# Cosmological tests of thermal axions

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based on:

Phys.Dark Univ. 52 (2026) 102335 [arXiv:2511.14864], MB, A. Gomułka, M. Laletin, K. Szafrański

JHEP 02 (2025) 014 [arXiv:2410.18186], MB, M. Laletin

JHEP 09 (2024) 136 [arXiv:2403.05621], MB, K. Harigaya, M. Łukawski, R. Ziegler

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# Outline

1. QCD axion
2. Thermal production of axions and the impact on CMB
3. Astrophobic QCD axions
4. Precise predictions for $\Delta N_{\rm eff}$ with full phase-space evolution
5. Cosmological bounds on axion-lepton couplings for massive axions

# QCD axion

## One of the best candidates for new physics

1

**Solves strong CP**

Predicted by the Peccei-Quinn (PQ) mechanism solving the strong CP problem

2

**Dark matter**

Constitutes a good dark matter candidate

3

**Pseudo-Goldstone boson**

The axion is the PNGB of a $U(1)_{\rm PQ}$ symmetry broken by non-perturbative QCD effects

# Axion models

Need **anomalous** breaking of PQ (fermion sector) and **spontaneous** PQ breaking (scalar sector)

$$U(1)_{\rm PQ} \times SU(3)_c^2$$

SM fermions

BSM fermions

2Higgs

2Higgs + Singlet

Higgs + Singlet

**PQWW**

[Peccei, Quinn, Wilczek, Weinberg '78]

**DFSZ**

[Dine, Fischler, Srednicki, Zhitnitsky '80]

**KSVZ**

[Kim, Shifman, Vainshtein, Zakharov '80]

**excluded: $J/\psi \to \gamma a$**

$\langle \text{Singlet} \rangle \gg v$: "Invisible" axion models

slide borrowed from R. Ziegler

# Axion effective Lagrangian

UV models can be described by an effective Lagrangian well below the PQ scale

$$\mathcal{L} = \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \frac{E}{N}\frac{a}{f_a}\frac{\alpha_{\rm em}}{8\pi}F\tilde{F} + \frac{\partial_\mu a}{2f_a}\bar{f}_i\gamma^\mu\left(C^V_{ij} + C^A_{ij}\gamma_5\right)f_j$$

**Gluon coupling**

Solves the strong CP problem and generates the axion mass:

$$m_a \approx 6\ \text{meV}\left(\frac{10^9\ \text{GeV}}{f_a}\right)$$

**Photon coupling**

Allows searches for axions in helioscopes, e.g. IAXO

**Fermion couplings**

In general flavor-violating

**The axion decay constant $f_a$ controls the size of axion couplings to SM particles**

# Thermal axions and $\Delta N_{\rm eff}$

- For small enough $f_a$ axions can be in thermal equilibrium with the SM plasma
- Thermally produced axions contribute to

$$\Delta N_{\rm eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{\frac{4}{3}} \left.\frac{\rho_a}{\rho_\gamma}\right|_{T_{\rm CMB}}$$

**leading to lower bounds on $f_a$ from BBN/CMB**

# Instantaneous decoupling

For instantaneous axion decoupling at $T_{\rm dec}$:

$$\Delta N_{\rm eff} \approx 0.03 \left( \frac{100}{g_{*s}(T_{\rm dec})} \right)^{4/3}$$

$\Delta N_{\rm eff} \lesssim 0.3$

Planck bound (2σ)

$\Delta N_{\rm eff} \approx 0.1$

Simons Observatory sensitivity (2σ)

$\Delta N_{\rm eff} \approx 0.05$

CMB-S4 sensitivity (2σ)

**$\Delta N_{\rm eff}$ sizeable if the axion decouples during or after the QCD phase transition**

*CMB-S4 status is uncertain; LiteBIRD and the proposed CMB-HD offer comparable reach.*

# Why we need Boltzmann equations

- Around the QCD phase transition $g_{*s}$ changes rapidly
- Precise determination of $\Delta N_{\rm eff}$ requires solving Boltzmann equations (freeze-in also taken into account)

$$\frac{dn_a}{dt} + 3Hn_a = \left(\sum_i \Gamma_i\right)\left(n_a^{\rm eq} - n_a\right)$$

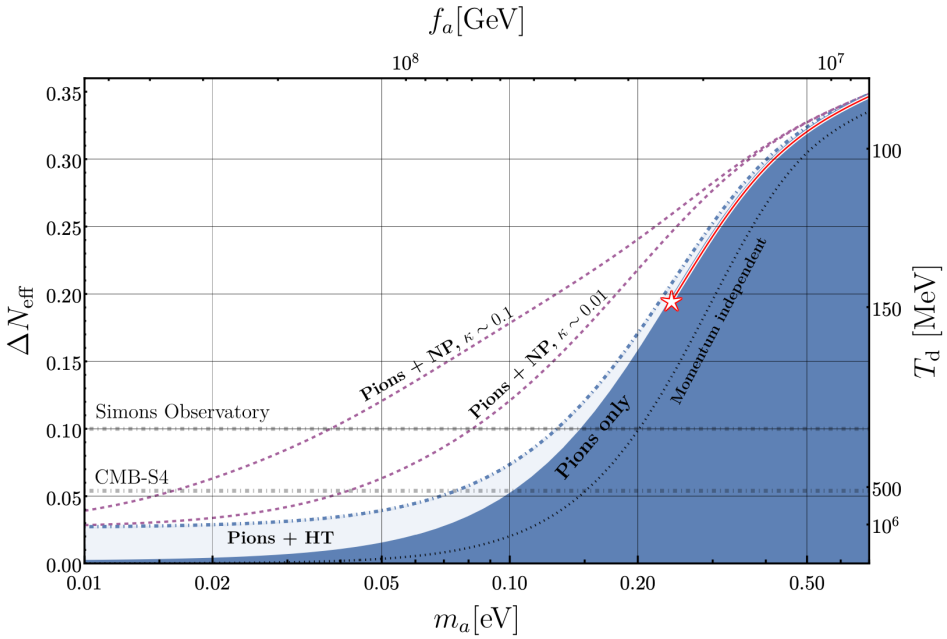
left: dilution by expansion

right: collisions drive $n_a$ toward $n_a^{\rm eq}$

$\Sigma_i \Gamma_i$: thermally averaged axion production rates

# Axion-pion scattering: $\Delta N_{\rm eff}$ vs $m_a$

Axion-pion scattering typically dominates for the QCD axion




$$\pi\pi \leftrightarrow \pi a$$

[Chang, Choi '93]

