Topological mass of neutrino

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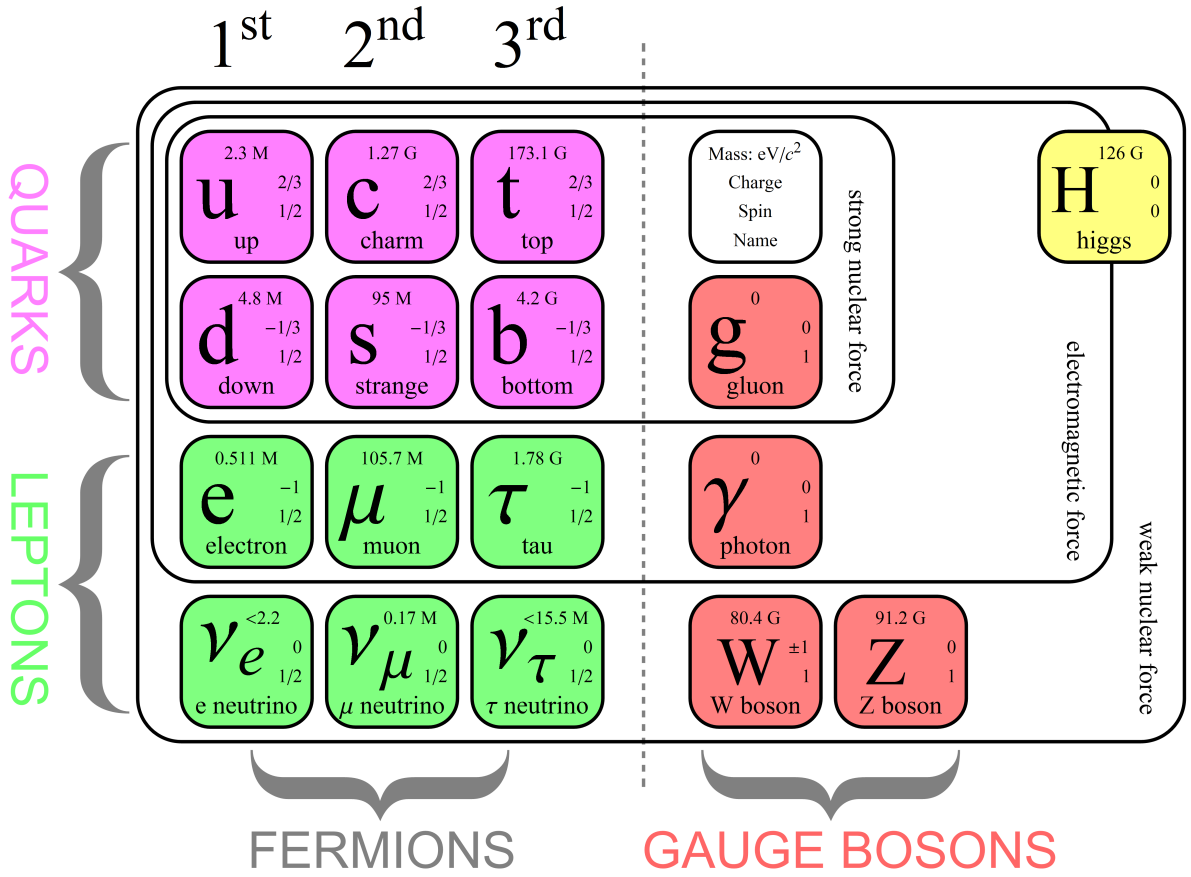
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Outline

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3. **Electroweak anomaly**
   - B+L anomaly
   - Neutrino vertex
   - Gap equation
4. **Overview and Conclusion**

Introduction



- No definite neutrino identity - Dirac or Majorana?
- Seesaw mechanism(s) is unproven
- LHC did not detect sterile neutrinos
- Neutrino oscillation requires further study

Instanton dynamics

- Instantons signify tunnelling between two distinct vacua of the quantum field
- These vacua are labelled by topological charge, which is defined as

$$Q = \frac{g^2}{32\pi^2}\int d^4x\,\frac{1}{2}\varepsilon_{\mu\nu\sigma\tau}F^{\sigma\tau}F^{\mu\nu} = \frac{g^2}{32\pi^2}\int d^4x\,\tilde{F}_{\mu\nu}F^{\mu\nu} \in \mathbb{Z} \quad (1)$$

- Instanton action is

$$S_E = \frac{1}{4}\int d^4x\,F_{\mu\nu}F^{\mu\nu} = \pm\frac{1}{4}\int d^4x\,\tilde{F}_{\mu\nu}F^{\mu\nu} = \pm\frac{8\pi^2}{g^2}Q$$

  where we have used self-duality $\tilde{F} = F$
- Instantons are suppressed as

$$\text{Amplitude} \propto e^{-S_E} = \exp\left[\frac{8\pi^2}{g^2}Q\right] = 10^{-150}$$

