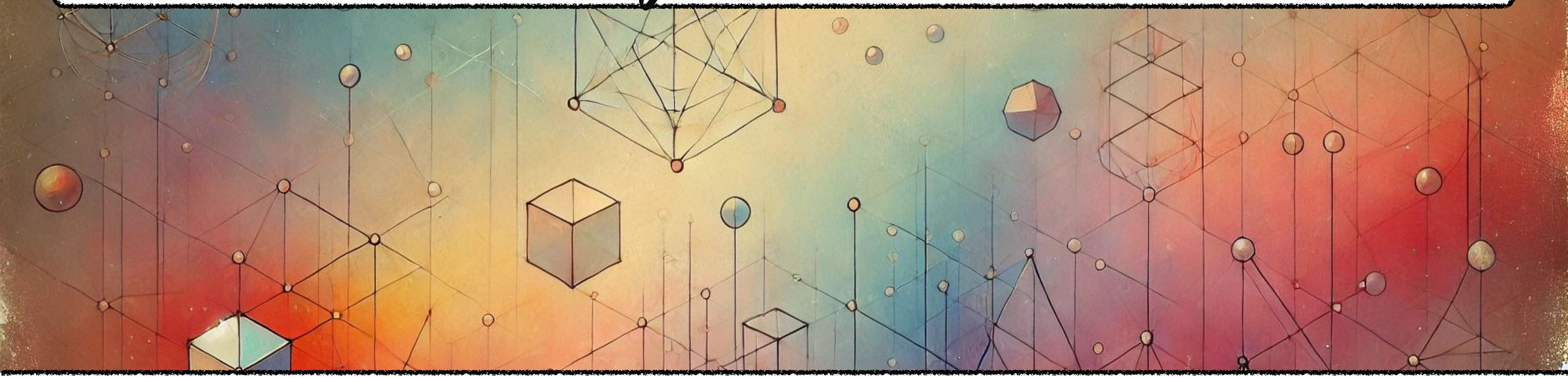
Topological Defects from Non-Abelian Discrete Symmetries



Jessica Turner

Institute for Particle Physics Phenomenology, Durham University

DISCRETE 2026, Bratislava

Talk based on work with B. Fu, S. King, L. Marsili, S. Pascoli & Y.L. Zhou

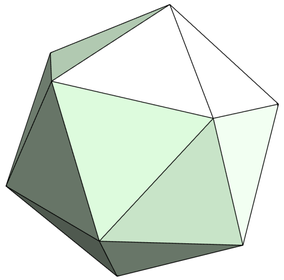
Based on 2409.16359 & 2512.13784

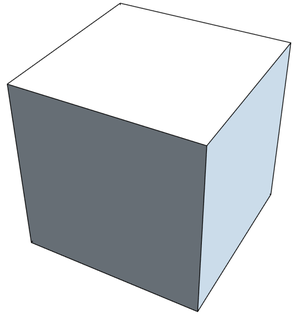
Durham University

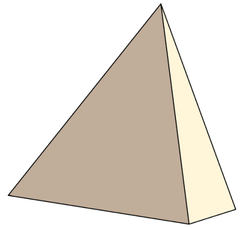
IPPP

Models of Flavour

Discrete non-Abelian symmetries successfully explain leptonic mixing







See talk by Claudia Hagedorn

non-Abelian symmetry breaks leaving residual Abelian symmetries

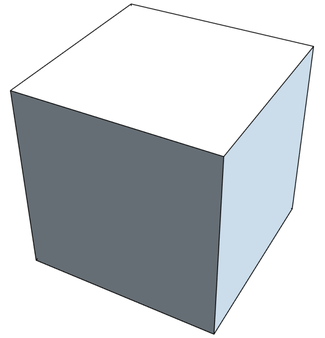
The same symmetry breaking produces domain walls

S4 Leptonic Flavour Symmetry

Hagedorn, King & Luhn, 1205.3114

Ding & Zhou, 1304.2645

Luhn, 1306.2358



real S_4 flavon triplet: $\{\phi_1, \phi_2, \phi_3\}$

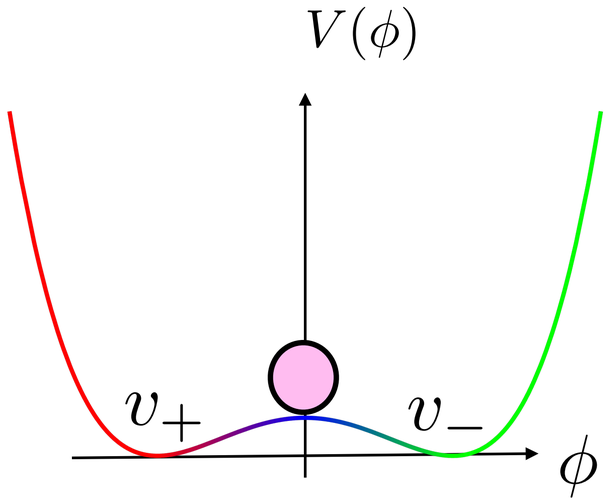
$$V(\boldsymbol{\phi}) = -\frac{\mu^2}{2}\sum_i \phi_i^2 + \frac{g_1}{4}\left(\sum_i \phi_i^2\right)^2 + \frac{g_2}{2}\sum_{i<j}\phi_i^2\phi_j^2$$

We focus on $\mathbb{Z}_2$ preserving vacuum

$$v = \frac{\mu}{\sqrt{g_1}} \qquad\qquad \beta = \frac{g_2}{g_1}$$

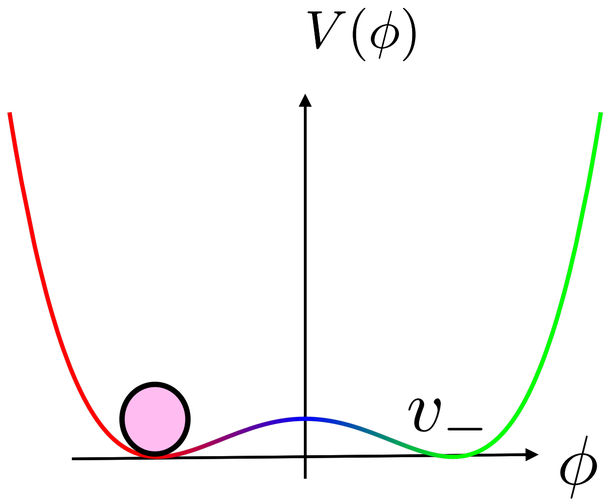
Domain Walls

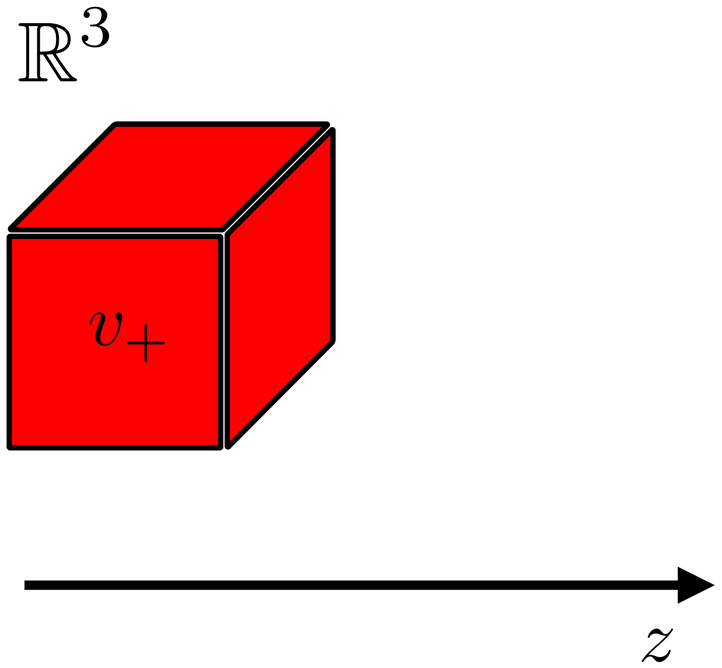
Consider a real scalar field with $\mathbb{Z}_2$-symmetric potential



Domain Walls

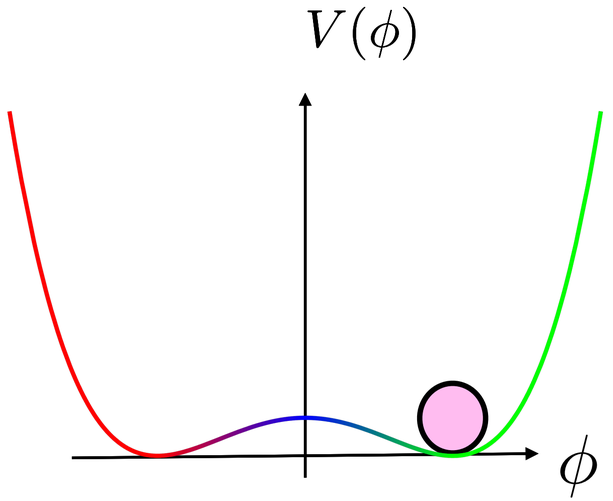
Consider a real scalar field with $\mathbb{Z}_2$-symmetric potential

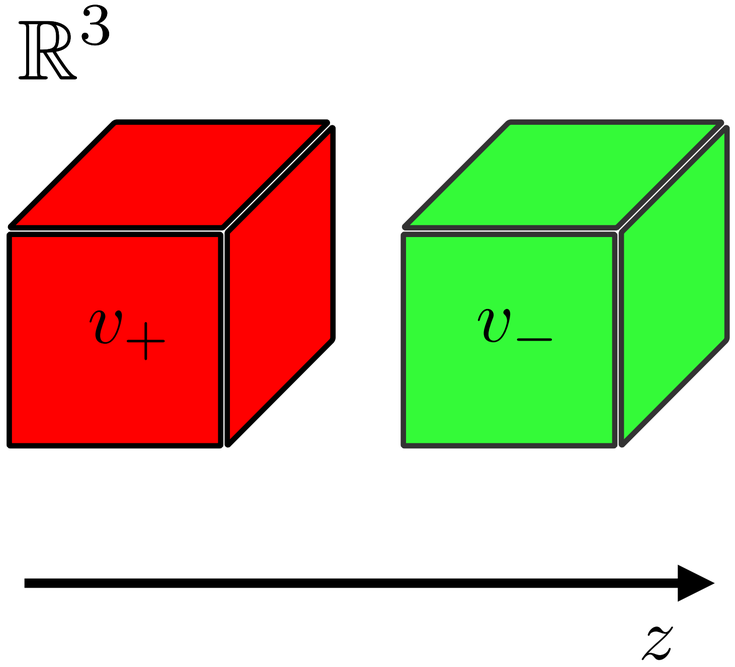




Domain Walls

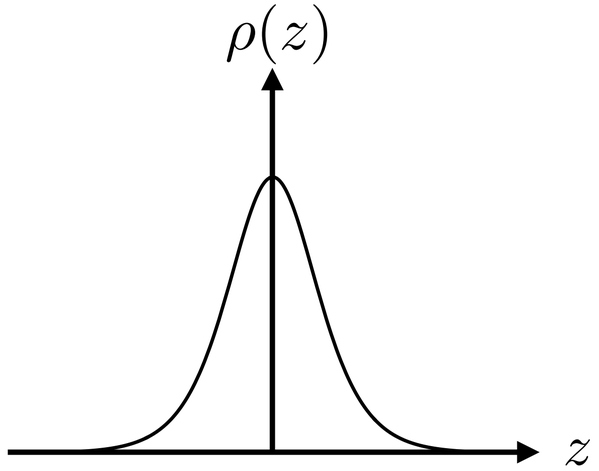
Consider a real scalar field with $\mathbb{Z}_2$-symmetric potential