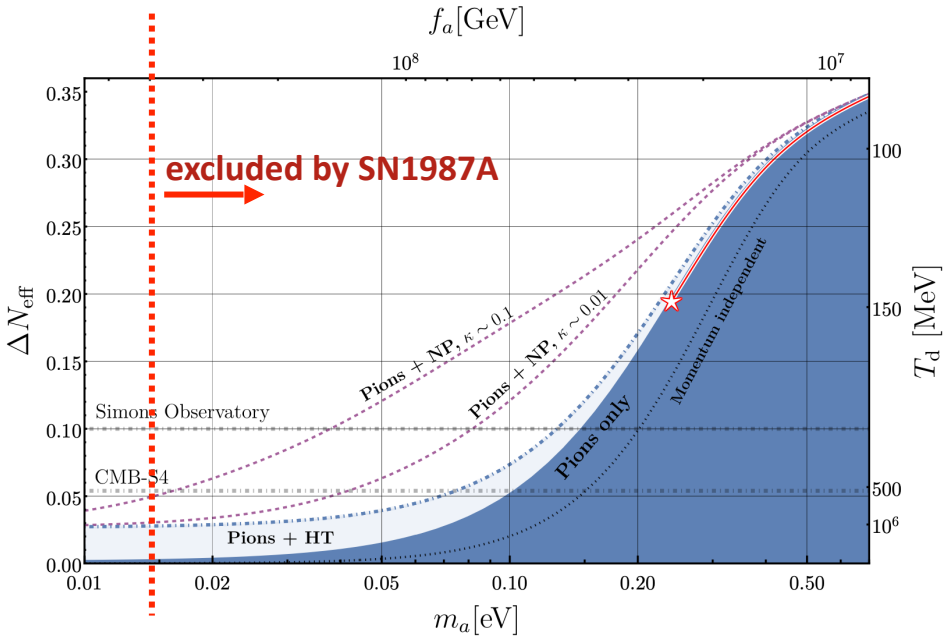
Plot: Notari, Rompineve, Villadoro '22

Hot dark matter bound for the KSVZ (hadronic) axion: $m_a \lesssim 0.24$ eV ($f_a \gtrsim 2\times10^7$ GeV).

**Note: highly model dependent!**

# Axion-pion scattering: astrophysics wins

The astrophysical bound on $f_a$ is much stronger than the CMB one



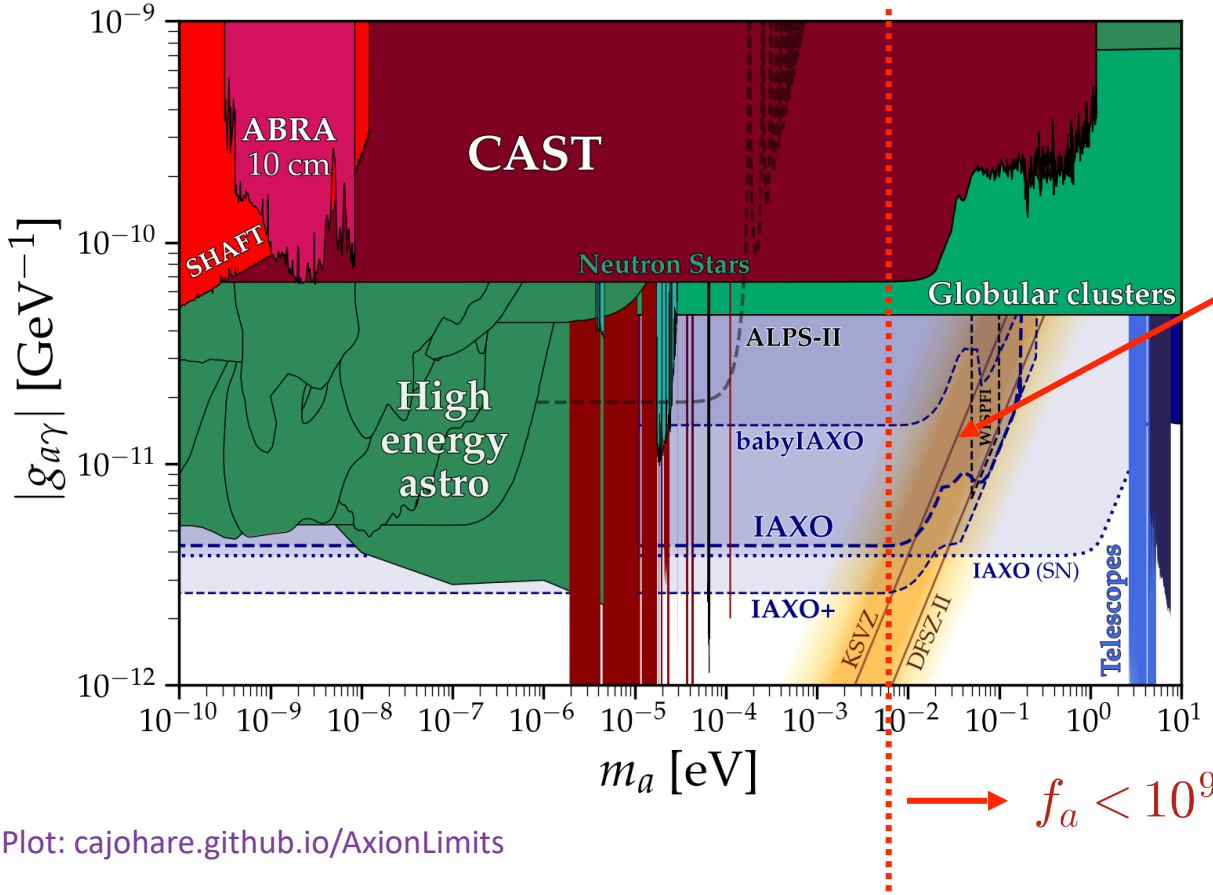

$$\pi\pi \leftrightarrow \pi a$$

[Chang, Choi '93]

Plot: Notari, Rompineve, Villadoro '22

**Astro bounds on $f_a$ in the KSVZ model imply negligible $\Delta N_{\rm eff}$, beyond the reach of future CMB experiments**

# Detecting axions in helioscopes




Large axion-photon couplings are excluded by astrophysics in minimal models

**No signal expected at IAXO unless the axion is astrophobic**

$f_a < 10^9$ GeV

Plot: cajohare.github.io/AxionLimits

# Nucleophobic axion models

- SN1987A and NS cooling bounds can be relaxed if the axion-nucleon coupling is suppressed, which happens for

$$C_u + C_d = 1 \qquad C_u \approx 2/3 \qquad C_i \equiv C_{ii}^A$$

[Di Luzio, Mescia, Nardi, Panci, Ziegler '17]

$C_u$ and $C_d$ give contributions to the proton and neutron couplings that cancel against the model-independent contribution from the axion-gluon coupling.