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B+L anomaly

- Applying instantons to the Electroweak Theory, they violate baryon $U(1)_B$ and lepton $U(1)_L$ numbers
- Allows for lepto- and baryogenesis - Important for cosmology
- The electroweak anomaly reads

$$\partial_\mu J^\mu_B = \partial_\mu J^\mu_L = \frac{N_f}{32\pi^2}\left[g^2 W^a_{\mu\nu}\tilde{W}^{a,\mu\nu} - g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\right], \qquad (2)$$

  Eq. 2 conserves B-L while violating B+L

- The topological charge of Eq. 2 reads

$$Q_{EW} = \frac{1}{32\pi^2}\int d^4x \left[g^2 W^a_{\mu\nu}\tilde{W}^{a,\mu\nu} - g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\right] \in \mathbb{Z}.$$

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Atiyah-Singer Index Theorem

Theorem

Let $i\not{D} = i\gamma^\mu D_\mu$ be the Dirac operator on a compact, even-dimensional spin manifold M (typically 4D spacetime). The fermions can be decomposed into left-handed and right-handed chiral components using the projectors $P_L = \frac{1-\gamma_5}{2}$ and $P_R = \frac{1+\gamma_5}{2}$. The Atiyah-Singer Index Theorem states:

$$ind(i\not{D}_L) = n_L - n_R$$

Where: n_L is the number of normalizable left-handed zero modes satisfying the following conditions:

$$i\not{D}\phi_{f,0} = 0 \quad and \quad P_L\phi_{f,0} = \phi_{f,0} \tag{3}$$

n_R is the number of normalizable right-handed zero modes satisfying the following conditions:

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B+L anomaly

- Matsubara (tempreture dependent) mode expansion:

$$\psi_f(\tau, \mathbf{x}) = \sqrt{T} a_{f,0} \phi_{f,0}(\mathbf{x}) + \sqrt{T} \sum_{n \neq 0} a_{f,n} \phi_{f,n}(\mathbf{x}) e^{i\omega_n \tau}$$

- Due to the Atiyah-Singer Index Theorem and the fact that neutrinos are only left-handed, we only consider Eq. 3.
- Split the gauge field into instanton $A^b_\mu(x; z, \rho, U)$ and quantum fluctuation δA^b_μ terms

$$A^b_\mu = A^b_\mu(x; z, \rho, U) + \delta A^b_\mu,$$

- After separating the non-zero contribution of the zero modes, we have

$$Z[\mathbf{A}_{\text{inst}}] = \int d^4 z \int d\rho \int dU \, n(\rho, T) \, \det{}' (i \not{D}) I_0$$

$$\text{where} \quad I_0 = \left[\prod_{f=1}^{12} \phi_{f,0}(\mathbf{z}; \mathbf{z}, \rho, U) \phi^\dagger_{f,0}(\mathbf{z}; \mathbf{z}, \rho, U) \right]$$

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- The Haar measure integration (Weingarten calculus and Schur orthogonality) reads

$$I_{\text{Haar}}[\psi, \phi_0] = \int dU \prod_{f=1}^{12} (\bar{\psi}_i U \phi_{f,0})(\phi^\dagger_{f,0} U^\dagger \psi_j)$$

  - The standard configuration is an isospin singlet
  - Breaking the isospin/gauge index symmetry, we get flavour indices
- The final coarse-graining gives the Lagrangian of

$$\mathcal{L}_{\text{eff}} = \frac{1}{3!} \int d\rho \, n(\rho, T) C(\rho) \langle \mathcal{O}_q \rangle \varepsilon_{abc} \bar{\nu}_{La} \bar{\nu}_{Lb} \bar{\nu}_{Lc} \, \varepsilon_{ijk} \nu_{Li} \nu_{Lj} \nu_{Lk},$$

  where we define a temperature-dependent coupling constant which reads

$$K(T) = \frac{1}{3!} \int d\rho \, n(\rho, T) C(\rho) \langle \mathcal{O}_q \rangle$$

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Neutrino vertex

- The new effective neutrino vertex reads

$$\mathcal{L}_{\nu-\text{eff}} = -K[\det(\bar{\nu}_{Li}\nu_{Lj}) + \text{h.c.}] \tag{4}$$

  which is completely analogous to the KMT determinant in QCD for the η' meson

- The full neutrino Lagrangian now becomes

$$\mathcal{L} = i\bar{\nu}_L\partial\!\!\!/\nu_L - |K|[e^{i\theta}\det(\bar{\nu}_L\nu_L) + \text{h.c.}] \tag{5}$$

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Neutrino vertex

- Mean-Field Approximation / Hartree-Fock Approximation
  - Hartree - diagonal terms, bulk of the condensate energies
  - Fock - off-diagonal terms, responsible for mixing and hierarchy
- The Majorana mass, from Eq. 4, reads

$$M_{gh}(\Sigma) = |K| e^{i\theta} [\Sigma_{gh} \mathrm{Tr}(\Sigma) - (\Sigma^2)_{gh}] \tag{6}$$

