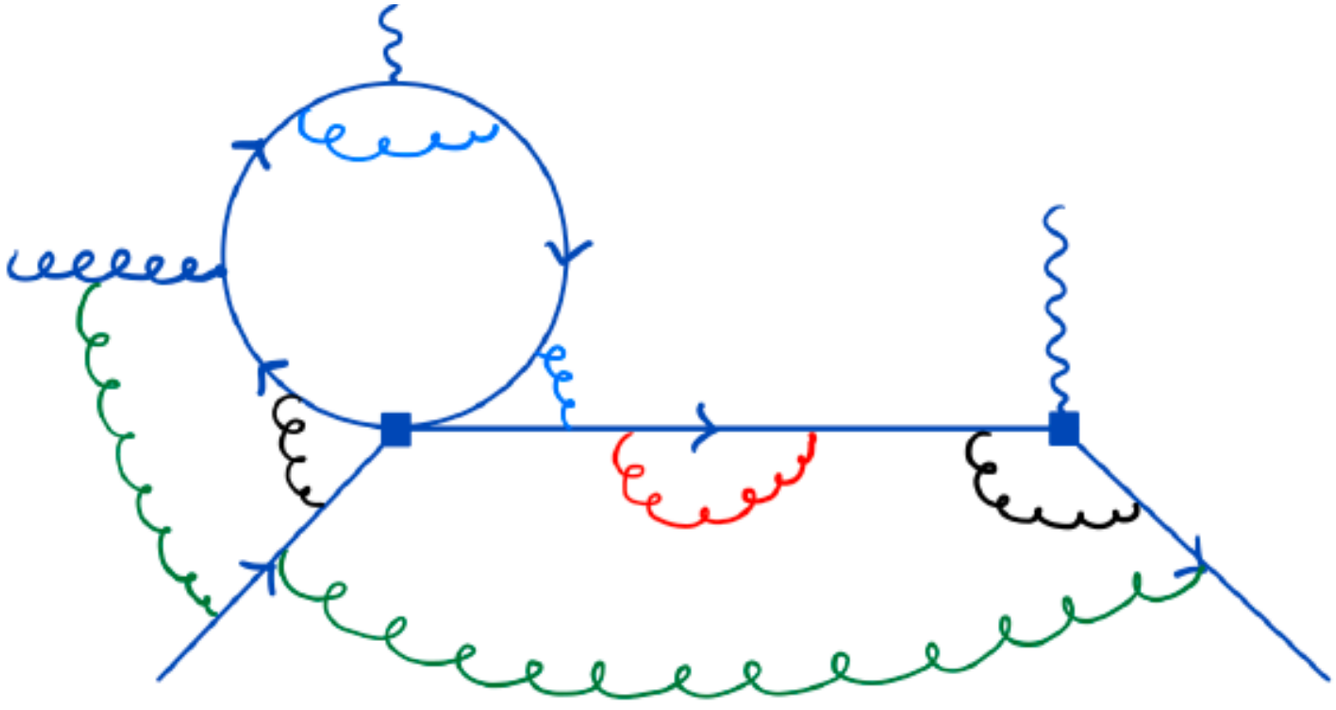


Additional complication at NLO

$$\mathrm{d}\Gamma\,(\bar{B} \to X_s\gamma) \sim \int_0^1 \mathrm{d}u\, H(u;\mu) \int_{-\infty}^{\infty} \mathrm{d}\omega_1\, J^{(\bar{n})}(u,\omega_1;\mu) \int_{-\infty}^{\bar{\Lambda}} \mathrm{d}\omega\, J^{(n)}(\omega;\mu) g_{17}(\omega,\omega_1;\mu)$$

Bartocci, Böer, Brune, Hurth

- **Task 2** Various steps of the NLL analysis Bartocci,Böer,Brune,Hurth



$$d\Gamma \propto H \times J \otimes S \otimes \bar{J}$$

Status quo

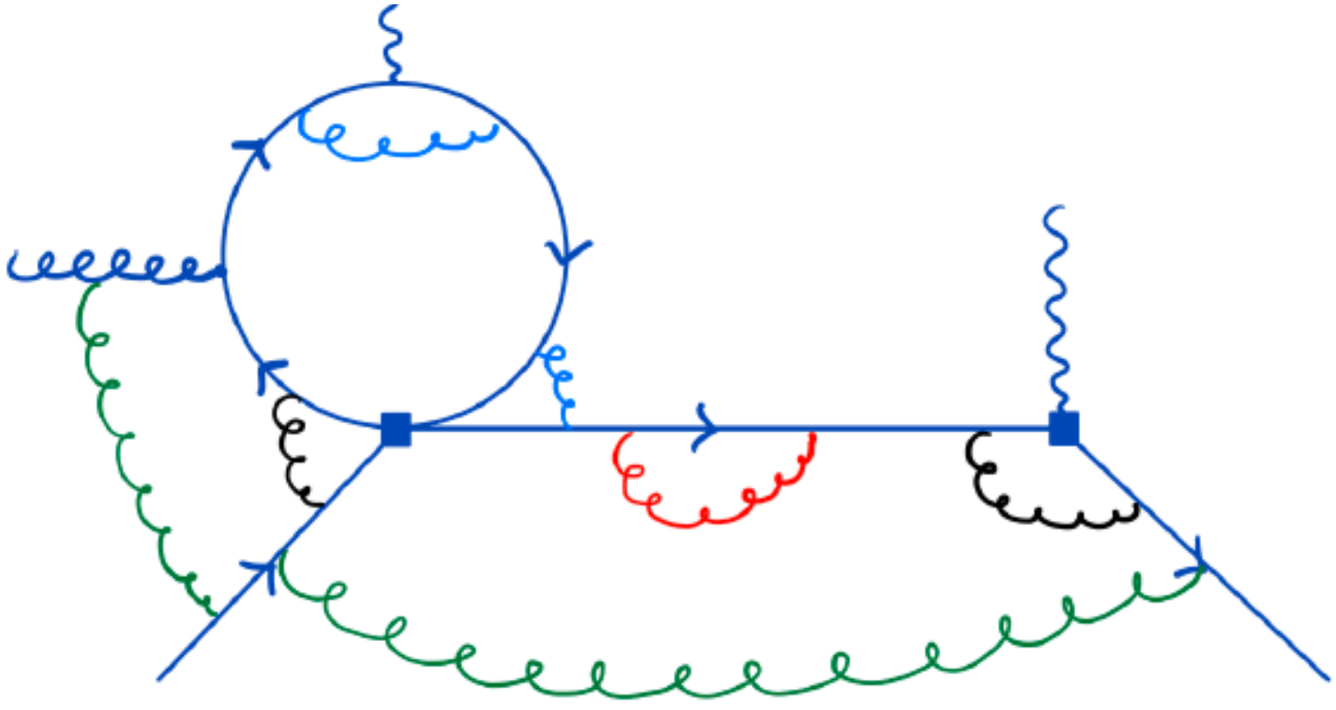
In SCET, we can compute gauge invariant pieces separately.