- Nucleophobia is realised in DFSZ-like models with non-universal PQ charges

**Nucleophobia ⇒ flavor-violating axion couplings!**

# DFSZ models

$$\text{SM fermions} + \text{2Higgs} + \text{Singlet} \left\{ \begin{array}{cc} \langle H_1 \rangle = c_\beta v & \langle H_2 \rangle = s_\beta v \\ \multicolumn{2}{c}{\langle \Phi \rangle = v_{\rm PQ} \gg v} \end{array} \right.$$

Construct a 2HDM Lagrangian invariant under a single U(1)

$$\mathcal{L}_{\rm yuk} = y^u_{ij} \overline{Q}_i U_j \left\{ \begin{array}{c} H_1 \\ H_2 \end{array} \right. \xrightarrow[\text{U(1) charges}]{\text{flavor-universal}} \begin{array}{l} \overline{Q}_i U_j \; H_1 \\ \overline{Q}_i D_j \; \tilde{H}_2 \\ \overline{L}_i E_j \; \tilde{H}_{1 \text{ or } 2} \end{array}$$

Break the residual U(1) by H-Singlet couplings

$$\mathcal{L} \sim H_1^\dagger H_2 \Phi$$

**Axion-fermion couplings are fixed by tan $\beta$**

# Nucleophobic non-universal DFSZ models

Generalized DFSZ-type models: **PQ charges universal only for two generations**

There are 4 nucleophobic charge assignments in the quark sector: non-universal in **q** or **u/d** sector

$$\underline{\text{e.g.}} \qquad \mathcal{L} \sim \begin{matrix} \overline{f}_{L3} f_{R3} \\ \overline{f}_{L3} f_{Ra} \end{matrix} \begin{cases} h_1 \; u \\ \tilde{h}_2 \; d \end{cases} + \begin{matrix} \overline{f}_{La} f_{Rb} \\ \overline{f}_{La} f_{R3} \end{matrix} \begin{cases} h_2 \; u \\ \tilde{h}_1 \; d \end{cases}$$

Each model in the quark sector can be combined with 4 models in the charged lepton sector

**16 nucleophobic models in total**

| Model | $E_Q/N$ | $C^A_{u_iu_i}$ | $C^A_{d_id_i}$ | $C^{V,A}_{u_i\neq u_j}$ | $C^{V,A}_{d_i\neq d_j}$ |
|---|---|---|---|---|---|
| Q1 | $2/3+6c_\beta^2$ | $c_\beta^2$ | $\xi^{d_R}_{ii}-c_\beta^2$ | $0$ | $\xi^{d_R}_{ij}$ |
| Q2 | $-4/3+6c_\beta^2$ | $c_\beta^2-\xi^{u_L}_{ii}$ | $-\xi^{d_L}_{ii}+s_\beta^2$ | $\pm\xi^{u_L}_{ij}$ | $\pm\xi^{d_L}_{ij}$ |
| Q3 | $-4/3+6c_\beta^2$ | $c_\beta^2-\xi^{u_R}_{ii}$ | $-\xi^{d_R}_{ii}+s_\beta^2$ | $-\xi^{u_R}_{ij}$ | $-\xi^{d_R}_{ij}$ |
| Q4 | $-10/3+6c_\beta^2$ | $-s_\beta^2+\xi^{u_R}_{ii}$ | $s_\beta^2$ | $\xi^{u_R}_{ij}$ | $0$ |

| Model | $E_L/N$ | $C^A_{e_ie_i}$ | $C^{V,A}_{e_i\neq e_j}$ |
|---|---|---|---|
| E1L | $2-6c_\beta^2$ | $-c_\beta^2+\xi^{e_L}_{ii}$ | $\mp\xi^{e_L}_{ij}$ |
| E1R | $2-6c_\beta^2$ | $-c_\beta^2+\xi^{e_R}_{ii}$ | $\xi^{e_R}_{ij}$ |
| E2L | $4-6c_\beta^2$ | $s_\beta^2-\xi^{e_L}_{ii}$ | $\pm\xi^{e_L}_{ij}$ |
| E2R | $4-6c_\beta^2$ | $s_\beta^2-\xi^{e_R}_{ii}$ | $-\xi^{e_R}_{ij}$ |

# Astrophobic DFSZ-like models

- DFSZ-like 2HDM models with 2+1 flavor structure of PQ charges: astrophobia (nucleophobia + electrophobia) ⇒ LFV axion couplings
- For $f_a \lesssim 10^8$ GeV all axion couplings in the lepton sector are fixed once astro and exp. ($\mu \to ea$) constraints are imposed; $\Delta N_{\rm eff}$ is then predicted as a function of $f_a$
- Astrophobic 2HDM models (different PQ charge assignments for SM fermions) give different predictions for $E/N$, which determines the axion-photon coupling

**Fixed lepton couplings**

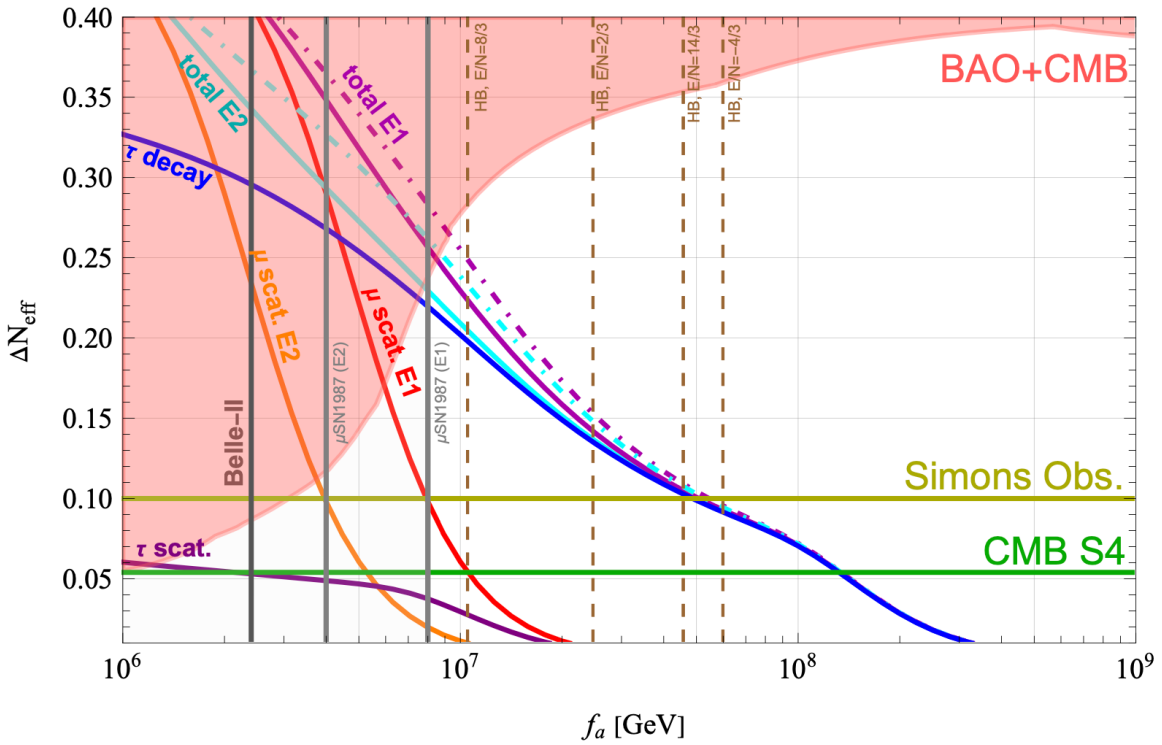
$$C_{\tau e} \approx \frac{2}{3}$$

$$C_{\tau} \approx -\frac{1}{3}$$

$$C_{\mu} \approx -\frac{2}{3}\ \left(\text{or } \frac{1}{3}\right)$$

[Di Luzio, Mescia, Nardi, Panci, Ziegler '17]