- After the Mean-Field Approximation, Eq. 5 becomes the linearised Lagrangian reads

$$\mathcal{L} = i\bar{\nu}_{L,g} \partial\!\!\!/ \nu_{L,g} - \frac{1}{2} \left[M_{gh} \bar{\nu}_{L,g} \nu_{L,h} + M^{\dagger}_{gh} (\bar{\nu}_{L,g} \nu_{L,h})^{\dagger} \right] + V(\Sigma) \tag{7}$$

where $\Sigma_{gh} = \langle \bar{\nu}_{L,g} \nu_{L,h} \rangle$ is the dineutrino condensate

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Gap equation

- Nambu-Jona-Lasinio-like gap equation

$$\langle \bar{\nu}_{L,g} \nu_{L,h} \rangle = \Sigma_{gh} = 2 \int \frac{d^4 k}{(2\pi)^4} \left[\frac{M^\dagger(\Sigma)}{k^2 + M(\Sigma) M^\dagger(\Sigma)} \right]_{gh}$$

- Preforming the integration results in

$$\Sigma_{gh} = \frac{|K| e^{i\theta}}{8\pi^2} \left[\Lambda^2 - M M^\dagger \log \left(1 + \frac{\Lambda^2}{M M^\dagger} \right) \right]_{ga} \mathrm{Adj}(\Sigma^\dagger)_{ah} \qquad (8)$$

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Condensation and vacumm minima

- After linearising Eq. 8, in the IR limit for the condensate, we get a critical value of

$$|K_c| = \frac{8\pi^2}{\Lambda^2 (\det \Sigma^\dagger)^{1/3}}$$

- $K < K_c$ - Symmetric phase ($\Sigma = 0$) - Massless neutrinos - Lepton number conserved
- $K \approx K_c$ - Second minima ($\Sigma \neq 0$) - Massive neutrinos - Out of equilibrium Universe
- $K > K_c$ - Condensed phase ($\Sigma = \Sigma_0$) - Massive neutrinos - Lepton number violated, large CP angles

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- Majorana neutrinos ***demanded*** by Atiyah-Singer Index Theorem
- Determinant vertex for neutrinos

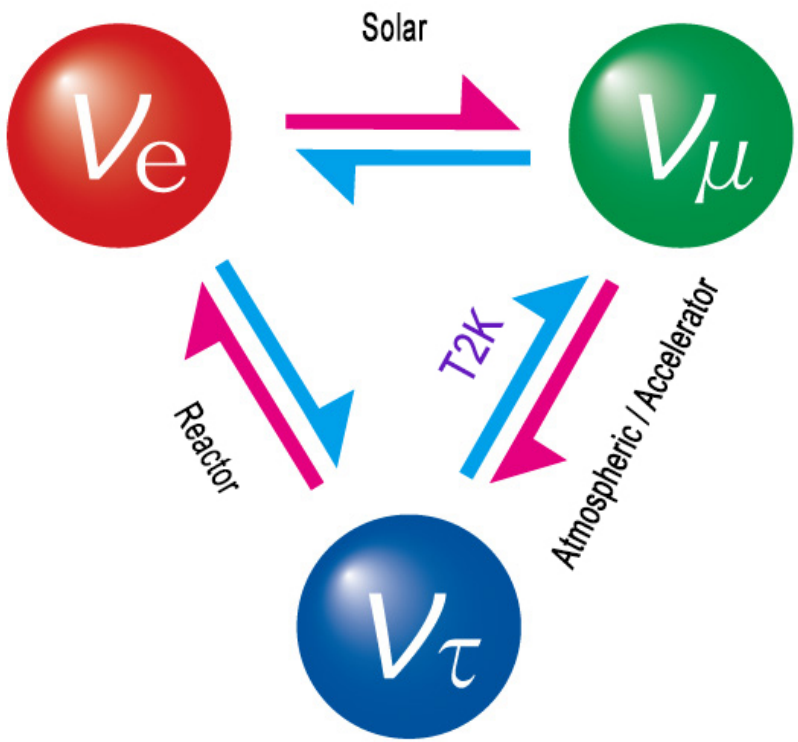
$$\mathcal{L} = -|K|[e^{i\theta}\det(\bar{\nu}_L\nu_L) + \text{h.c.}]$$

- Hartree-Fock Approximation - (di)neutrino condensate
- Majorana mass generated by NJL-like gap equation

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- Neutrino oscillation - generated by Fock terms in MFA

Overview and Conclusion



Neutrino oscillation between three generations

- From topology, we have that the neutrino is a ***Majorana*** particle
- The ***Majorana*** mass is generated by an effective 't Hooft vertex
- Neutrino ***hierarchy*** emerges due to the Hartree-Fock approximation
- Neutrino ***oscillation/mixing*** is described by the Fock terms

Thank you very much
for your attention!

References



Noel Vincze.
On the topological origins of neutrinos, 2026.
hal-05774248.



Peter Arnold and Larry McLerran.
Sphalerons, small fluctuations, and baryon-number violation in electroweak theory.
Phys. Rev. D, 36:581–595, Jul 1987.



G. 't Hooft.
How instantons solve the u(1) problem.
Physics Reports, 142(6):357–387, 1986.



Steven Weinberg.
Gauge and global symmetries at high temperature.
Phys. Rev. D, 9:3357–3378, 1974.