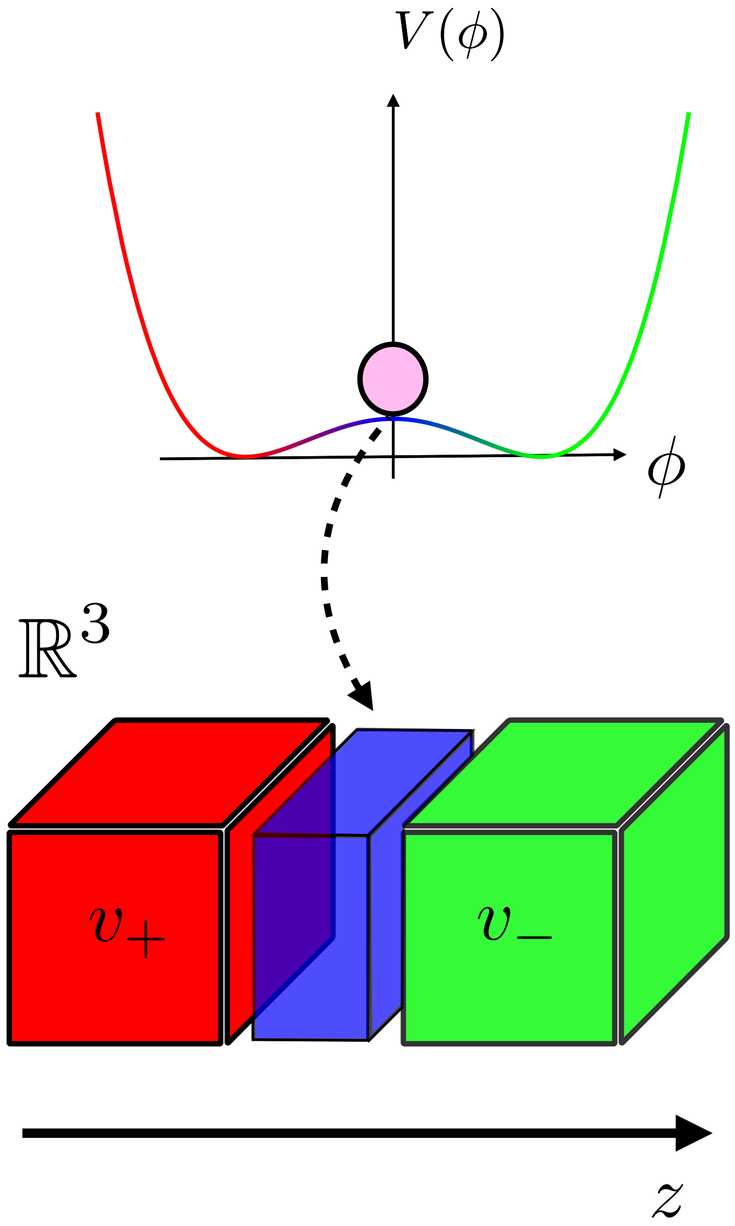
Domain Walls

Consider a real scalar field with $\mathbb{Z}_2$-symmetric potential



$$\rho(z) = \sum_i \underbrace{\frac{1}{2}\left[\phi_i'(z)\right]^2}_{\text{Gradient}} + \underbrace{\Delta V(\phi(z))}_{\text{Potential}}$$

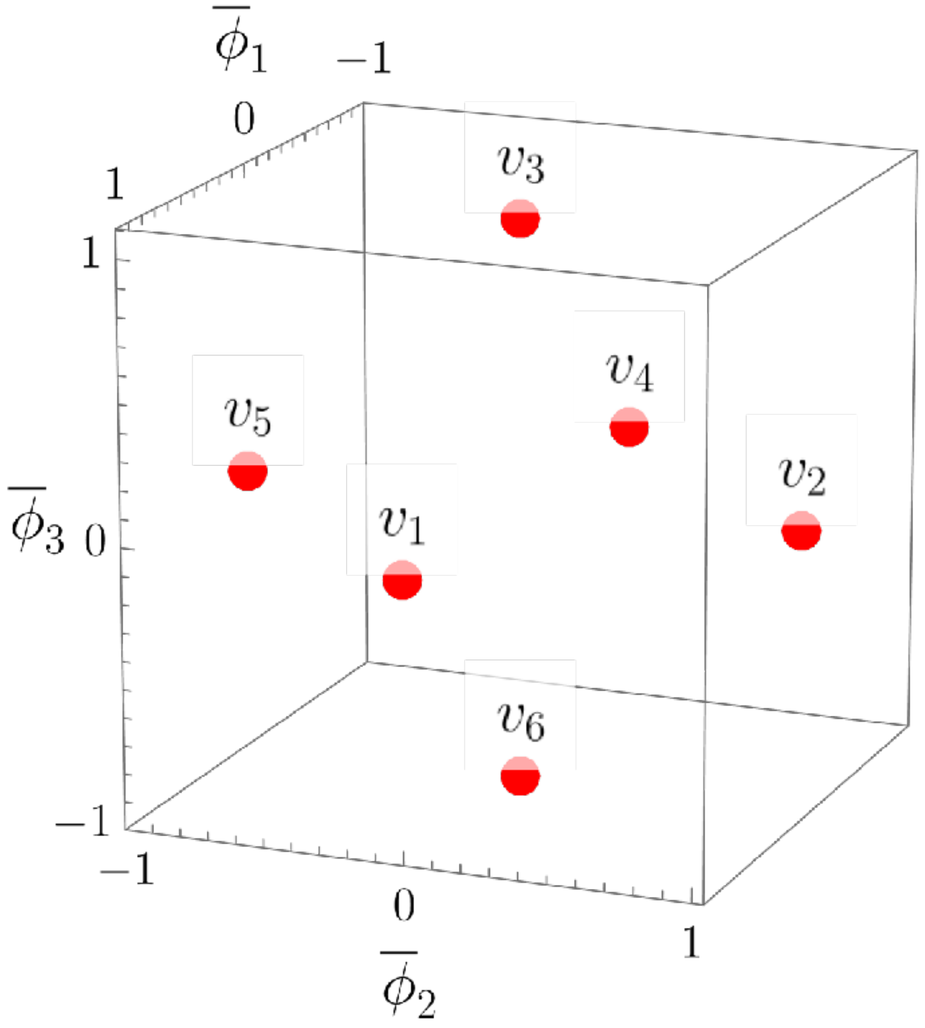
$$\sigma = \int_{-\infty}^{+\infty} dz \rho(z)$$



S4 Leptonic Flavour Symmetry

Six degenerate vacua preserve $\mathbb{Z}_2$ subgroup

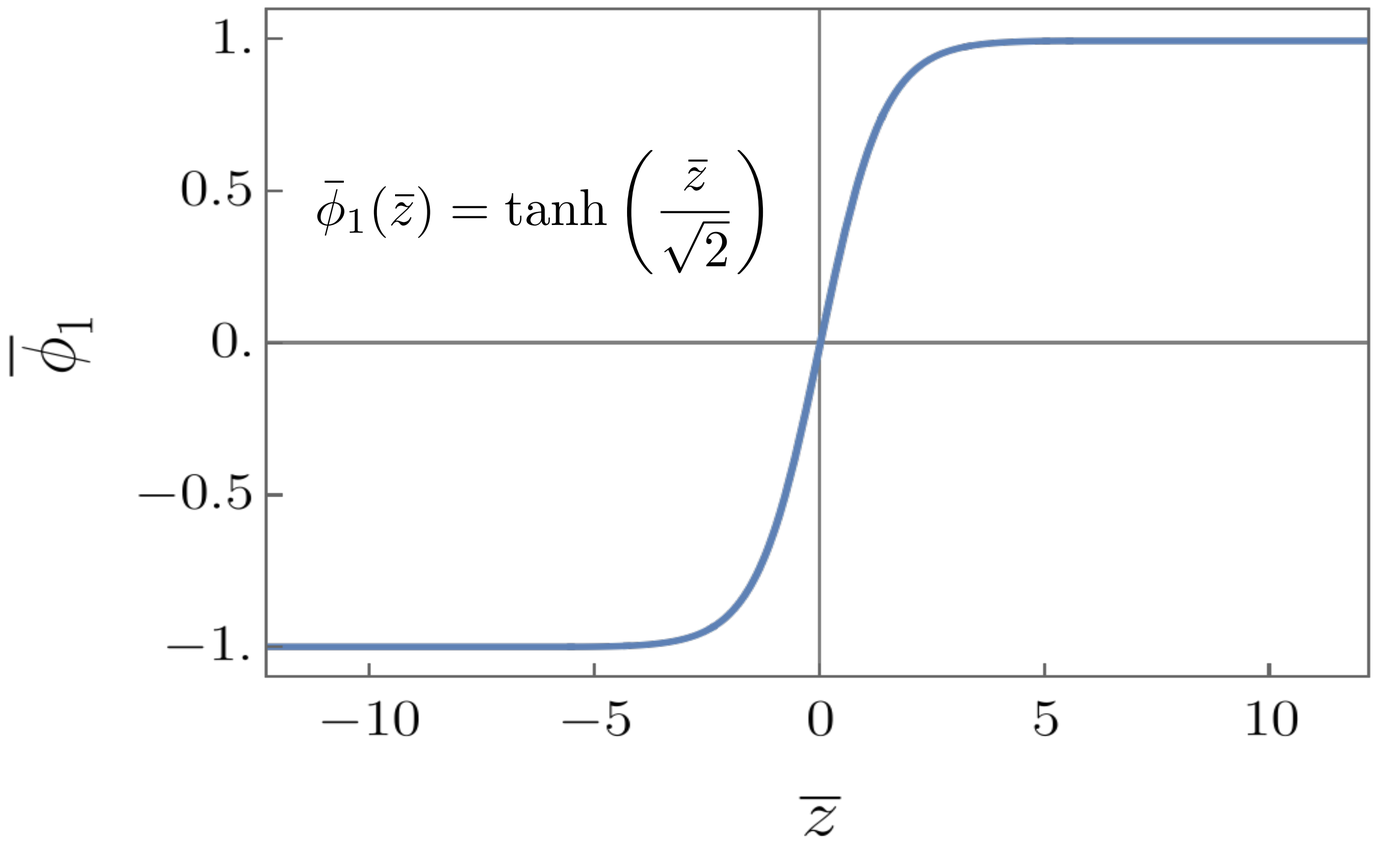
$$\langle \boldsymbol{\phi} \rangle \in \left\{ \pm v \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \pm v \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \pm v \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