# $\Delta N_{\rm eff}$ in astrophobic DFSZ-like models




$$C_\mu \approx -\frac{2}{3}\ (\mathrm{E1})$$

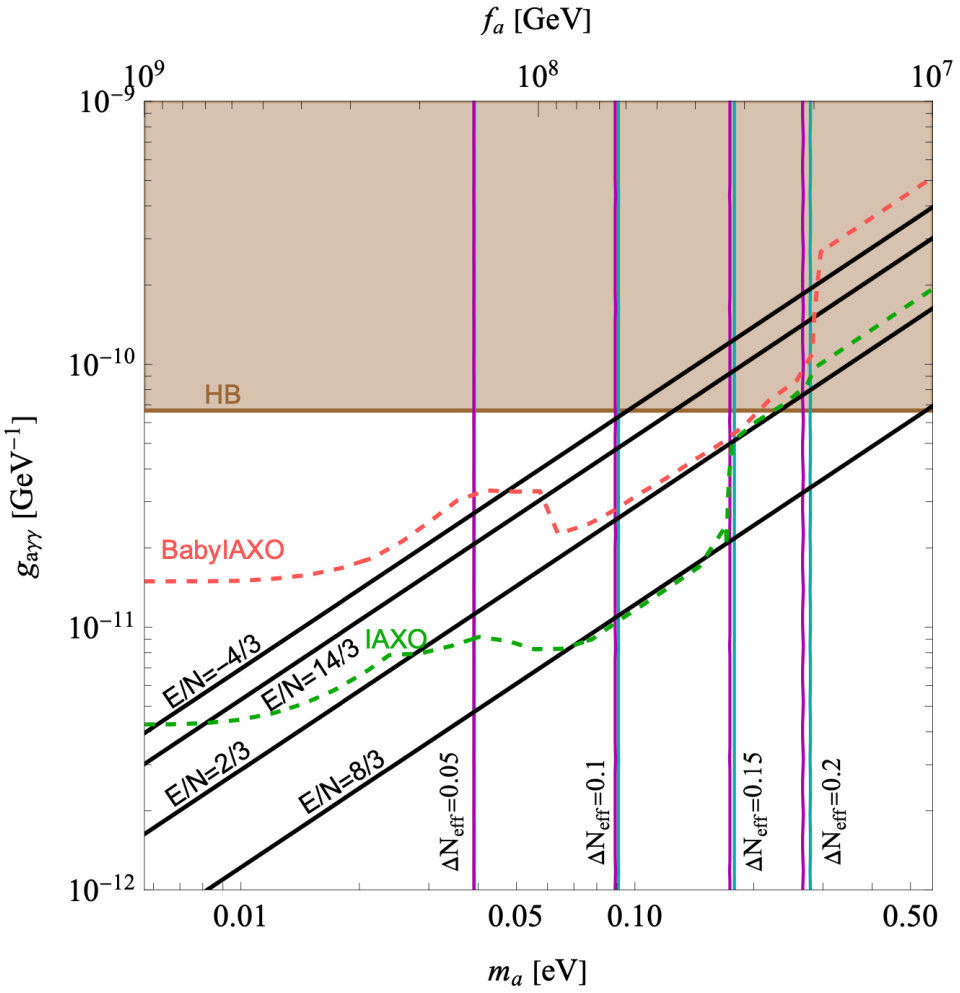
$$C_\mu \approx \frac{1}{3}\ (\mathrm{E2})$$

[MB, Harigaya, Łukawski, Ziegler 2403.05621]

**$\Delta N_{\rm eff}$ as large as 0.25 consistent with astro constraints**

**Good prospects for discovery at future CMB exp. for $f_a$ up to $10^8$ GeV**

# Complementarity of IAXO and CMB




**Strong correlations between $\Delta N_{\rm eff}$ and the axion-photon coupling**

If non-zero $\Delta N_{\rm eff}$ is measured, (Baby)IAXO will confirm or rule out 2HDM astrophobic models

[MB, Harigaya, Łukawski, Ziegler 2403.05621]

# Cosmological constraints on leptophilic ALP

# Leptophilic ALP

- For general ALPs, couplings to gluons and photons can be suppressed and the axion mass is independent of the axion decay constant

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}(\partial_\mu a)^2 - \frac{1}{2}m_a^2 a^2 + \frac{\partial_\mu a}{2f}\,\bar{l}_i\gamma^\mu\left(C_{ij}^V + C_{ij}^A\gamma_5\right)l_j$$

axion-lepton couplings (in general flavor-violating)

I will present model-independent bounds on axion-lepton couplings using state-of-the-art techniques to compute axion momentum distributions.

# Full phase-space evolution

[MB, Laletin 2410.18186] see also D'Eramo, Lenoci 2410.21253

- Axions produced around the QCD phase transition may have a momentum distribution $f_a(x, q)$ different from the thermal one
- Precise prediction for $\Delta N_{\rm eff}$ requires full phase-space analysis:

$$\tilde{H}(x\partial_x - \tilde{g}q\partial_q) f_a(x,q) = C[f_a]$$

left: free streaming and expansion (momentum redshift)