- analysis of renormalisation properties of **the soft function** ✓
- α_s (two-loop) corrections to **anti-jet function**

  We already calculated all diagrams for $m_c \to m_u = 0$ ✓
- α_s corrections to **quark jet function** known ✓
- **hard** matching at order α_s known ✓
- use RG techniques to run various functions to a common scale.

We already checked the pole cancellation for $m_c \to m_u = 0$ ✓

- **Task 2** Various steps of the NLL analysis Bartocci,Böer,Brune,Hurth



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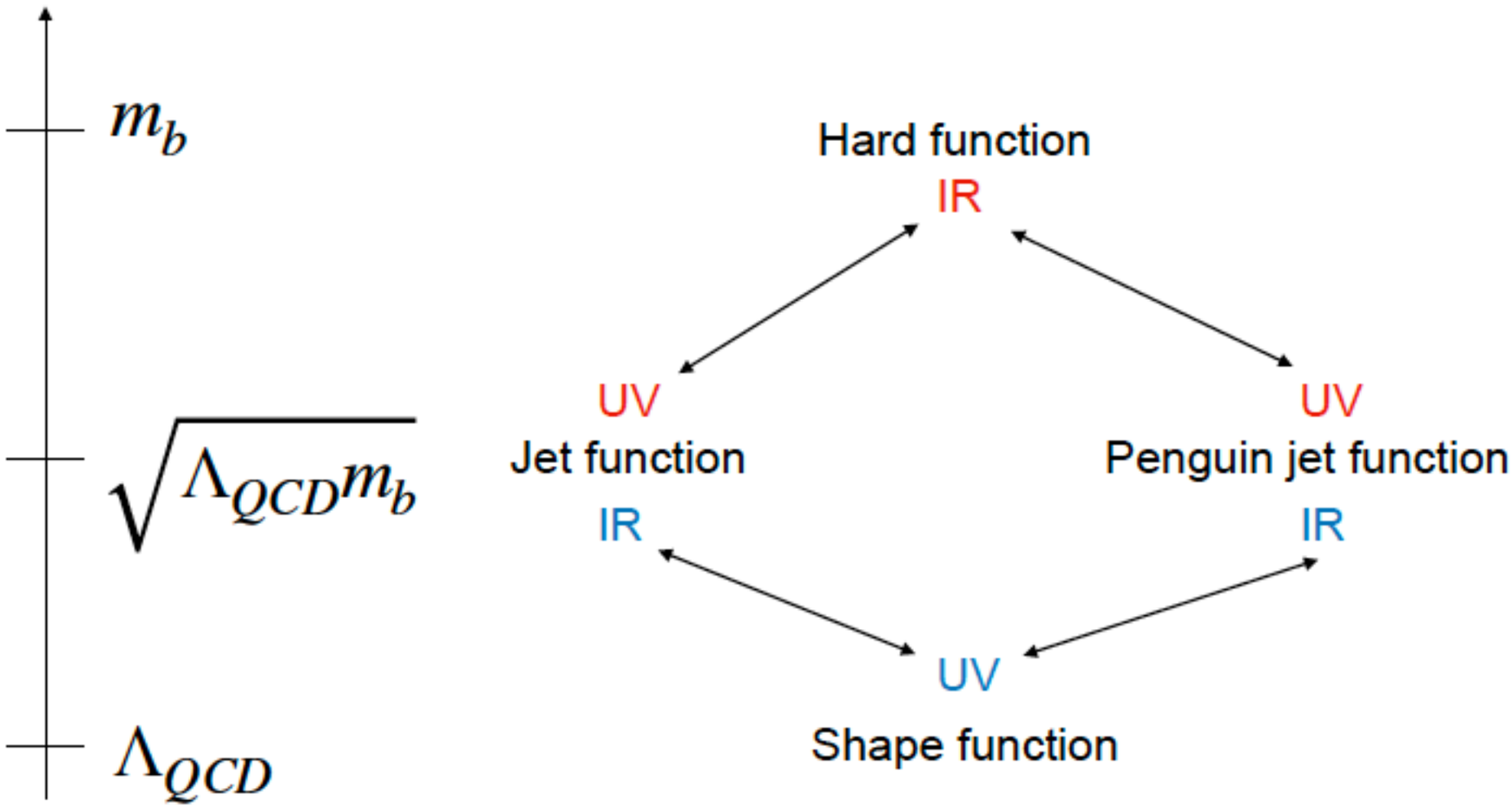
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We already checked the pole cancellation for $m_c \to m_u = 0$

$$d\Gamma(\bar{B} \to X_s\gamma) \sim H \cdot J \otimes g_{17} \otimes \bar{J}$$



All poles cancel !