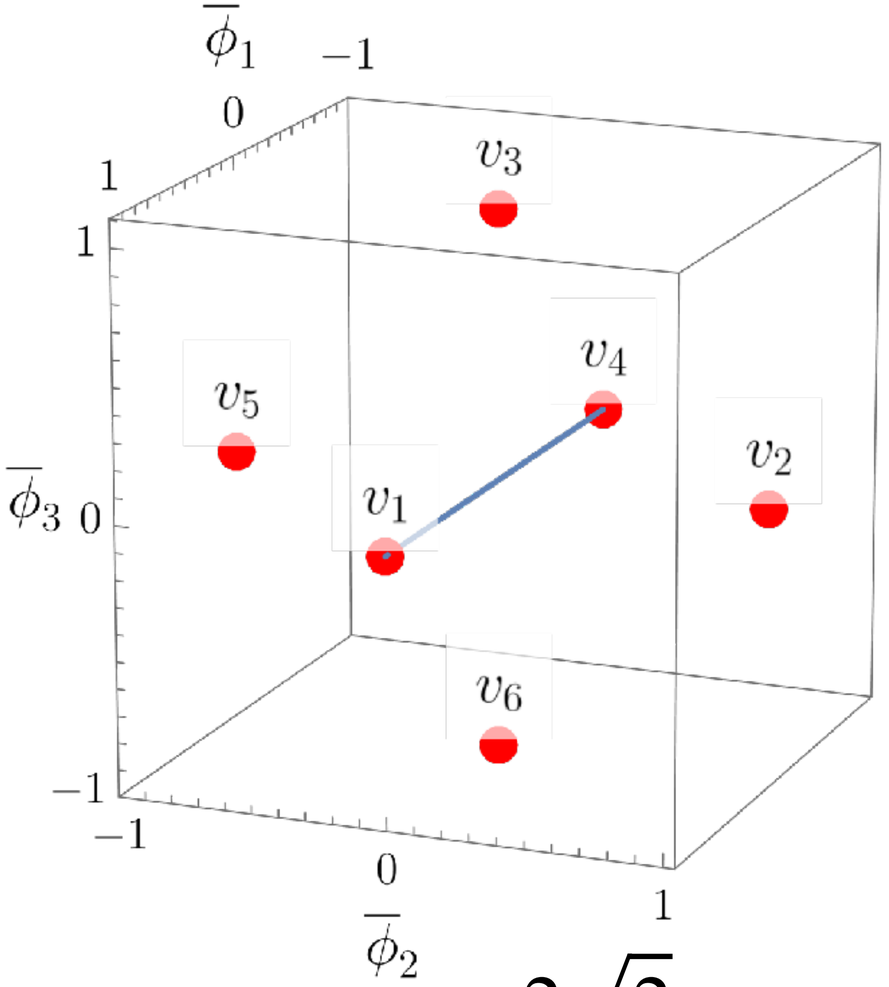


SI walls connect opposite vacua

SII walls connect neighbouring vacua

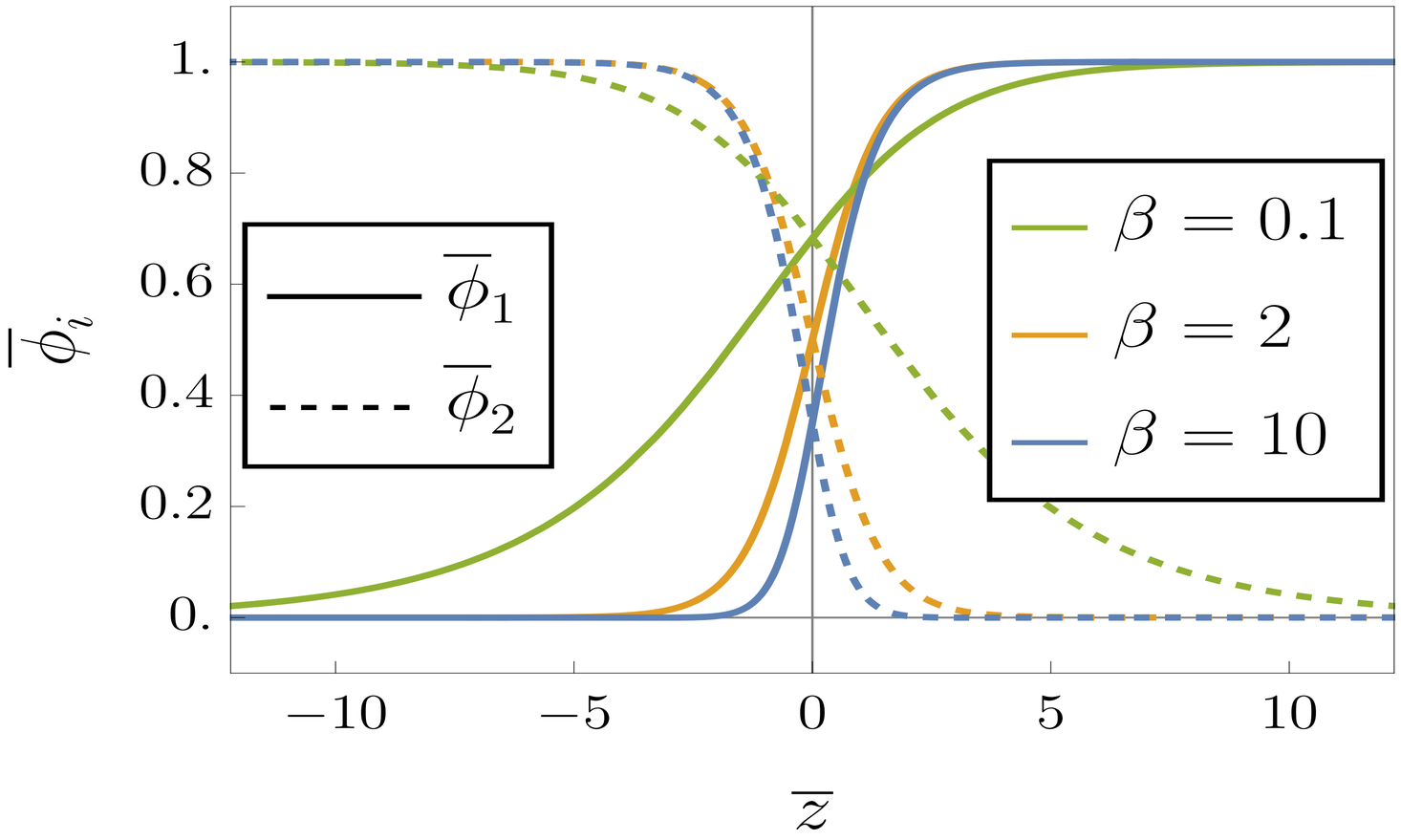
SI Walls

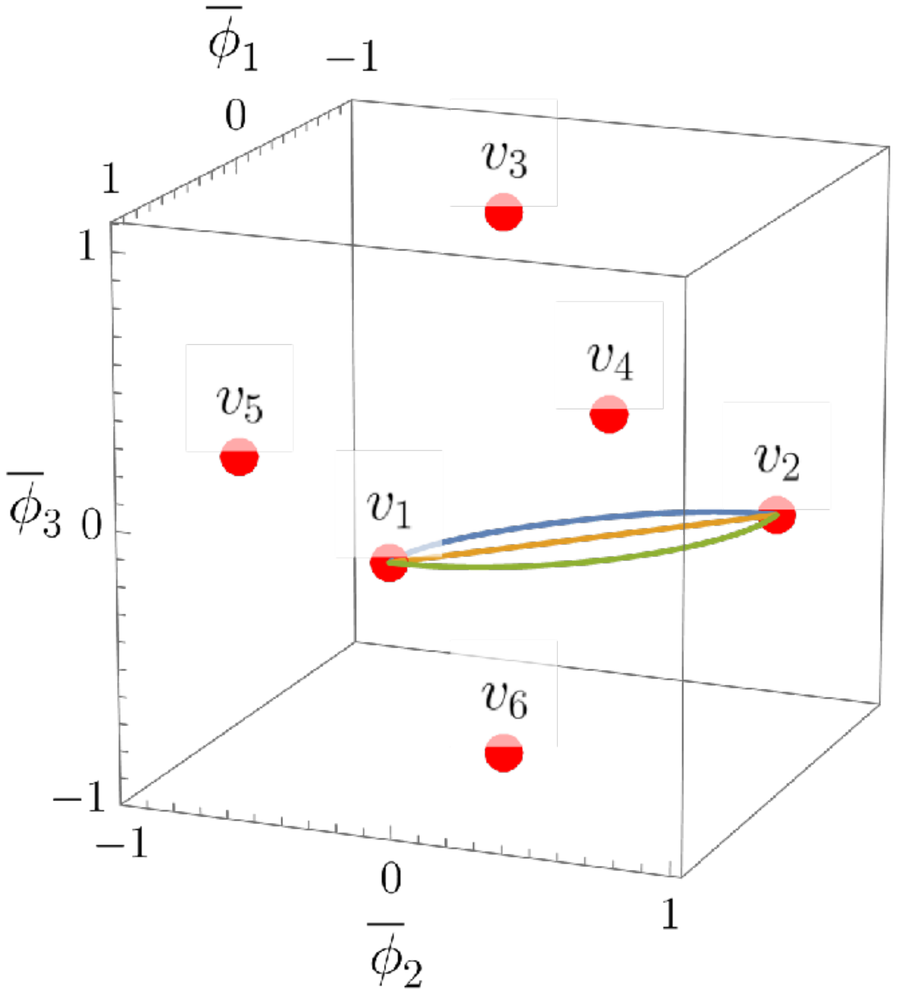




$$\sigma = \frac{2\sqrt{2}}{3}$$

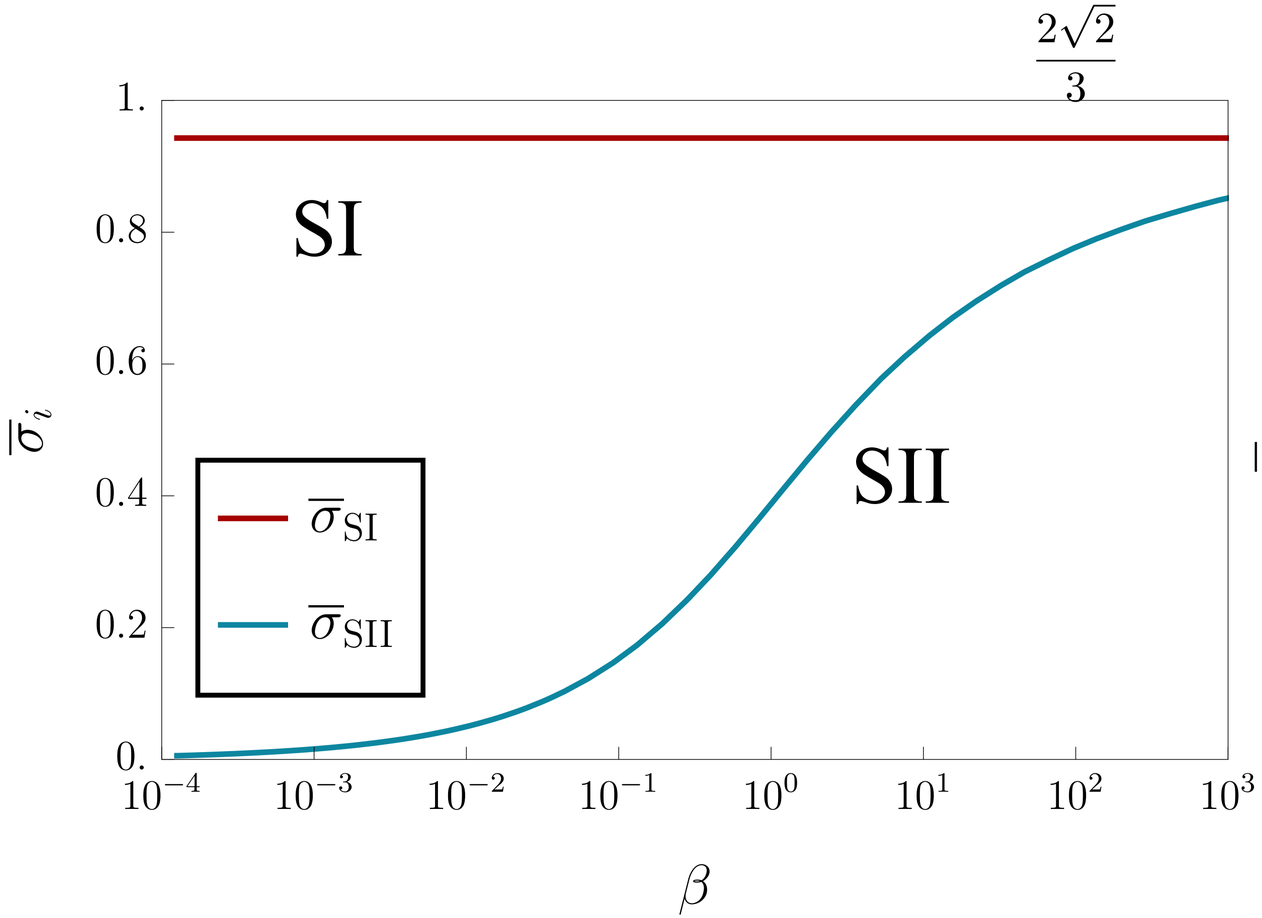
SII-type

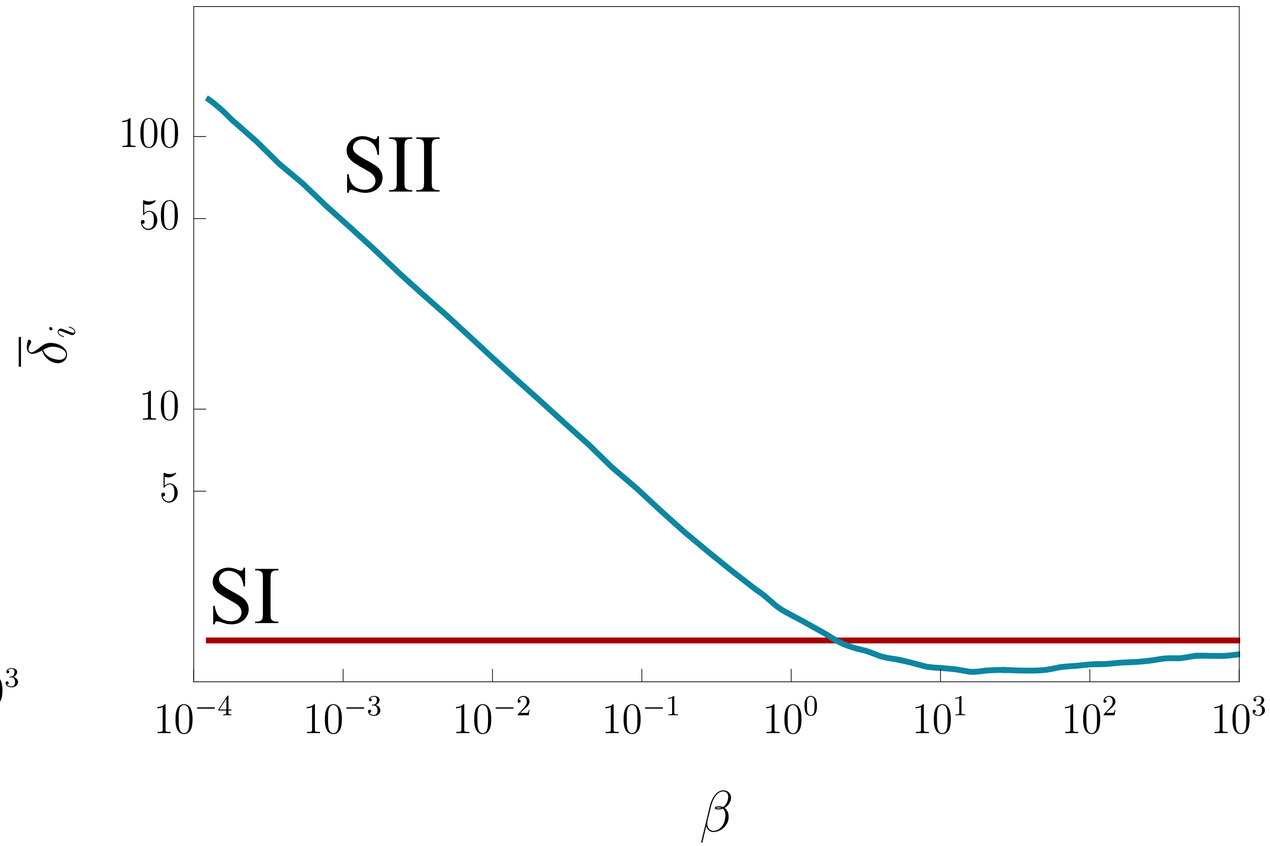




Profiles & tension depend on $\beta = \dfrac{g_2}{g_1}$