right: collision term, for every momentum q

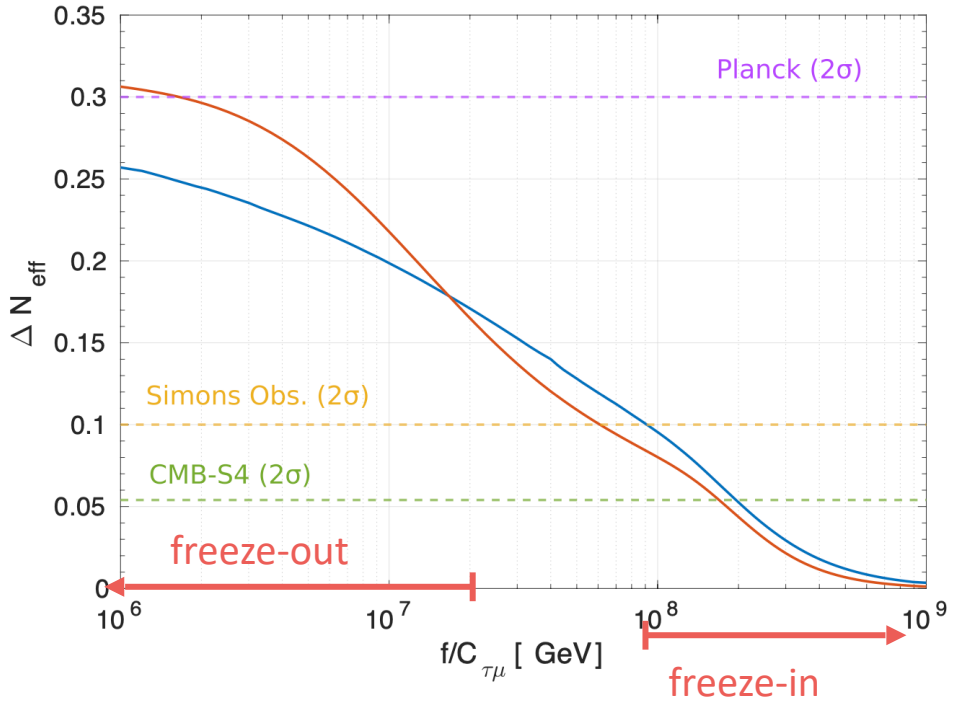
$$\tilde{g} \equiv -\frac{1}{3}\frac{d\ln h_s}{d\ln x}, \quad \tilde{H} \equiv H/(1+\tilde{g})$$

$$x \equiv m/T, \quad q \equiv p/T$$

# Tau decays: simplified vs full Boltzmann

$\tau \to \mu a$ decay

[MB, Laletin 2410.18186]




**nBE (simplified)**

**fBE (full)**

**At $f/C = 10^6$ GeV: nBE reaches the Planck $2\sigma$ bound ($\Delta N_{\rm eff} \approx 0.3$), fBE gives only $\approx 0.26$. The Planck bound is strongly relaxed.**

# Impact of axion mass

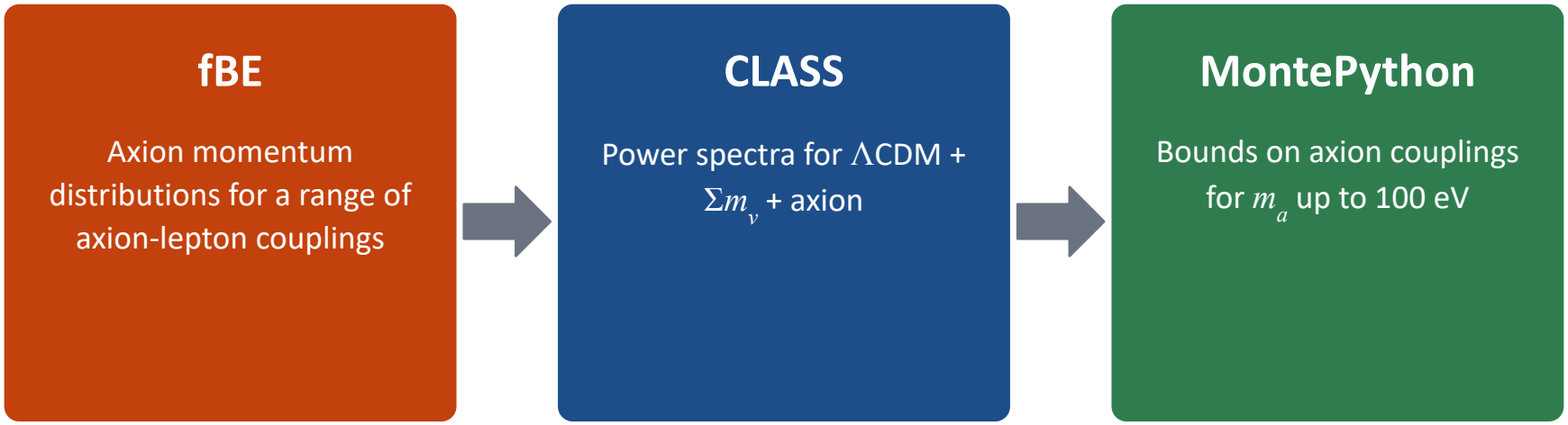
Energy density is enhanced at CMB scales if the axion mass exceeds the recombination temperature:

$$\rho_a = \frac{g_a T^4}{2\pi^2} \int dq \sqrt{q^2 + \left(\frac{m_a}{T}\right)^2} \cdot q^2 f_a(q)$$

**The $(m_a/T)^2$ term enhances the energy density when $m_a$ exceeds the temperature**

Constraints become stronger than those derived from $\Delta N_{\rm eff}$, and a dedicated cosmological analysis is necessary to reliably set constraints.