Non-trivial crosscheck of the factorisation theorem and of the calculations

Summary

- Several persisting deviations from the *theory* predictions in $b \to s\ell\ell$ transitions since 2013.

- Ambiguities in soft formfactors and unknown nonfactorisable power corrections in *theory* predictions.

- Measurements of the inclusive decay $\bar{B} \to X_s\ell\ell$ can crosscheck these deviations.

- Refactorisation techniques allow to solve the problem of endpoint divergences, in particular in subleading $\bar{B} \to X_s\gamma$ and $\bar{B} \to X_s\ell\ell$ (perhaps also in exclusive modes).

- Nonlocal power corrections presently belong to the largest uncertainties in the inclusive modes $\bar{B} \to X_s\gamma$ and $\bar{B} \to X_s\ell\ell$ including a large scale ambiguity. Higher moments of shape functions and NLL corrections are needed.

- RGE of soft function has been fully calculated offering key insights into generalised light-cone distributions amplitudes.

- RGEs of the hard functions have now also been calculated.

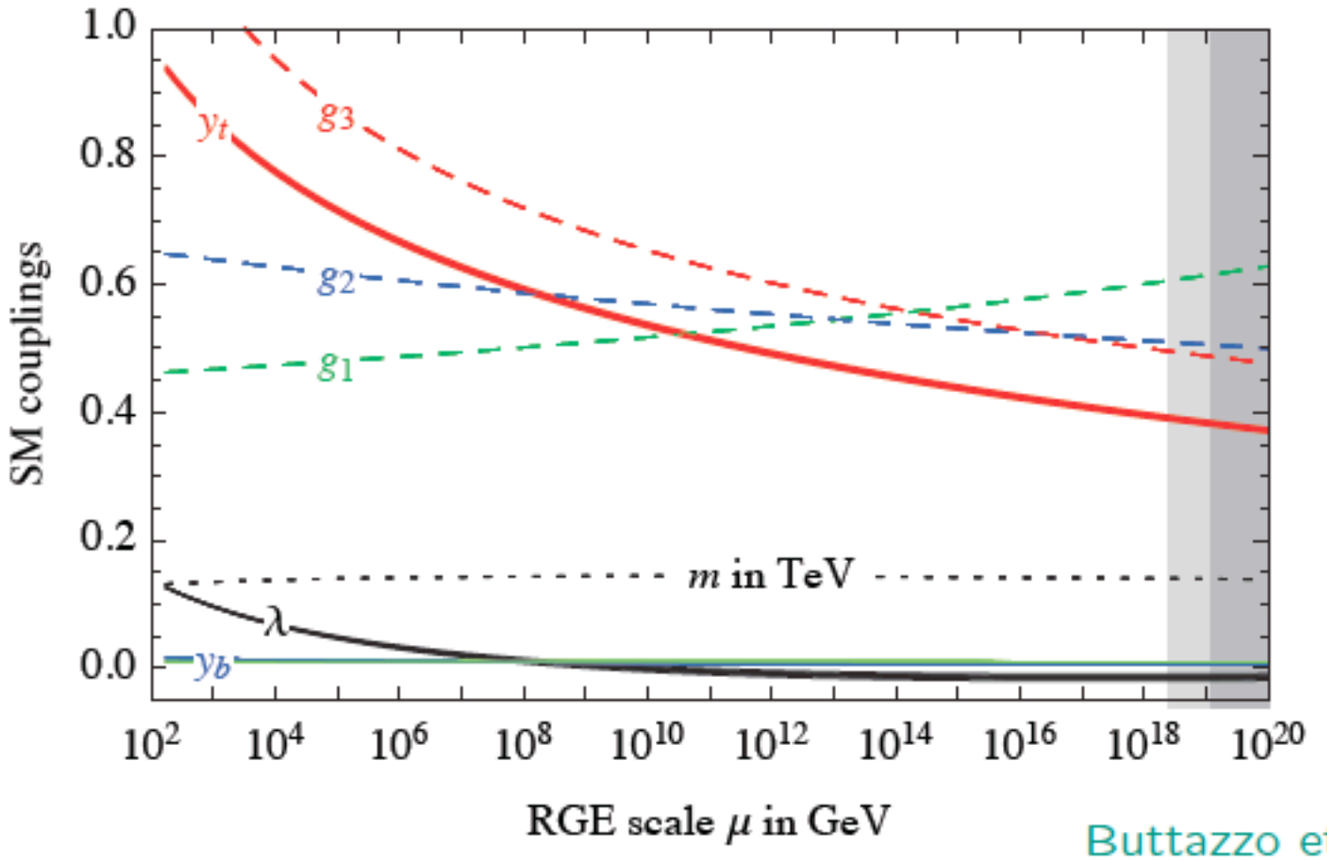
- Two-loop radiative jet function computed for consistency checks (pole cancellation) in the massless case. After adding mass corrections, all the ingredients for phenomenological analysis are ready.

Epilogue

Self-consistency of the SM

Do we need new physics beyond the SM ?

- It is possible to extend the validity of the SM up to the M_P as weakly coupled theory.



Buttazzo et al. arXiv:1307.3536

High-energy extrapolation shows that the Yukawa couplings, weak gauge couplings and the Higgs self coupling remain perturbative in the entire energy domain between the electroweak and Planck scale (no Landau poles !).

- Renormalizability implies no constraints on the free parameters of the SM Lagrangian.

Experimental evidence beyond SM

- Dark matter (visible matter accounts for only 4% of the Universe)
- Neutrino masses (Dirac or Majorana masses ?)
- Baryon asymmetry of the Universe (new sources of CP violation needed)

Caveat:

Answers perhaps wait at energy scales which we do not reach with present experiments.