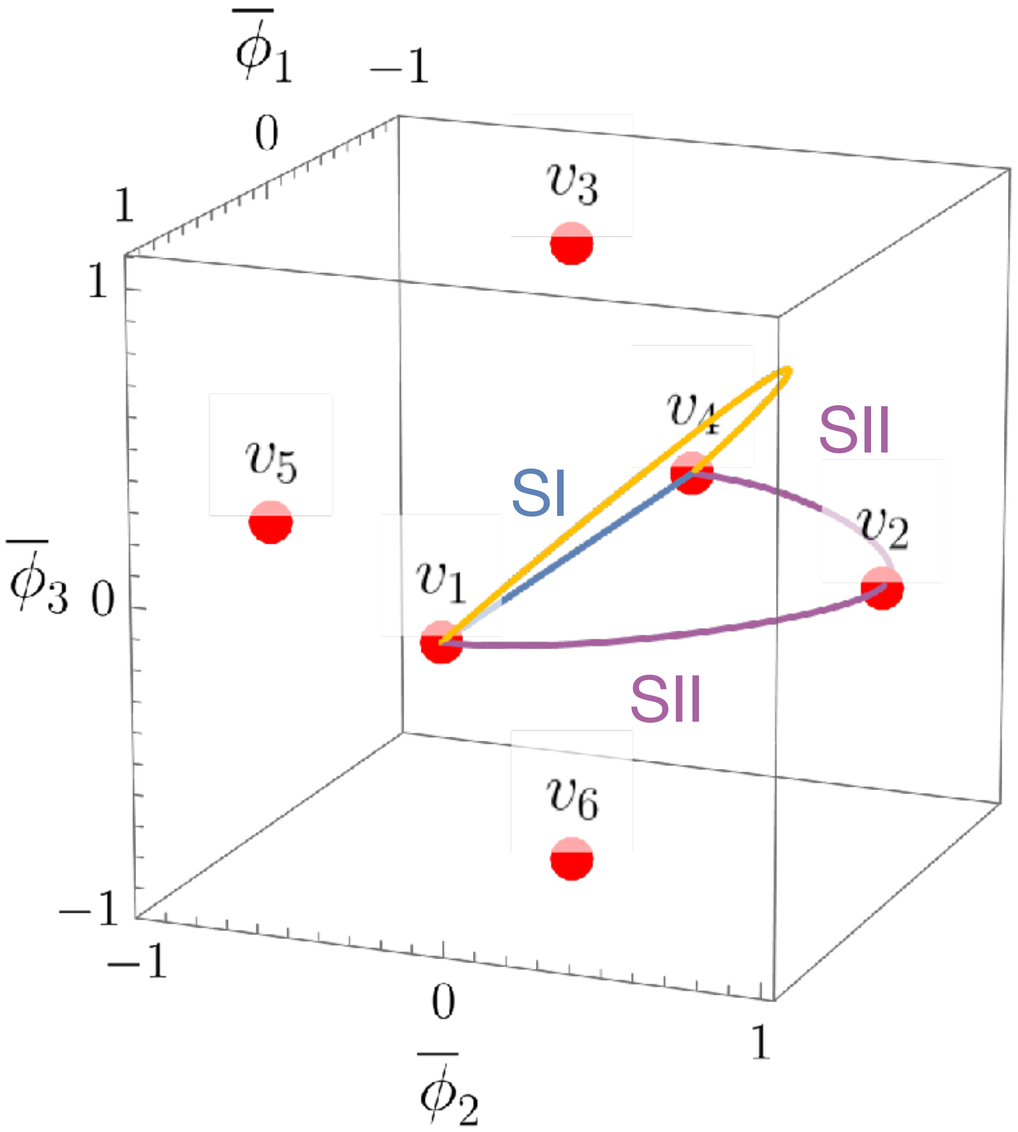
Domain Wall Tensions

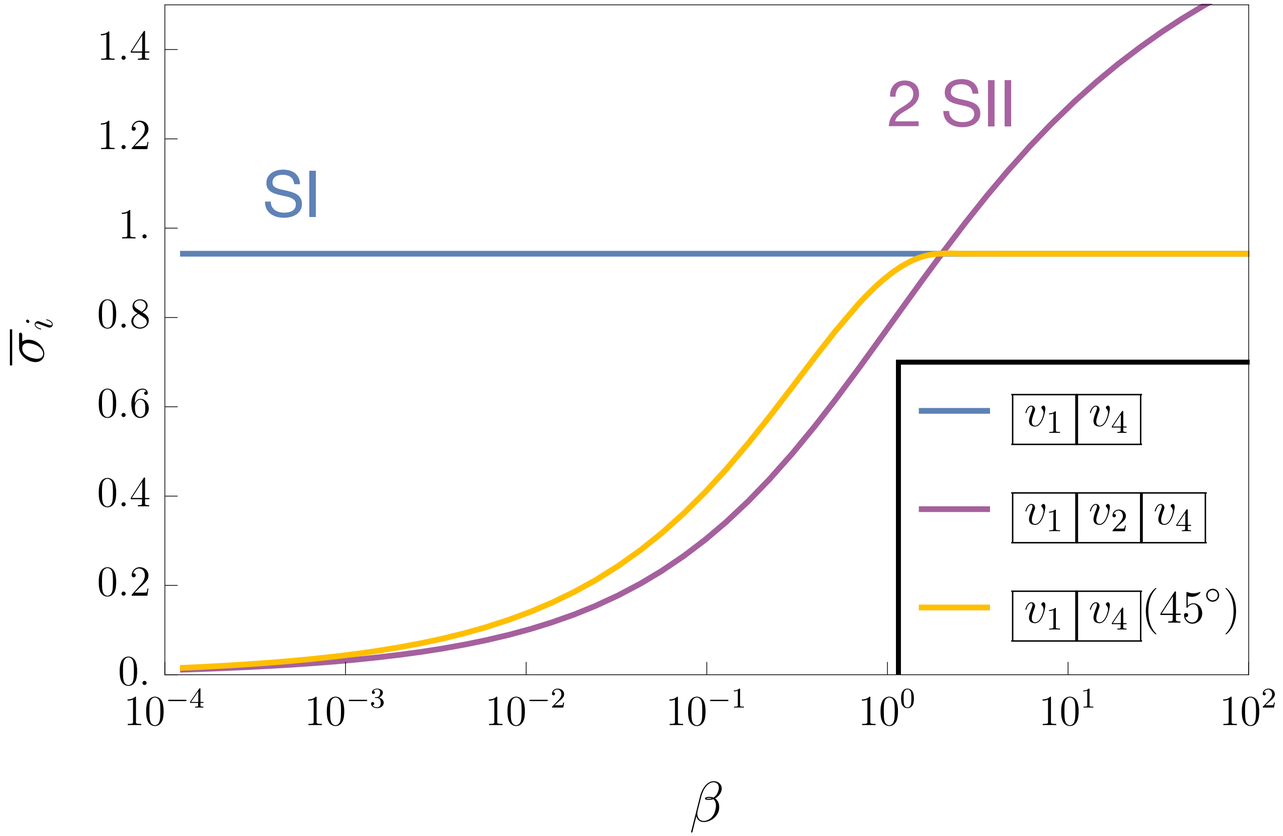




Relative tensions determine is a wall can split

When SI Walls split

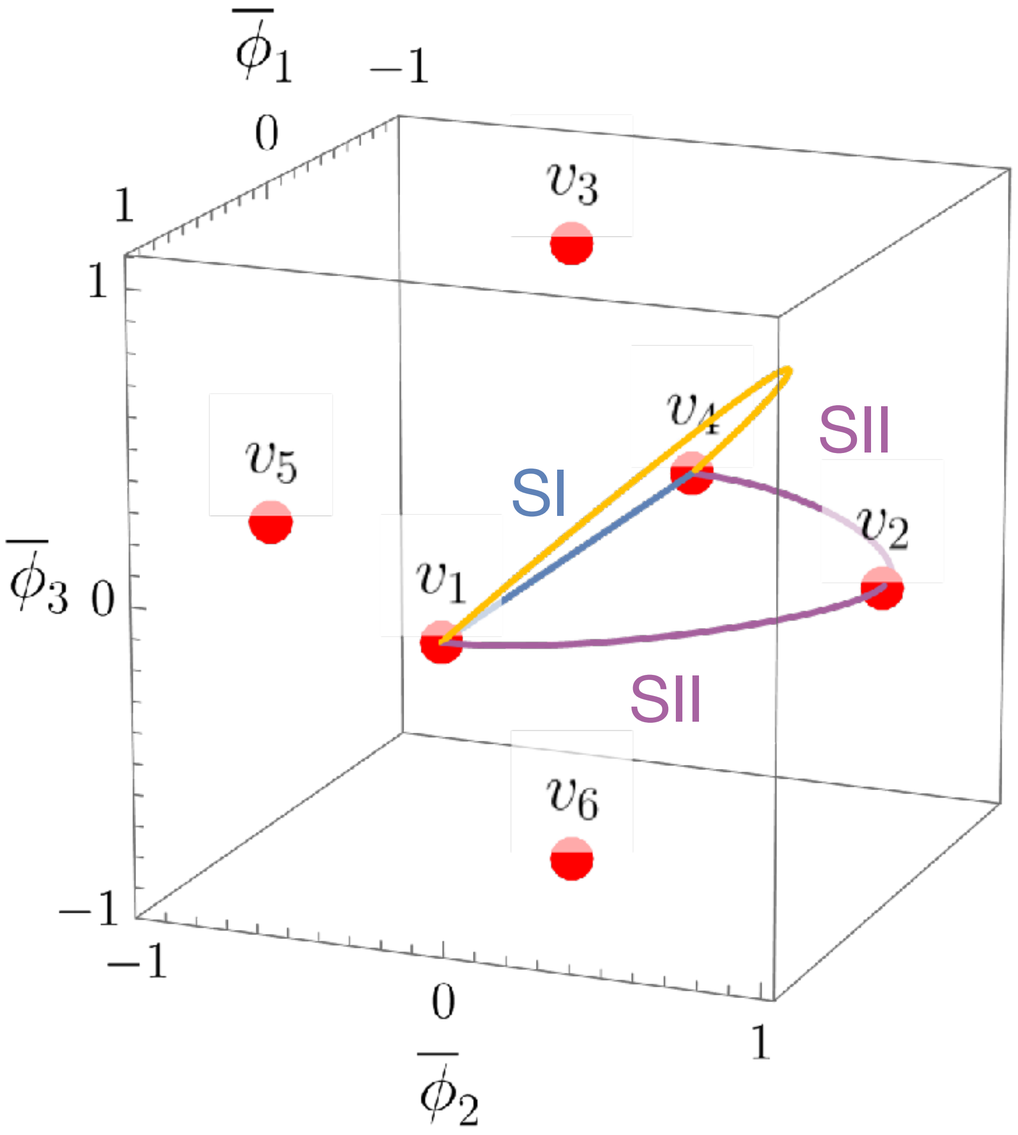


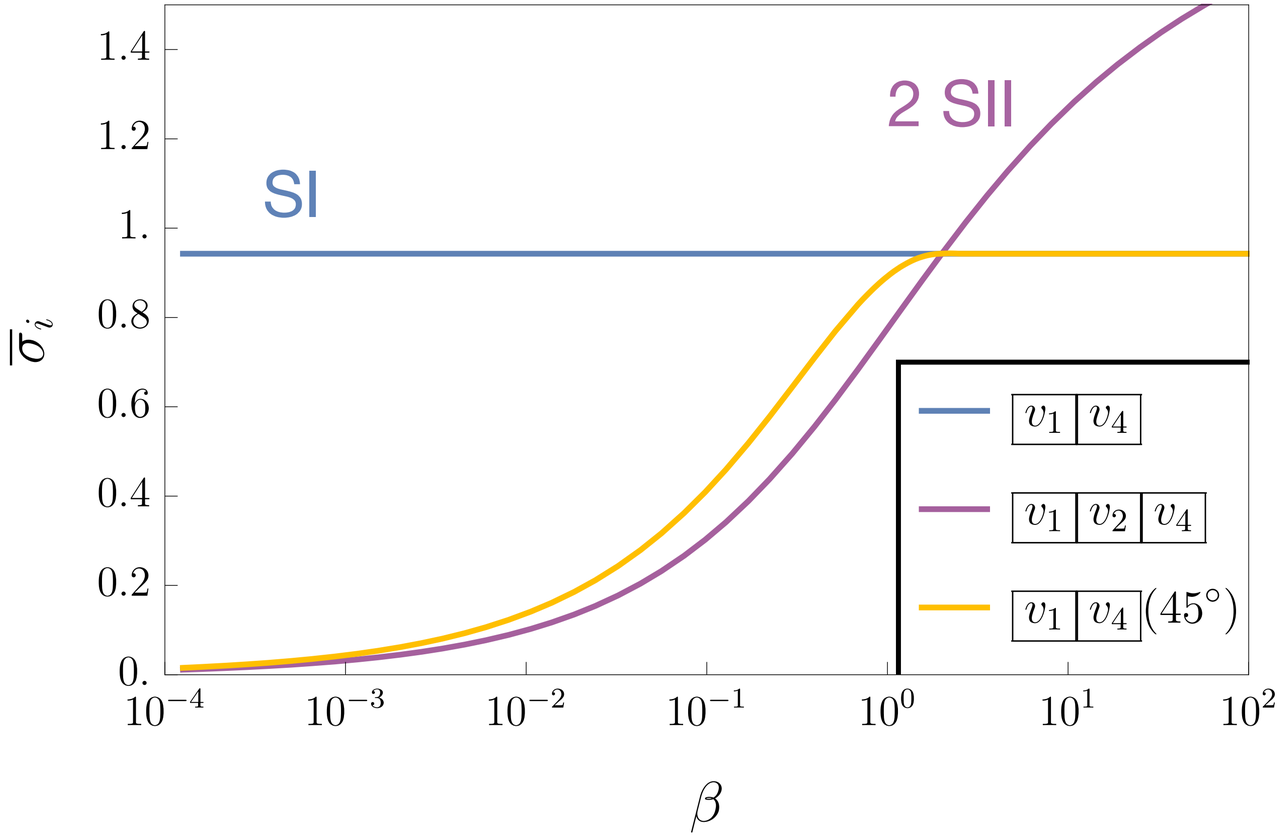


$$\beta < 2 : \quad \sigma(\mathrm{SI}) > 2\sigma(\mathrm{SII})$$

SI wall can split into 2 SII

When both walls survive