# Methodology



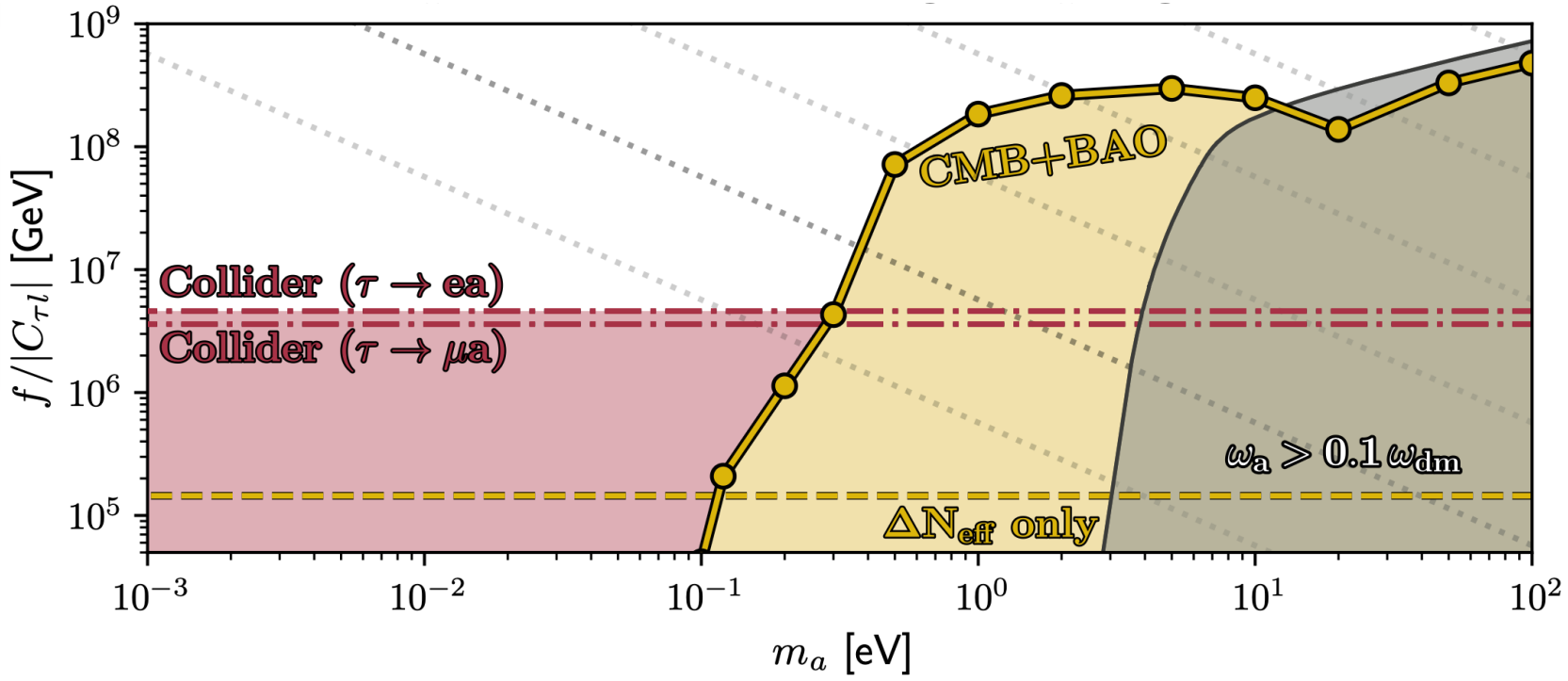

## Data

Planck: CMB temperature, polarisation and lensing power spectra

BAO measurements from DESI DR2

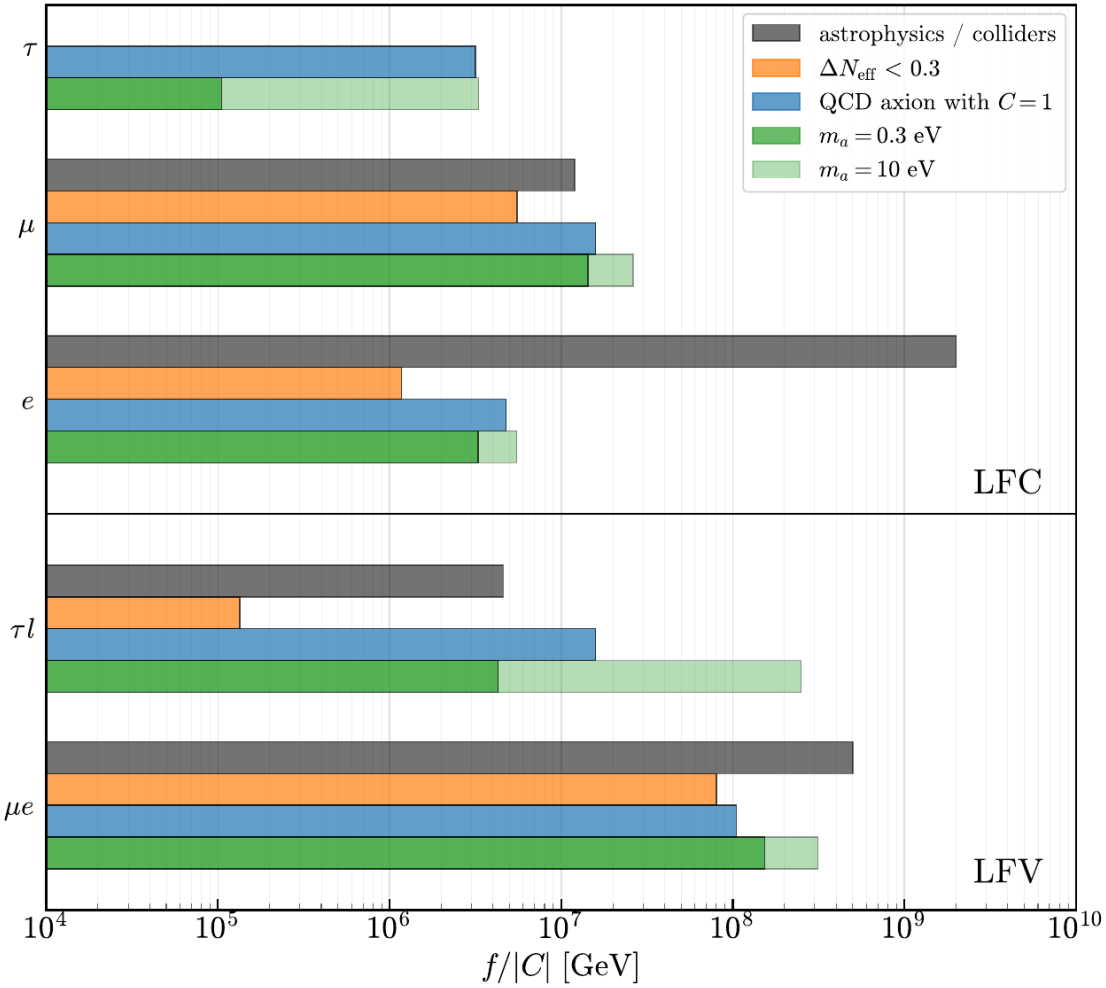
# Bounds on LFV axion-tau couplings

Grey region: $\omega_a > 0.1\ \omega_{\rm dm}$, used as a proxy for the Ly-$\alpha$ constraint.




**For $m_a \gtrsim 0.1$ eV the bounds are stronger than those derived from $\Delta N_{\rm eff}$**

# Bounds on axion-lepton couplings




**Massive axions ($m_a \gtrsim 1$ eV)**

Cosmology gives the strongest bounds on the LFC $\tau$ and $\mu$ couplings, ahead of astrophysics, and on the LFV $\tau$ couplings, ahead of colliders.

# Summary

1

## Astrophobic QCD axions give observable $\Delta N_{\rm eff}$

Only astrophobic models predict observable $\Delta N_{\rm eff}$ without conflict with astrophysical constraints.

2

## Full phase-space evolution changes the answer

Reliable comparison with CMB data requires Boltzmann equations for momentum distributions, not just number densities.