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Answers perhaps wait at energy scales which we do not reach with present experiments.



Courtesy of Sophie Renner

Take more data...



Courtesy of Sophie Renner

Look for indirect evidence...



Courtesy of Sophie Renner

Intermezzo II

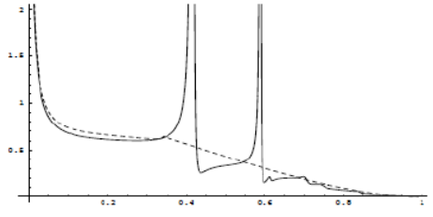
Complete angular analysis of inclusive $B \to X_s \ell\ell$

- Phenomenological analysis to NNLO QCD and NLO QED for all angular observables

Huber,Hurth,Lunghi, arXiv:1503.04849

$$\frac{d^2\Gamma}{dq^2\,dz} = \frac{3}{8}\left[(1+z^2)\,H_T(q^2) + 2\,z\,H_A(q^2) + 2\,(1-z^2)\,H_L(q^2)\right] \qquad (z = \cos\theta_\ell)$$

$$\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2) \qquad \frac{dA_{\rm FB}}{dq^2} = 3/4\,H_A(q^2)$$



- Dependence on Wilson coefficients

Lee,Ligeti,Stewart, Tackmann hep-ph/0612156

$$H_T(q^2) \propto 2s(1-s)^2\left[\left|C_9 + \frac{2}{s}\,C_7\right|^2 + |C_{10}|^2\right]$$

$$H_A(q^2) \propto -4s(1-s)^2\ \mathrm{Re}\left[C_{10}\left(C_9 + \frac{2}{s}\,C_7\right)\right]$$

$$H_L(q^2) \propto (1-s)^2\left[\left|C_9 + 2\,C_7\right|^2 + |C_{10}|^2\right]$$

- Electromagnetic effects due to energetic photons are large and calculated analytically and crosschecked against Monte Carlo generator events

$$\alpha_{\rm em}\, log(m_b^2/m_\ell^2) \qquad q^2 = (p_{\ell+} + p_{\ell-}) \ \Rightarrow\ q^2 = (p_{\ell+} + p_{\ell-} + p_\gamma)$$

Huber,Hurth,Lunghi, arXiv:1503.04849

- In the ratio of the inclusive $b \to s\ell\ell$ decay rate in the high-q^2 region and the semileptonic decay rate large part of the nonperturbative effects cancel out:

Ligeti,Tackmann, arXiv:0707.1694

$$R^{(\ell)}_{\rm incl}(q_0^2) = \frac{\displaystyle\int_{q_0^2}^{m_B^2} dq^2 \frac{d\Gamma(B \to X_s \bar{\ell}\ell)}{dq^2}}{\displaystyle\int_{q_0^2}^{m_B^2} dq^2 \frac{d\Gamma(B \to X_u \bar{\ell}\nu)}{dq^2}}$$

Tensions in the inclusive high q^2 decay rate ??

Isidori,Polonsky,Tinari,arXiv:2305.03076
Isidori, arXiv:2308.11612

$$R^{SM}_{\rm incl}(15) = \frac{\displaystyle\int_{15}^{m_B^2} dq^2 \frac{d\Gamma(B\to X_s\bar\ell\ell)}{dq^2}}{\displaystyle\int_{15}^{m_B^2} dq^2 \frac{d\Gamma(B\to X_u\bar\ell\nu)}{dq^2}} \quad x \quad \mathcal{B}(B\to X_u\bar\ell\nu)^{\rm exp}_{[15]} = (1.50\pm 0.24)\times 10^{-4}$$