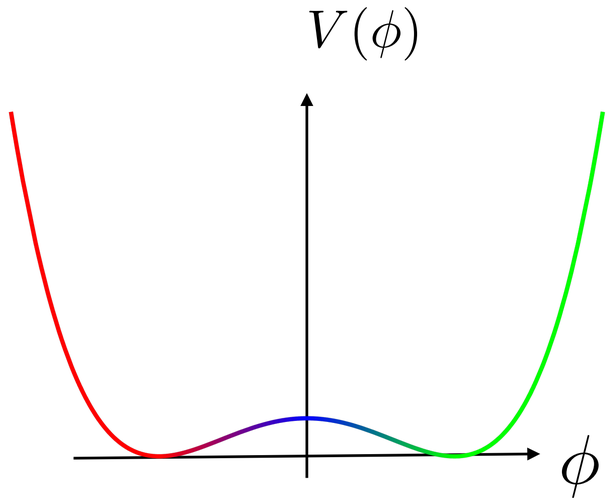




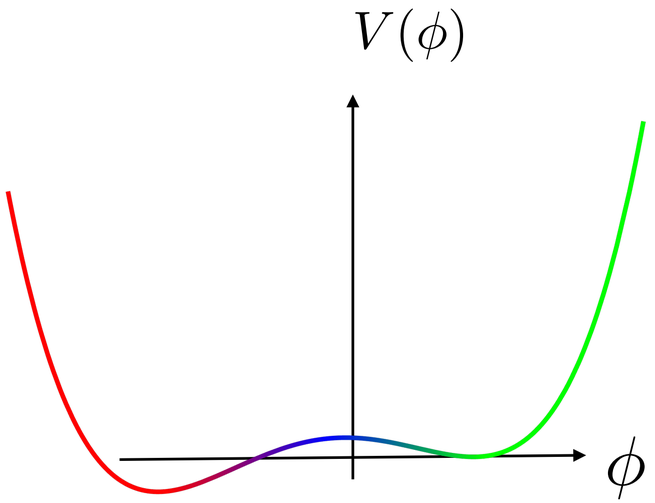
$$\beta > 2 : \sigma(\mathrm{SI}) < 2\sigma(\mathrm{SII})$$

Both walls survive until annihilation

Annihilation of Domain Walls



Static Domain Walls



$$V_{\text{bias}}^{ij} = \epsilon^{ij} v^4$$

Bias $\Longrightarrow$ may correct
LO mixing pattern

Bias $\Longrightarrow$ accelerating DWs
$\Longrightarrow$ Collisions & GW Emission