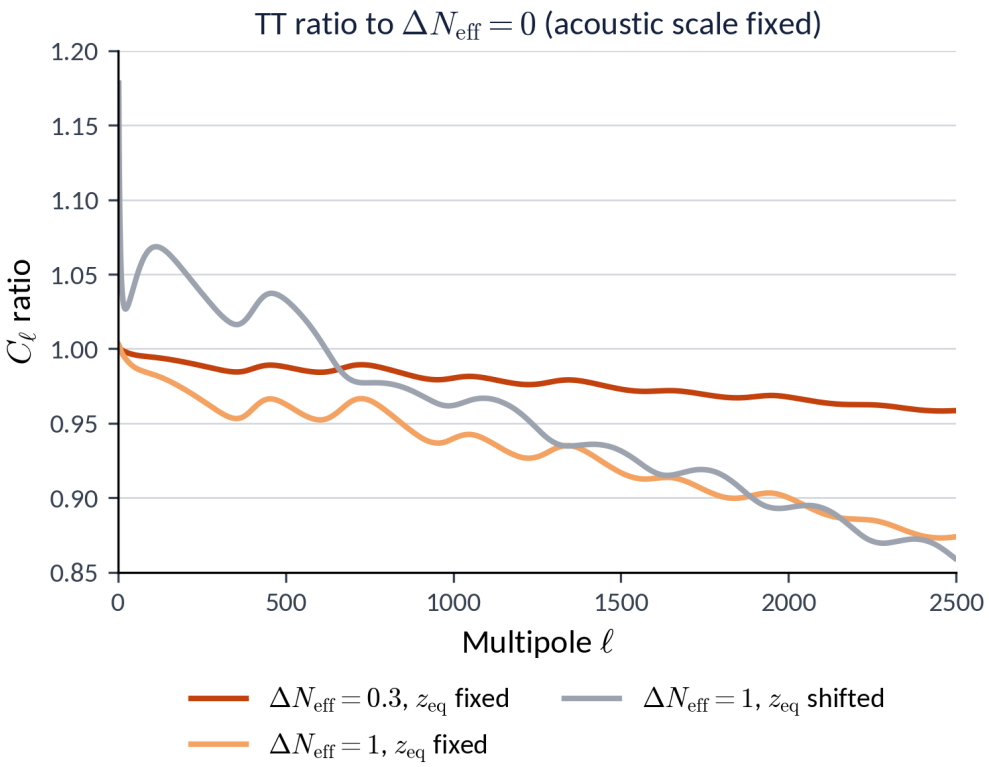
3

## Axion mass matters

For $m_a$ above the recombination temperature, cosmology gives the strongest constraints on LFC couplings to $\tau$, $\mu$ and LFV ones to $\tau\mu$, $\tau e$, orders of magnitude beyond $\Delta N_{\rm eff}$-only bounds.

# Backup

# $\Delta N_{\rm eff}$ in the CMB power spectrum




## Damping tail (main effect)

Extra radiation raises $H$ before recombination: $r_D/r_s \propto H^{1/2}$ grows, so at fixed peak positions high-$\ell$ power is suppressed, more and more towards high $\ell$.

$r_s$: sound horizon, the distance sound travels in the plasma before recombination. $r_D$: photon diffusion length. $D_M$: comoving distance to last scattering.

First peak: delayed matter-radiation equality boosts it through the early ISW effect (grey). The peak heights fix $z_{\rm eq}$, so a fit raises $\omega_m$ and the boost disappears.

**Peak positions fix $\theta_* = r_s/D_M$, so $H_0$ rises with $N_{\rm eff}$. Fixing $z_{\rm eq}$ and $\theta_*$ leaves the damping tail as the clean $N_{\rm eff}$ signature.**

# Naturally astrophobic axion

[MB, Harigaya '23]

- Non-universal DFSZ-like models with 2 Higgs doublets (2HDM) require tuning of parameters to suppress axion couplings to nucleons and electrons
- Astrophobia can be realised without tuning for a specific PQ charge assignment for the SM fermions:

| | $\bar{u}$ | $\bar{d}$ |
|---|---|---|
| $Q_f$ | 2 | 1 |

$$\frac{E}{N} = 2$$

$$C_u = 2/3 \quad C_d = 1/3$$

**axion couplings to nucleons and photons naturally suppressed!**

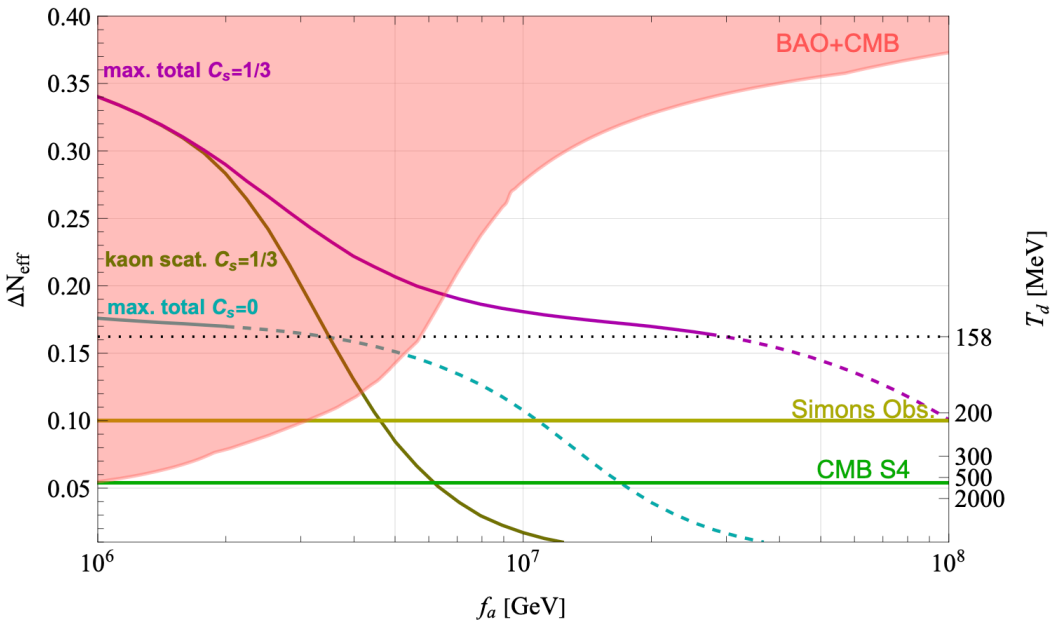
$$C_\gamma = E/N - 1.92(4) = 2 - 1.92(4) = 0.08(4)$$

**photon coupling compatible with zero**

$$f_a \gtrsim 2 \times 10^7 \, |C_\gamma| \ \text{GeV} \quad \text{(from horizontal branch stars)}$$

# $\Delta N_{\rm eff}$ from naturally astrophobic axion

$f_a < 10^7$ GeV is possible due to suppressed axion couplings also to photons and muons




Estimate of maximal $\Delta N_{\rm eff}$ from axion-quark scattering above the QCD PT, taking $g_s = 4\pi$ (perturbative calculation not reliable for the relevant temperatures)

[MB, Harigaya, Łukawski, Ziegler 2403.05621]