Belle,arXiv:2107.13855

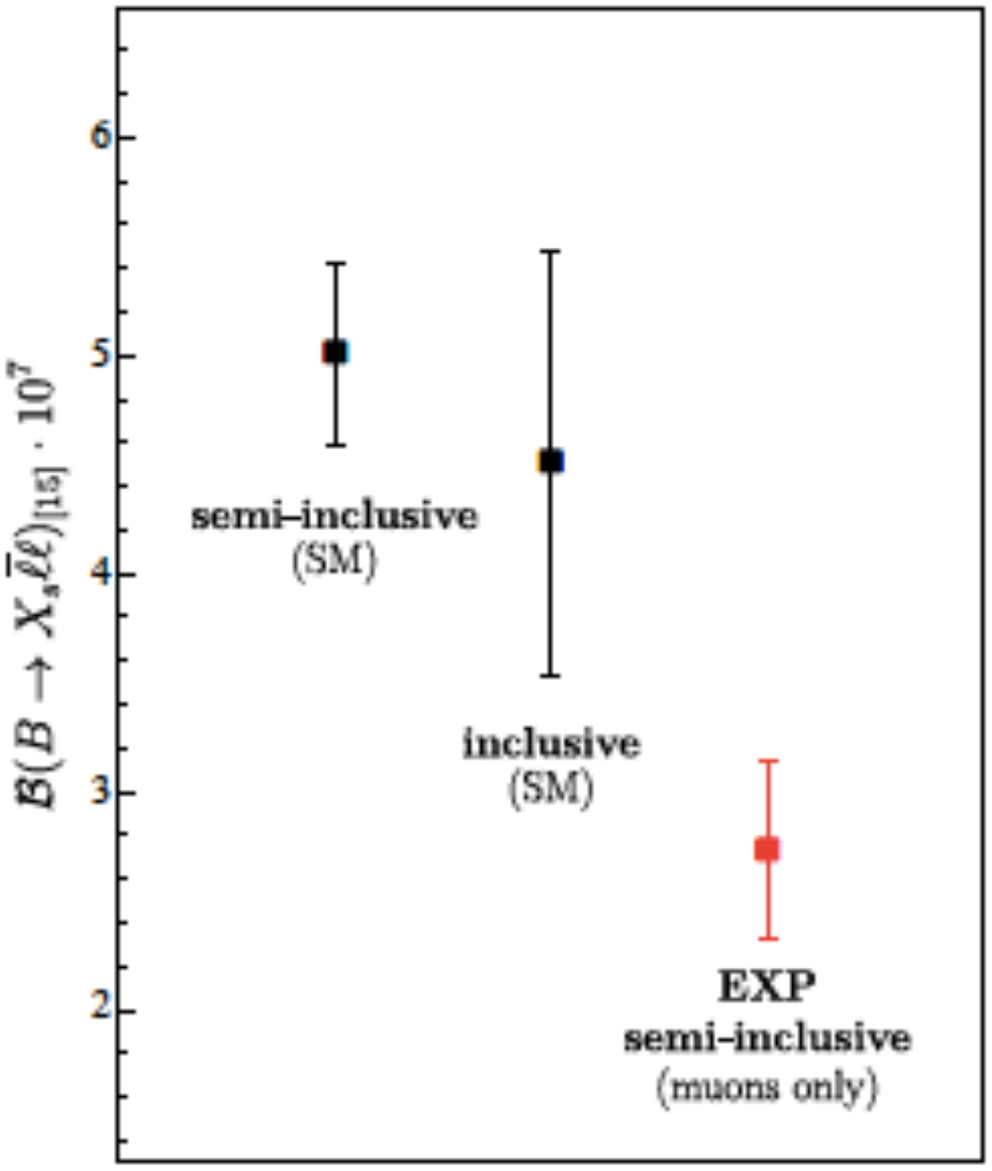
$$= \quad "\mathcal{B}(B\to X_s\bar\ell\ell)^{SM}_{[15]}" \quad \stackrel{!}{=} \quad \sum_i \mathcal{B}(B\to X_s^i\bar\mu\mu)^{\rm exp}_{[15]} = (2.74\pm 0.41)\times 10^{-7}$$

Isidori,Polonsky,Tinari,arXiv:2305.03076

- Experimental semi-inclusive rate is estimated by the sum of the $B\to K$ and $B\to K^*$ modes and a correction factor for the two-body final states $B\to K\pi$.

- Isidori et al. claim a tension up to 2σ – confirming analogous results in the exclusive modes.

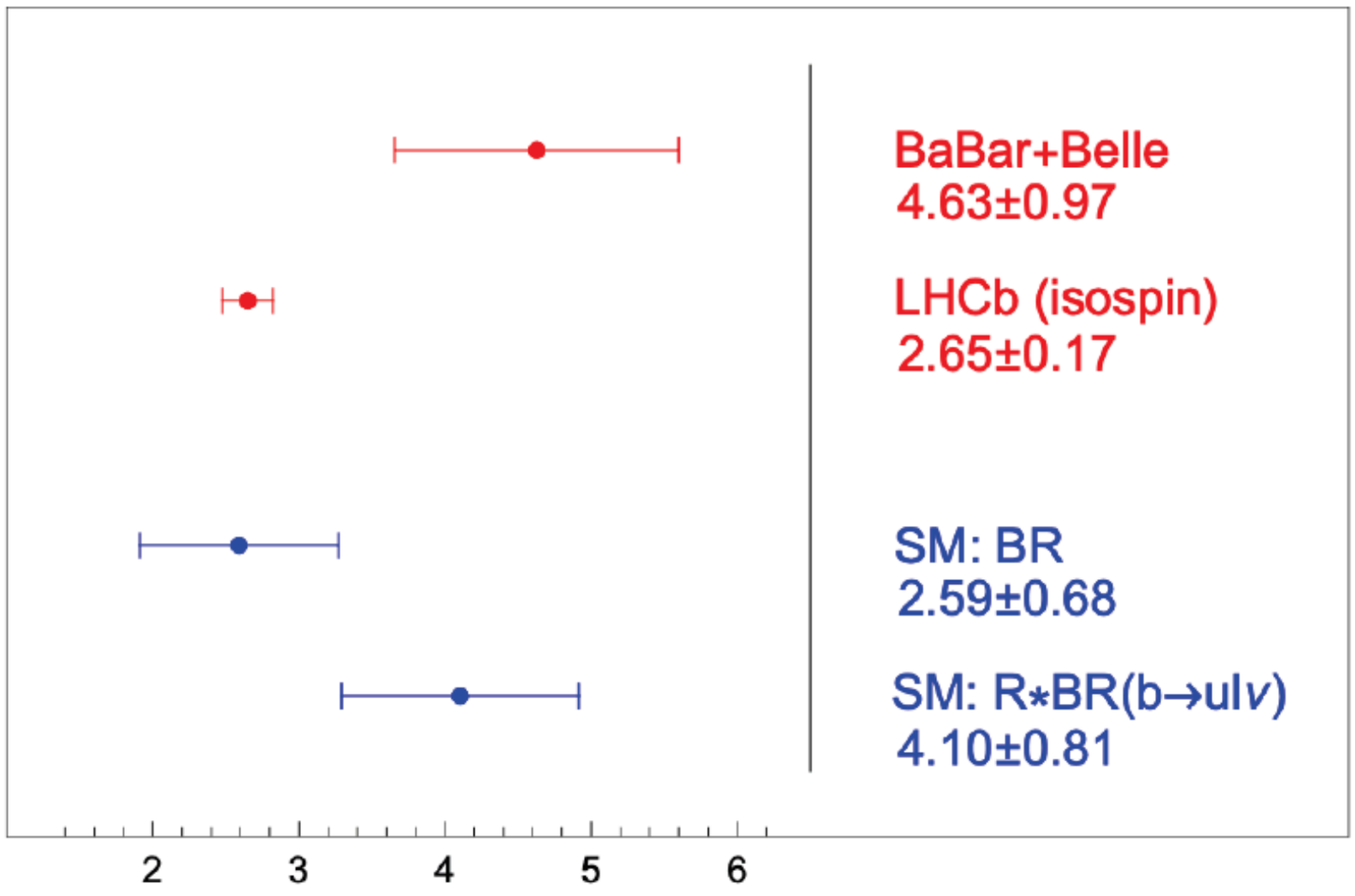
Isidori,Polonsky,Tinari,arXiv:2305.03076
Isidori, arXiv:2308.11612



- We do not find any tension if we also consider our direct result for the branching $\mathcal{B}(B \to X_s \ell\ell)^{\text{SM}}_{[15]}$ and the Babar/Belle measurements.