Gravitational Waves

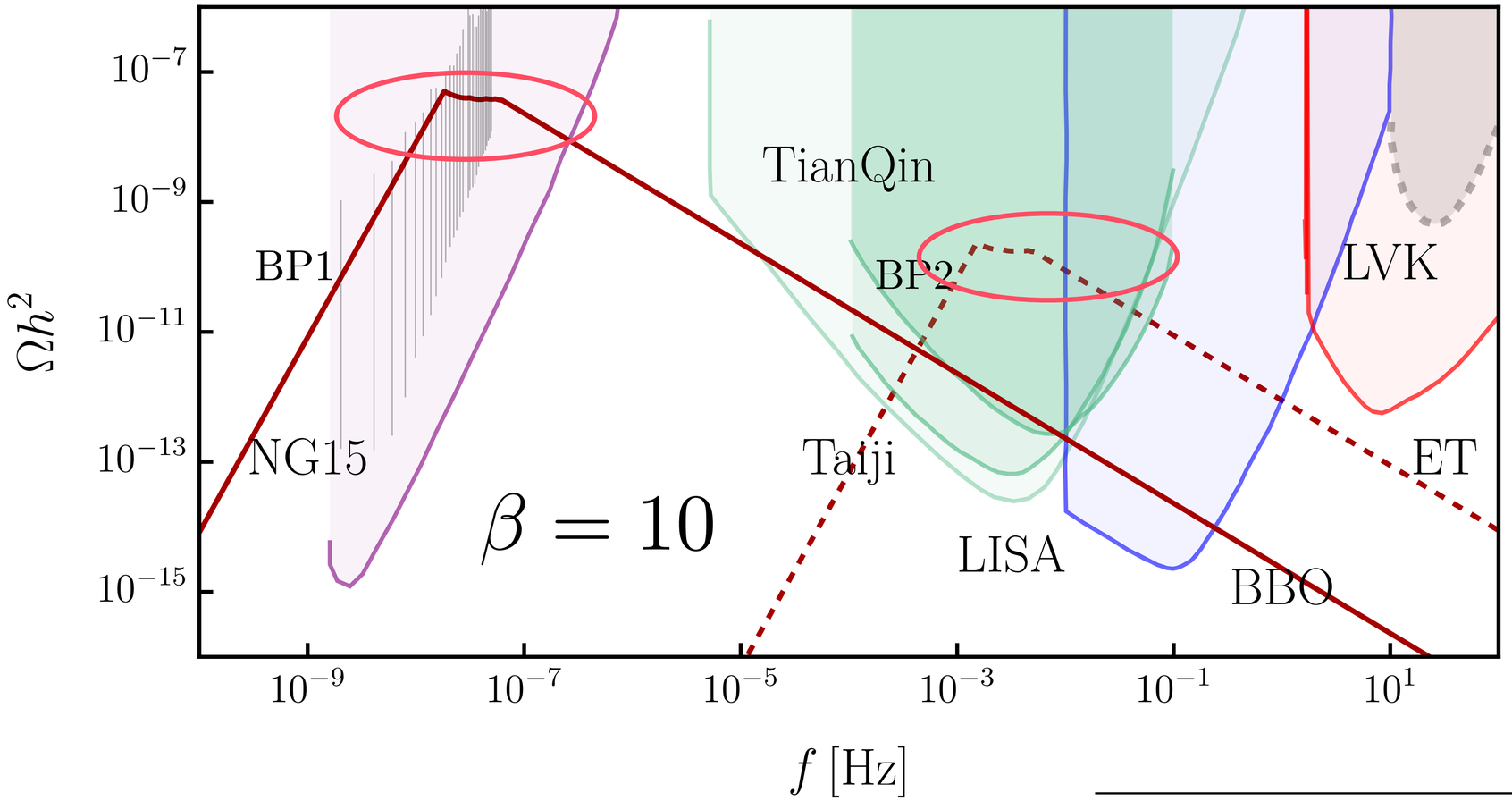
Bias and tension set annihilation time

$$f_{\rm peak} \simeq 3 \times 10^3\,{\rm Hz} \left(\frac{10}{{\rm g}_*({\rm T}_{\rm ann})}\right)^{1/2} \left(\frac{{\rm V}_{\rm bias}}{\sigma\,{\rm TeV}}\right)^{1/2}$$

$$\Omega_{\rm gw} h^2 \simeq 0.9 \times 10^{-67} \left(\frac{10}{g_*\left(T_{\rm ann}\right)}\right)^{1/3} \left(\frac{\sigma}{{\rm TeV}^3}\right)^4 \left(\frac{{\rm TeV}^4}{V_{\rm bias}}\right)^2$$

Gravitational Waves: S-type, $\mathbb{Z}_2$

Multipeak structure from different Domain walls i.e. SI, SII



	$\sigma^{1/3}$/TeV	$V_{\rm bias}^{1/4}$/TeV
BP1	3×10^2	$10^{-3.75}$
BP2	3×10^5	10^1

Domain walls in A4 flavour models (2512.13784)

The flavon sector determines the wall structure and its evolution.

Real flavons

Cubic interaction splits vacuum energies.
Composite “oreo” walls can form.

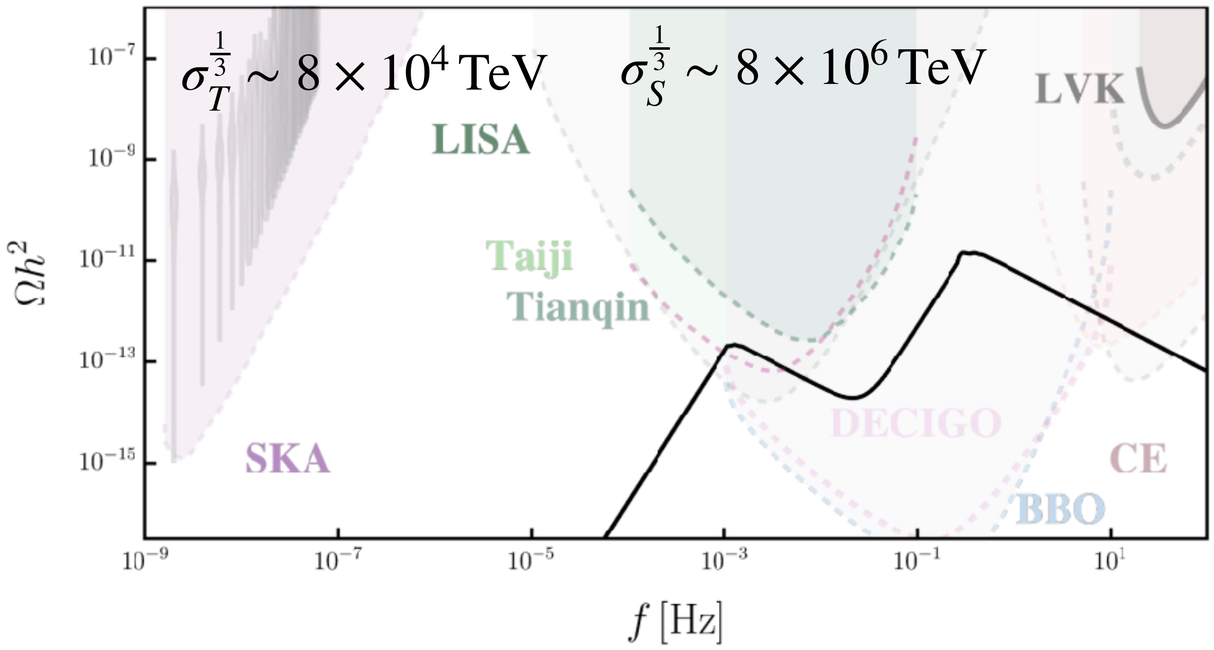
Complex flavons

CP-related vacua give new wall types.
A broken global U(1) also gives strings.

Supersymmetric flavons

Soft SUSY breaking can trigger a first stage of wall annihilation.

Two flavon sectors, two GW peaks



Different wall tensions and biases can produce separated GW peaks.

Summary

Breaking non-Abelian flavour symmetries can produce domain walls

S_4: distinct wall types with stability controlled by scalar couplings

A_4: composite and CP-related walls, SUSY wall dynamics,
and global strings when an extra global U(1) breaks

Different wall tensions and annihilation times can produce multiple GW peaks

Ongoing work: lattice simulations and quantitative links between
flavour physics and GW signatures

Backup slides

S4 irreps $1^2 + 1'^2 + 2^2 + 3^2 + 3'^2$

$$\left(M_\phi^2\right)_{ij} = \left.\frac{\partial^2 V(\phi)}{\partial\phi_i \partial\phi_j}\right|_{\langle\phi\rangle}$$

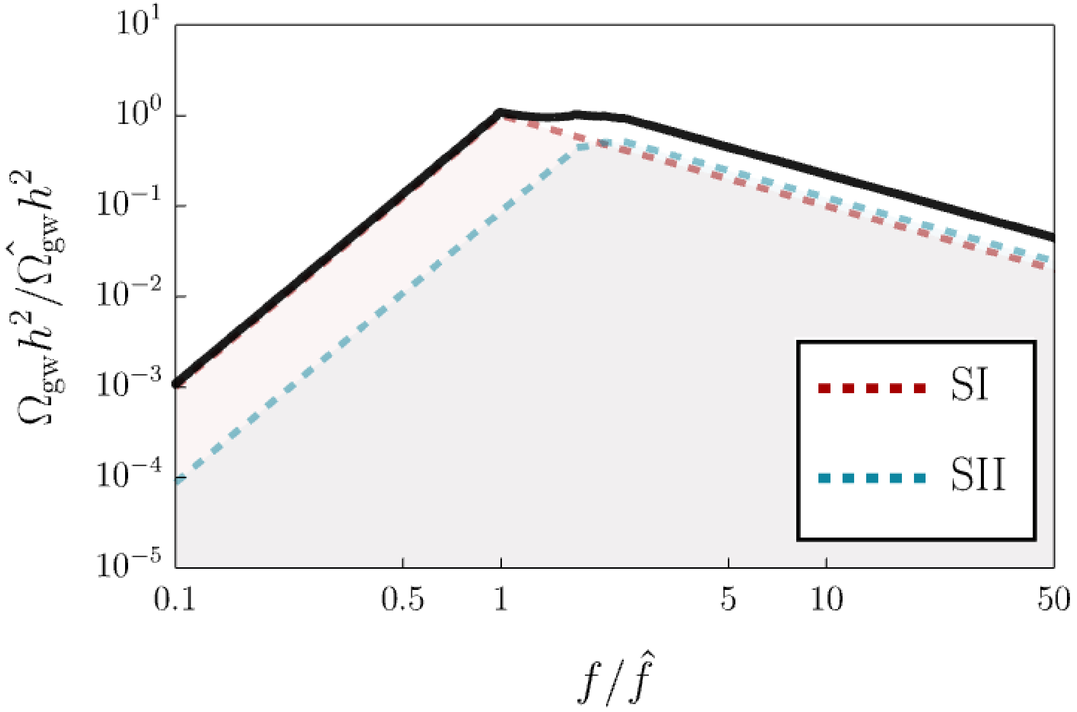
$$m_1^2 = 2g_1 v^2, \quad m_2^2 = m_3^2 = g_2 v^2;$$