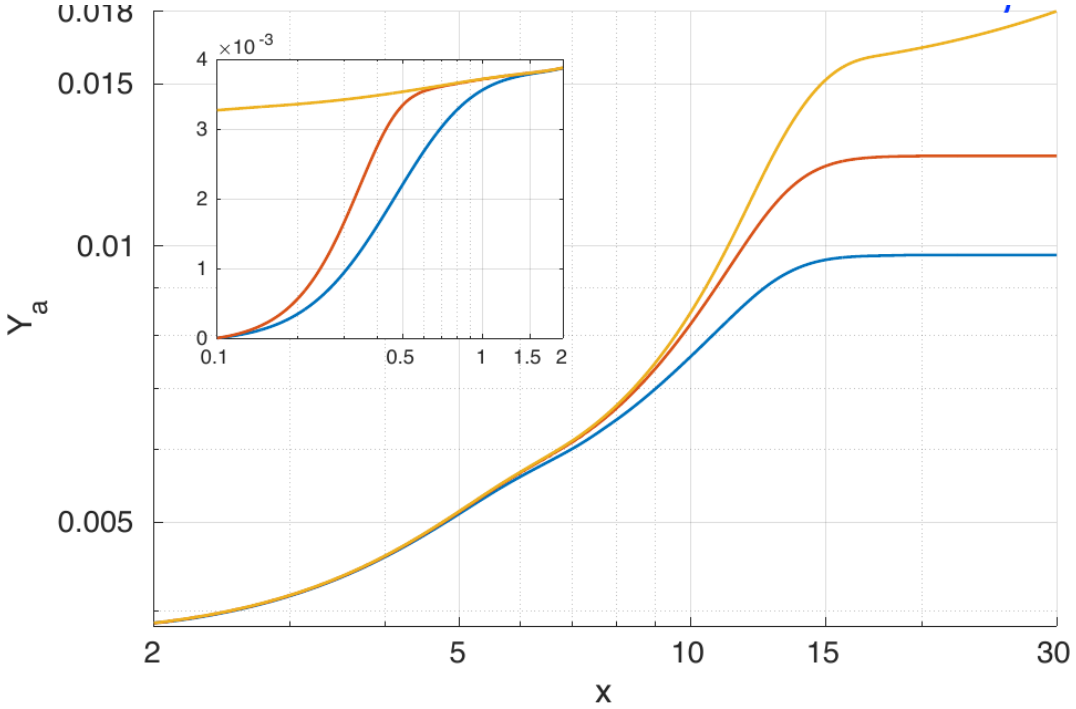
Minimal model predicts negligible $\Delta N_{\rm eff}$ from processes below the QCD phase transition

**Even an aggressive estimate of $\Delta N_{\rm eff}$ above the QCD PT allows $f_a \approx 5\times10^6$ GeV ($m_a \approx 1$ eV)**

# Tau decays: why fBE gives less

$\tau \to \mu a$ for $f/C_{\tau\mu} = 10^7$ GeV (freeze-out)

[MB, Laletin 2410.18186]



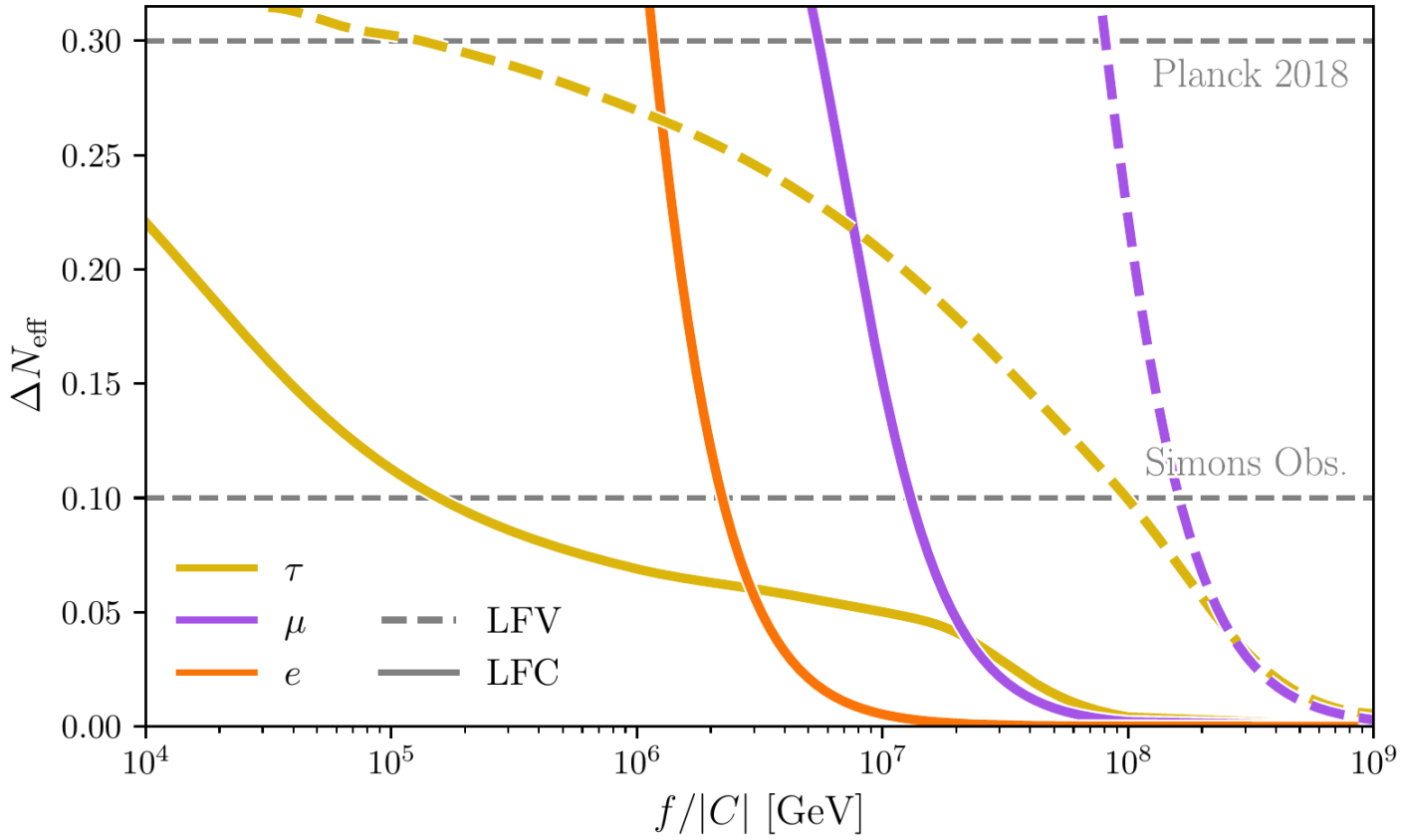

**nBE (simplified)**

**fBE (full)**

**Freeze-out at T from ~300 to 100 MeV**

Lower momentum modes decouple earlier (inverse decays suppressed) and their abundance is suppressed due to the rapidly changing $g_{*_S}$ around the QCD phase transition

# fBE results for $\Delta N_{\rm eff}$




Solid: lepton-flavor conserving (LFC). Dashed: lepton-flavor violating (LFV). Horizontal lines: Planck 2018 and Simons Obs.