Huber,Hurth,Jenkins,Lunghi,Qin Qin,Vos,arXiv:2007.04191

Talk by T.H. at FPCP23 and arXiv:2404.03517



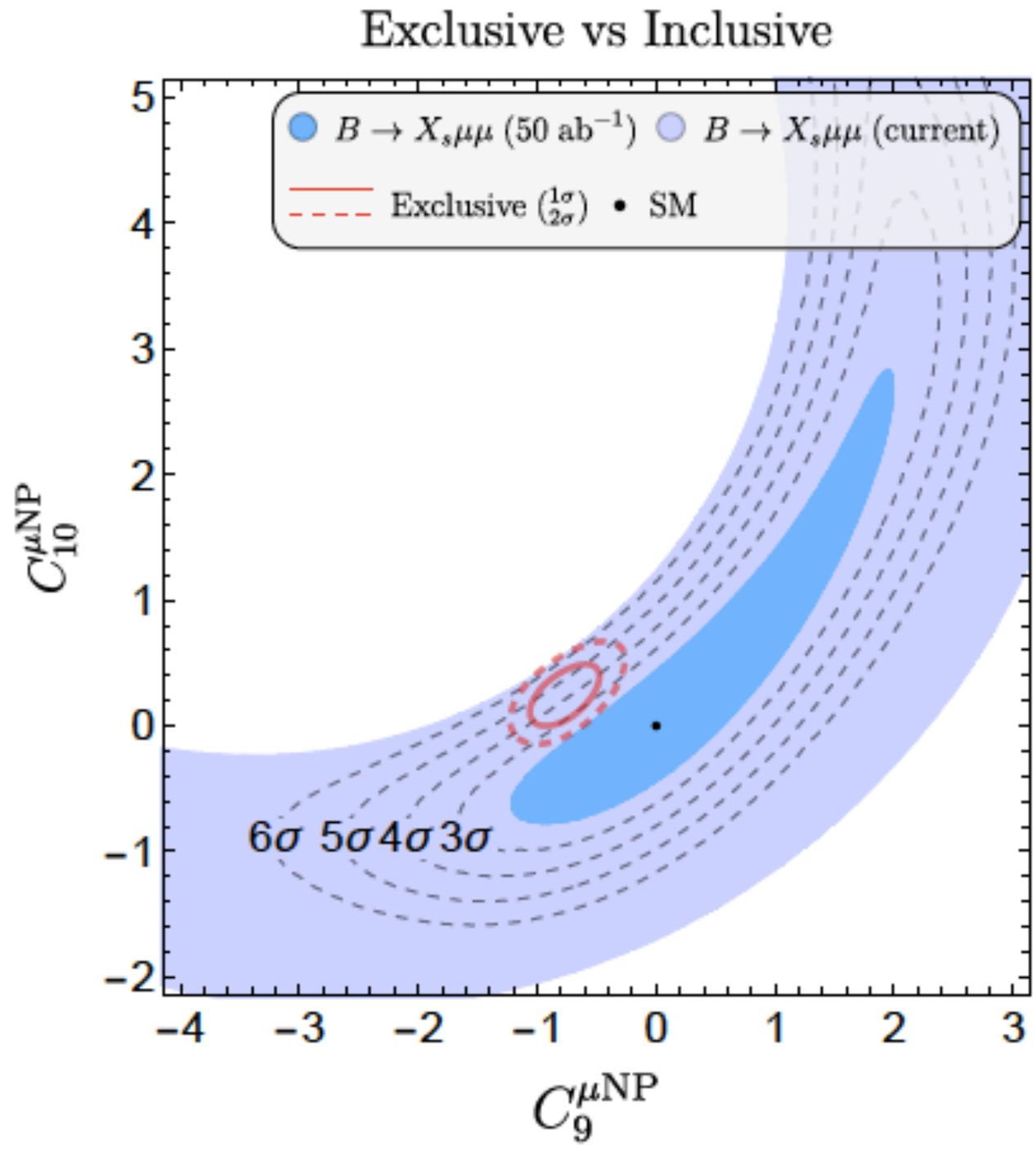
$$\text{BR}(B \to X_s \ell^+ \ell^-)[> 15] \times 10^7$$