$$m_1^2 = 2\left(3g_1 + 2g_2\right) u^2, \quad m_2^2 = m_3^2 = -2g_2 u^2;$$

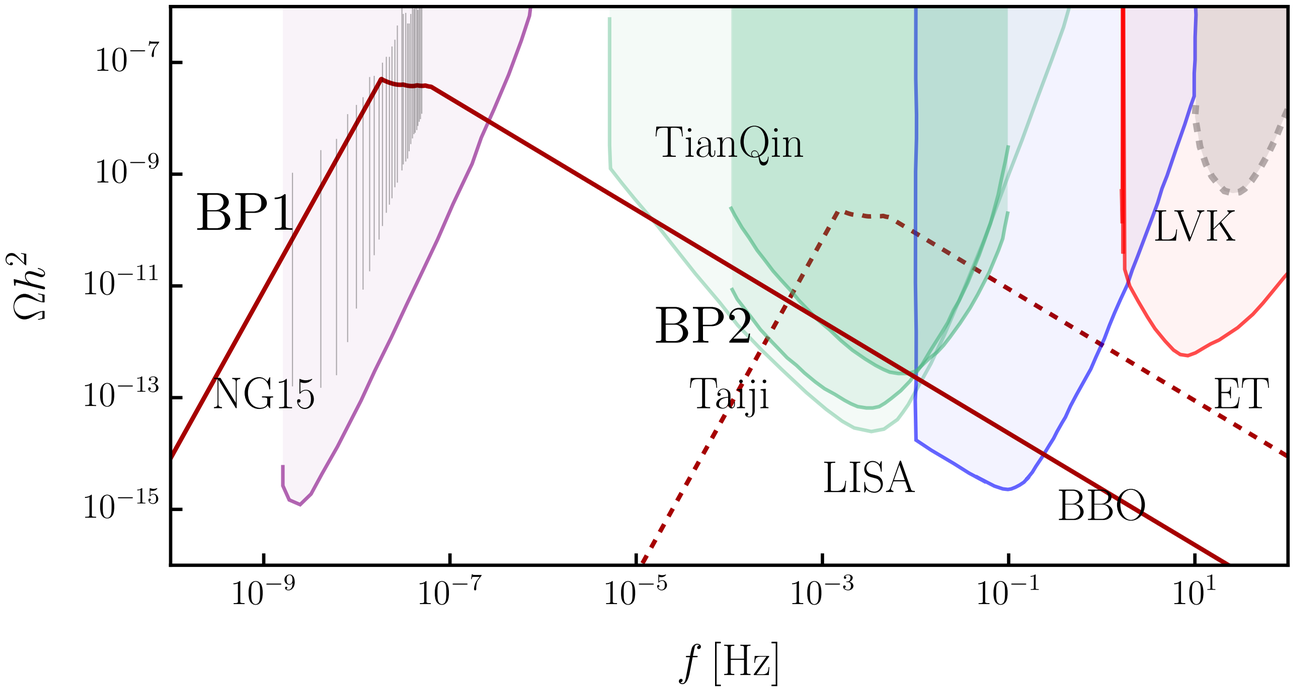
$g_2 < 0 \implies$ flavon breaks $\mathbb{Z}_2$, $g_2 > 0 \implies$ flavon breaks $\mathbb{Z}_3$

Gravitational Waves: S-type, $\mathbb{Z}_2$

$\beta = 10$



$\beta = 10$

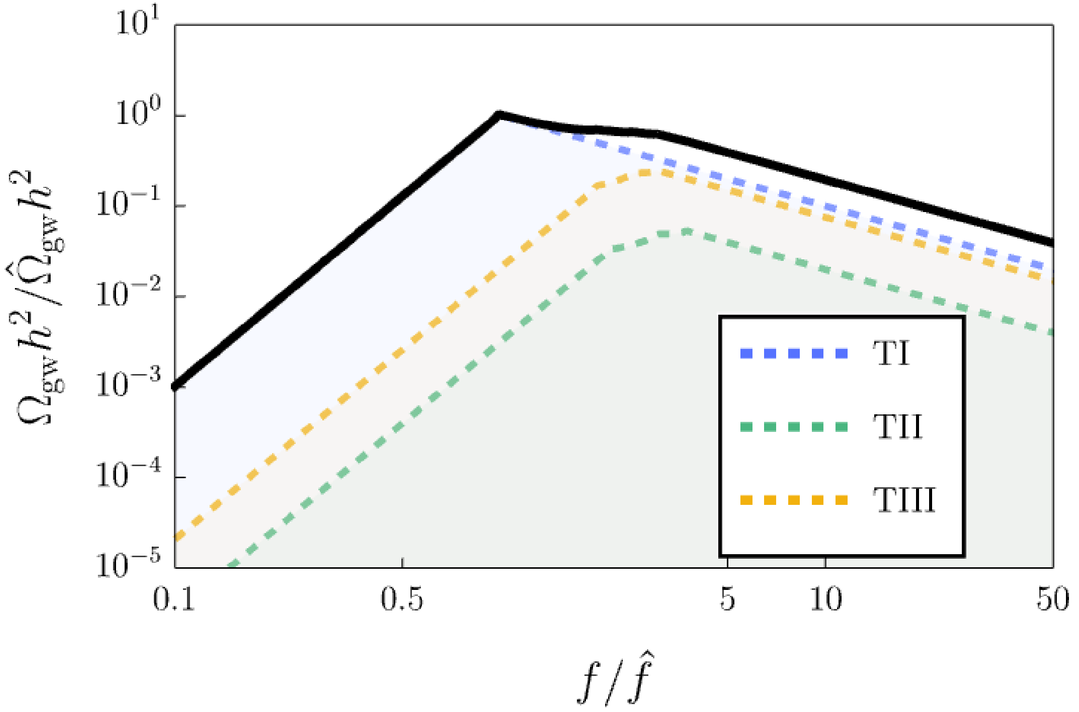


$$n_{\rm SI} : n_{\rm SII} = 3 : 12$$

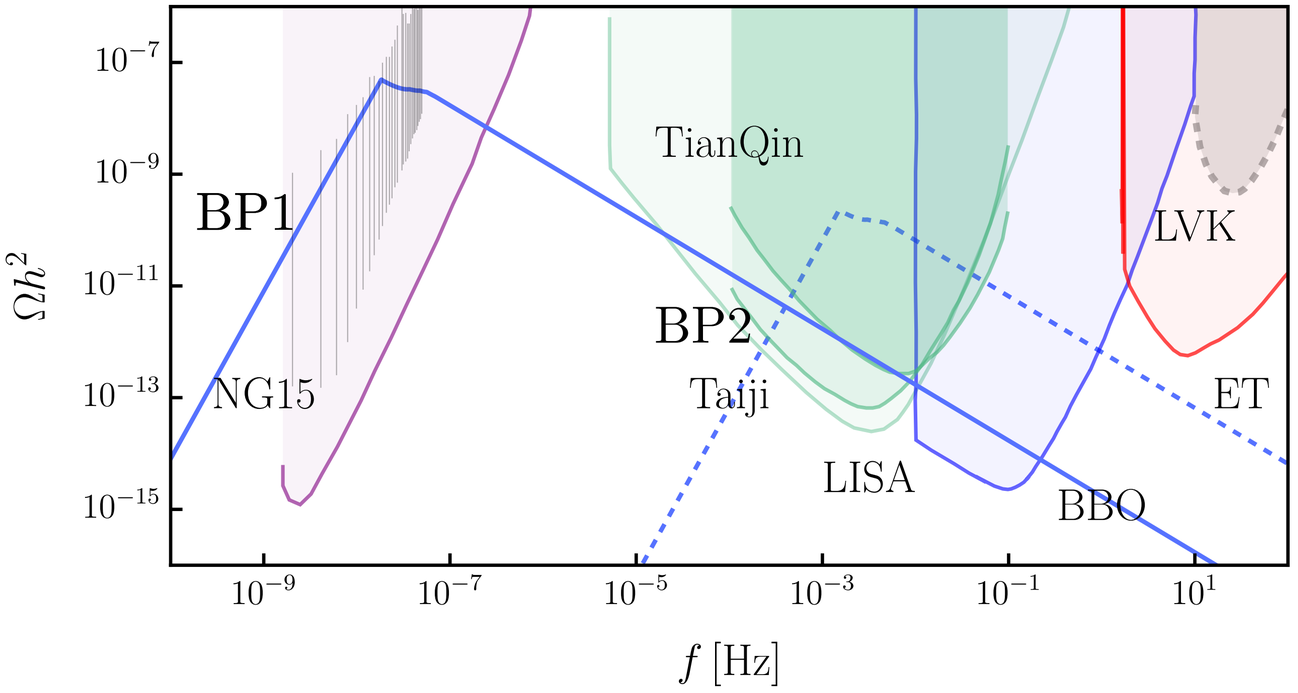
	$\sigma^{1/3}$/TeV	$V_{\rm bias}^{1/4}$/TeV
BP1	3×10^2	$10^{-3.75}$
BP2	3×10^5	10^1

Gravitational Waves: T-type, $\mathbb{Z}_3$

$\beta = -1.2$



$\beta = -1.2$



$$n_{\rm TI} : n_{\rm TII} : n_{\rm TIII} = 4 : 12 : 12$$

	$\sigma^{1/3}$/TeV	$V_{\rm bias}^{1/4}$/TeV
BP1	3×10^2	$10^{-3.75}$
BP2	3×10^5	10^1

Domain wall characteristics

Domain walls annihilate when pressure from the wall tension equals pressure from bias

$$p_T \sim \frac{\sigma}{t} \approx V_{\mathrm{bias}} \implies t_{\mathrm{ann}} = \frac{\sigma}{V_{\mathrm{bias}}}$$

We took the curvature to be equal to time, $R \sim t$

$$t_{\mathrm{ann}} = \frac{f_\sigma v^3}{\epsilon v^4} = \frac{f_\sigma}{\epsilon v}$$

Assuming radiation domination, $H \propto 1/(2t)$

$$H \simeq 1.66\sqrt{g_*}\frac{T^2}{M_{\mathrm{Pl}}} \implies t = \frac{1}{2H} = \frac{1}{2 \cdot 1.66\sqrt{g_*}} \cdot \frac{M_{\mathrm{Pl}}}{T^2} = \frac{0.301}{\sqrt{g_*}} \cdot \frac{M_{\mathrm{Pl}}}{T^2}$$

Domain wall characteristics

Invert to find annihilation temperature:

$$T = \left(\frac{0.301 M_{\rm Pl}}{\sqrt{g_*} t} \right)^{1/2}$$

Plug in constants and take $g_* \sim 10$

$$T_{\rm ann} \simeq 3 \times 10^7 {\rm TeV} \left(\frac{\epsilon_b}{f_\sigma} \right)^{1/2} \left(\frac{v}{\rm TeV} \right)^{1/2}$$

For BP1: $\epsilon \sim 10^{-25}$ $\quad T_{\rm ann} \sim 0.18$ GeV

For BP2: $\epsilon \sim 10^{-18}$ $\quad T_{\rm ann} \sim 18$ TeV

Domain wall characteristics

The observed peak frequency is the redshifted Hubble scale at annihilation:

$$f_{\rm peak} = \frac{a\left(T_{\rm ann}\right)}{a_0} \cdot H\left(T_{\rm ann}\right) \qquad H \approx 1.66\sqrt{g_*}\frac{T_{\rm ann}^2}{M_{\rm Pl}}$$

Redshift factor used $g_*(T_{\rm ann}) \sim 10\,, T_0 \sim 10^{-13}$ GeV, $g_*(T_0) = 3.91$

$$\frac{a\left(T_{\rm ann}\right)}{a_0} = \left(\frac{g_{*s}\left(T_0\right)}{g_*\left(T_{\rm ann}\right)}\right)^{1/3} \cdot \frac{T_0}{T_{\rm ann}}$$

$$f_{\rm peak} \approx \frac{T_0}{M_{\rm Pl}}\left(\frac{g_{*s}\left(T_0\right)}{g_*\left(T_{\rm ann}\right)}\right)^{1/3} \cdot T_{\rm ann} \quad \Rightarrow \quad f_{\rm peak} \approx 1.1 \times 10^{-7}\ {\rm Hz/GeV} \cdot T_{\rm ann}$$

Using $T_{\rm ann}$ expression from before: $f_{\rm peak} \simeq 3 \times 10^3\ {\rm Hz} \cdot \left(\frac{\epsilon_b v}{f_\sigma\,{\rm TeV}}\right)^{1/2}$