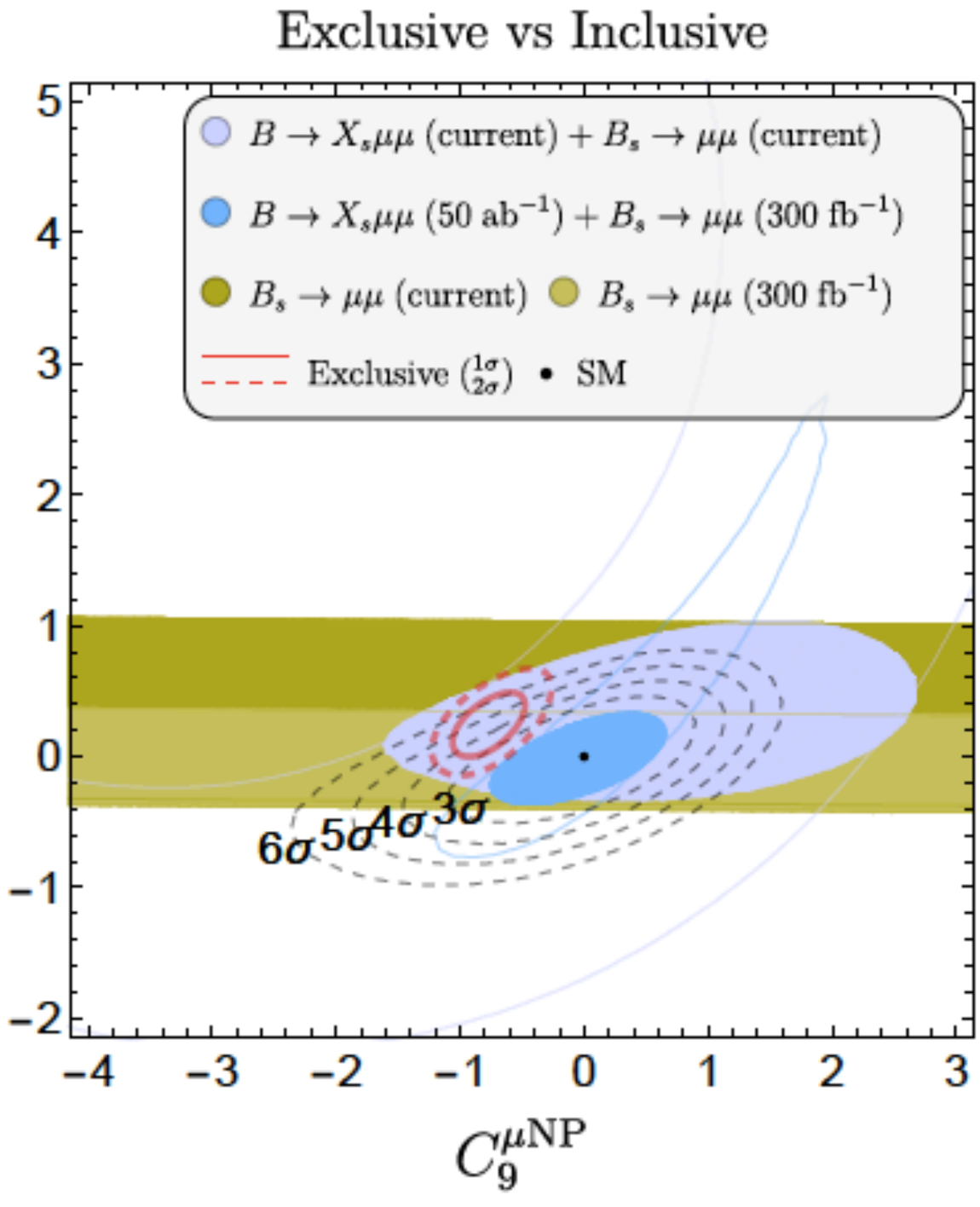
- We find a slight tension between the two theoretical and also between the two experimental results. We have to be patient!

New Physics Reach of Semi-leptonic Penguin Decays

Assuming Belle II measures SM values

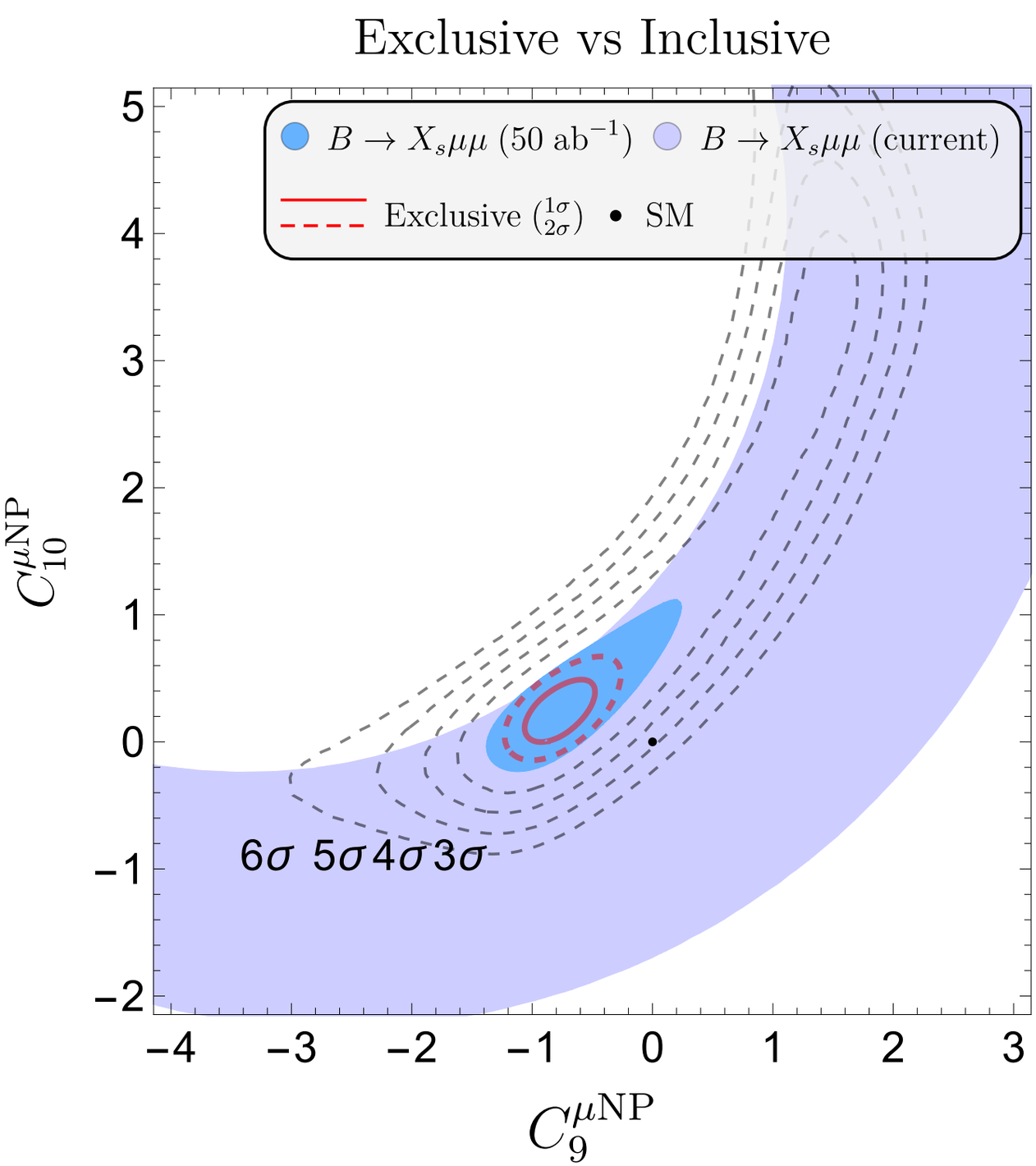
Huber,Hurth,Jenkins,Lunghi,Qin Qin,Vos,arXiv:2007.04191

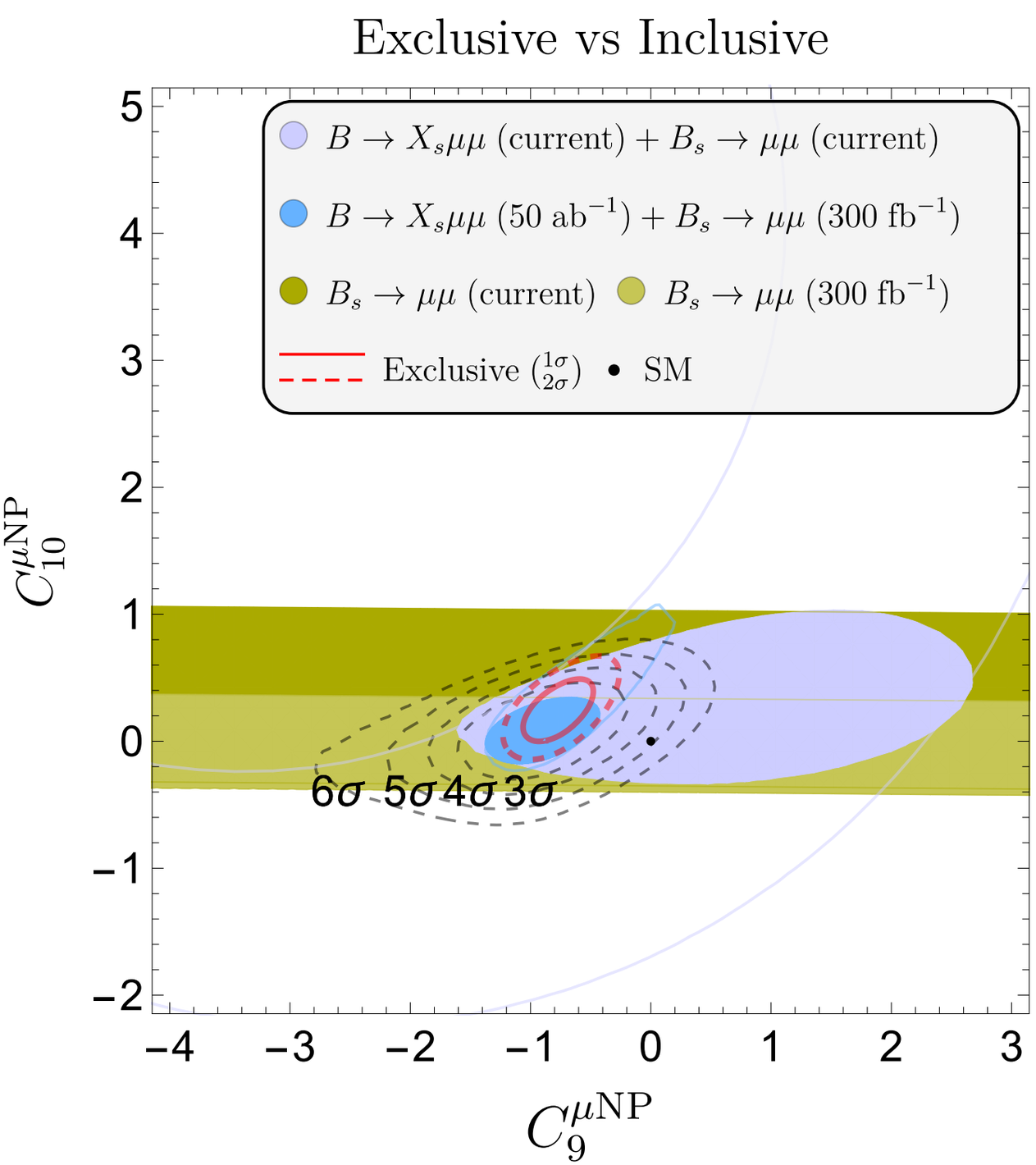




Assuming Belle II measures best fit point of exclusive fit

Huber,Hurth,Jenkins,Lunghi,Qin Qin,Vos,arXiv:2007.04191





Assuming Belle II measures SM values

Huber,Hurth,Jenkins,Lunghi,Qin Qin,Vos,arXiv:2007.04191

Update for post-R_K era arXiv:2404.03517