Domain wall characteristics

Energy Density in GW from domain wall collapse at the time of annihilation:

$$\rho_{\mathrm{GW}}\left(T_{\mathrm{ann}}\right) \sim G\sigma^2 H^2\left(T_{\mathrm{ann}}\right)$$

$$\rho_{\mathrm{GW}} \sim \frac{1}{M_{\mathrm{Pl}}^2} \cdot \left(f_\sigma^2 v^6\right) \cdot \left(\frac{T_{\mathrm{ann}}^4}{M_{\mathrm{Pl}}^2}\right) = \frac{f_\sigma^2 v^6 T_{\mathrm{ann}}^4}{M_{\mathrm{Pl}}^4}$$

$$T_{\mathrm{ann}} \sim \left(\frac{V_{\mathrm{bias}}}{\sigma}\right)^{1/2} M_{\mathrm{Pl}}^{1/2} \sim \left(\frac{\epsilon_b v}{f_\sigma}\right)^{1/2} M_{\mathrm{Pl}}^{1/2}$$

$$\rho_{\mathrm{GW}} \sim \frac{\epsilon_b^2 v^8}{M_{\mathrm{Pl}}^2}$$

Domain wall characteristics

Need to redshift this to today:

$$\rho_{\mathrm{GW},0} = \rho_{\mathrm{GW}}\left(T_{\mathrm{ann}}\right) \cdot \left(\frac{T_0}{T_{\mathrm{ann}}}\right)^4 \cdot \left(\frac{g_{*s}\left(T_0\right)}{g_{*s}\left(T_{\mathrm{ann}}\right)}\right)^{4/3}$$

Also convert to fractional energy density per logarithmic frequency:

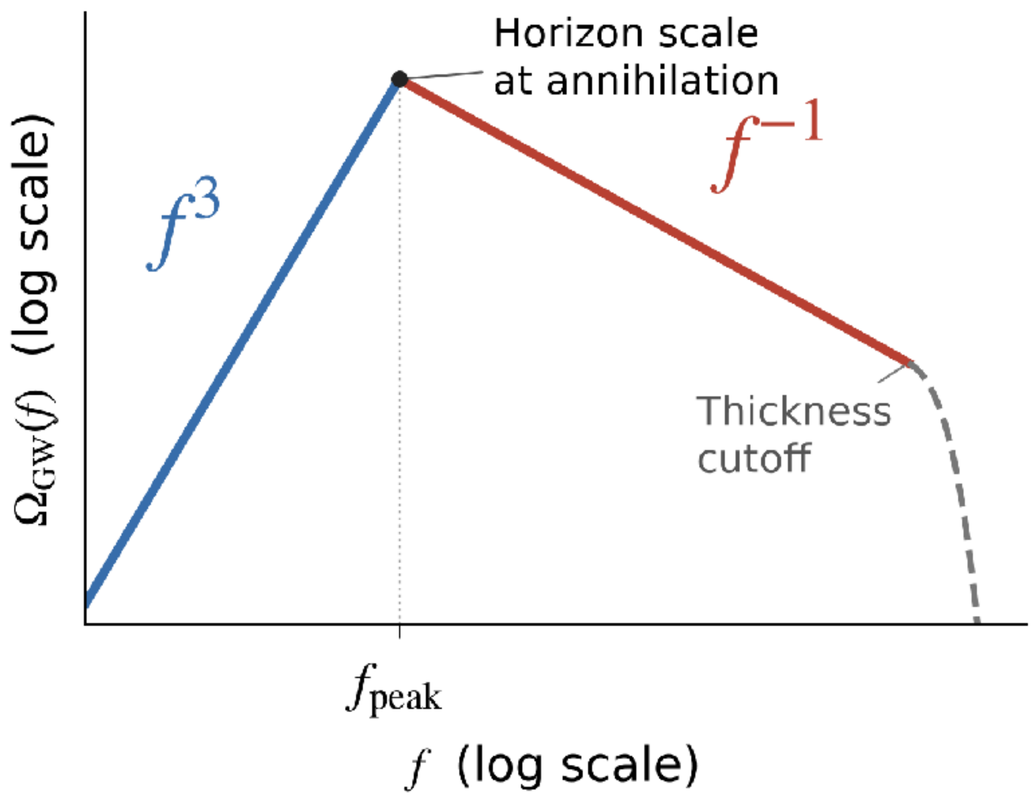
$$\Omega_{\mathrm{GW}} h^2 = \frac{h^2}{\rho_c} \cdot \rho_{\mathrm{GW},0} \sim \frac{h^2}{\rho_c} \cdot \frac{\epsilon_b^2 v^8}{M_{\mathrm{Pl}}^2} \cdot \left(\frac{T_0}{T_{\mathrm{ann}}}\right)^4$$

Plug in appropriate numerical factors:

$$\Omega_{\mathrm{gw}} h^2 \simeq 0.9 \times 10^{-67} \left(\frac{10}{g_*\left(T_{\mathrm{ann}}\right)}\right)^{1/3} \left(\frac{\sigma}{\mathrm{TeV}^3}\right)^4 \left(\frac{\mathrm{TeV}^4}{V_{\mathrm{bias}}}\right)^2$$

Why does the GW spectrum rise and fall?

The two slopes have different physical origins.



Schematic; cutoff position is illustrative.

IR: f^3 rise

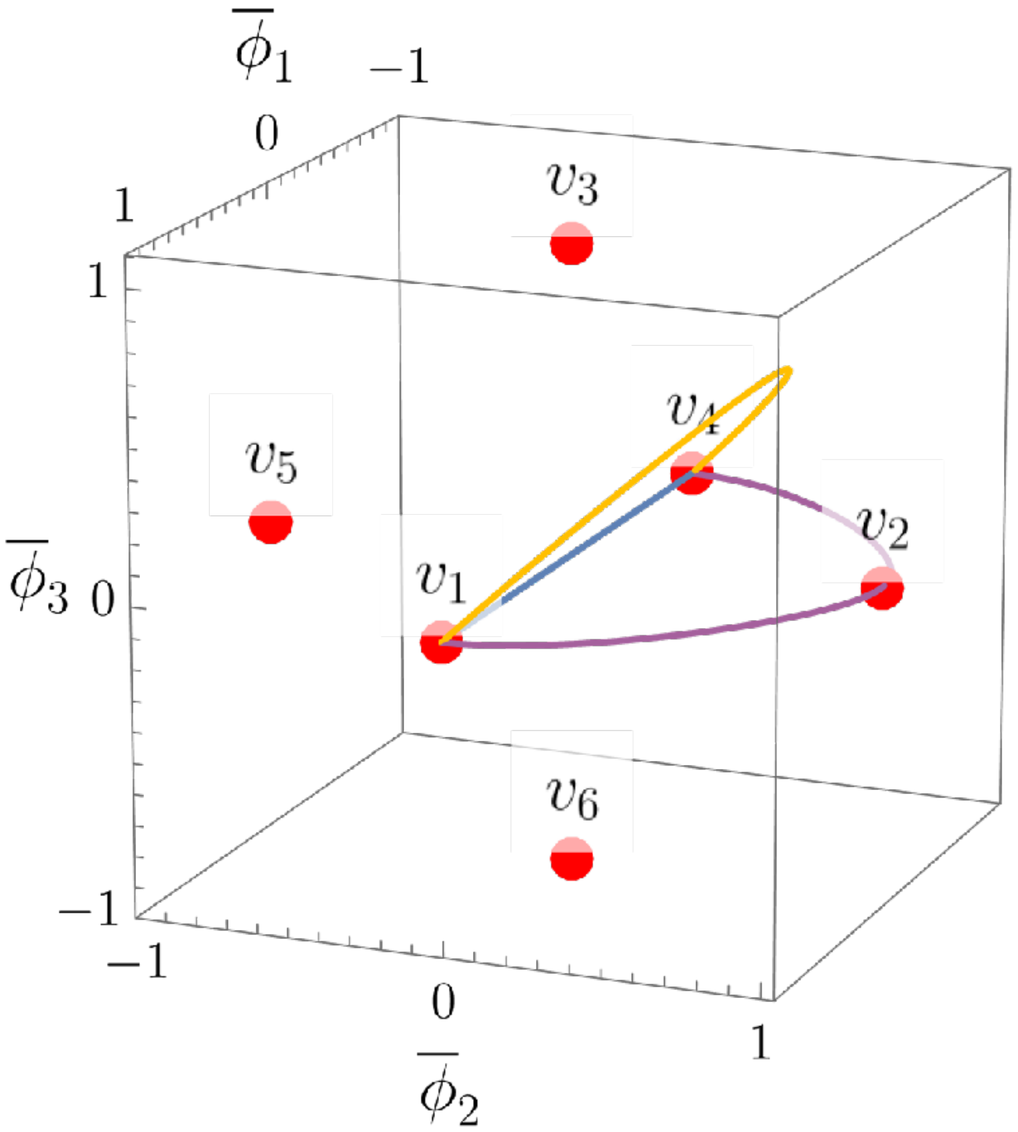
Distant wall patches are uncorrelated.
Long-wavelength stress power is flat.
Modes per log interval scale as f^3.

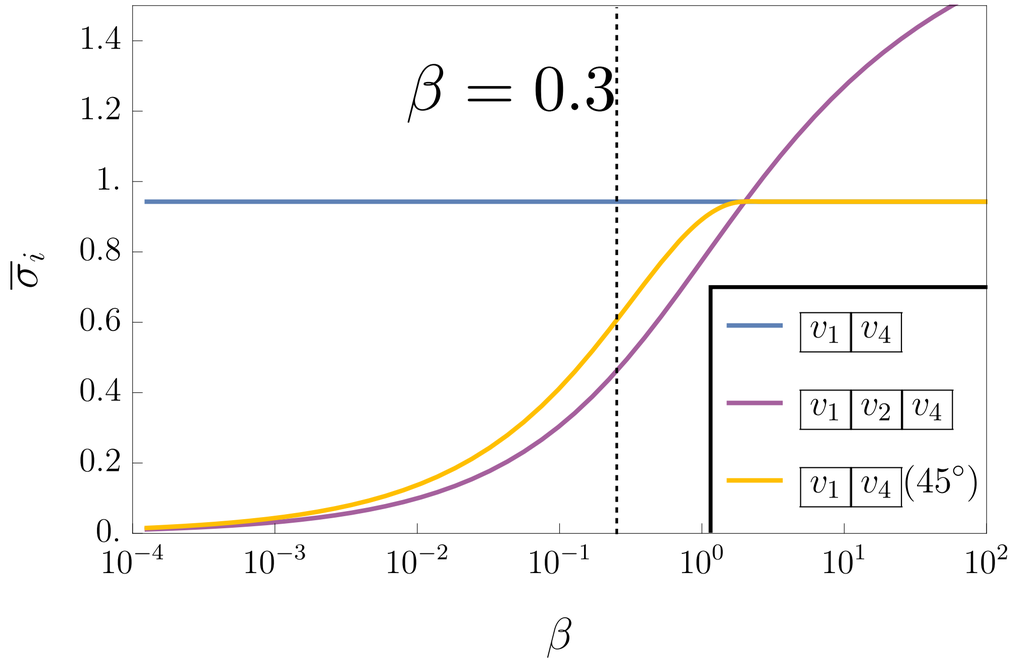
UV: approximately f^{-1}

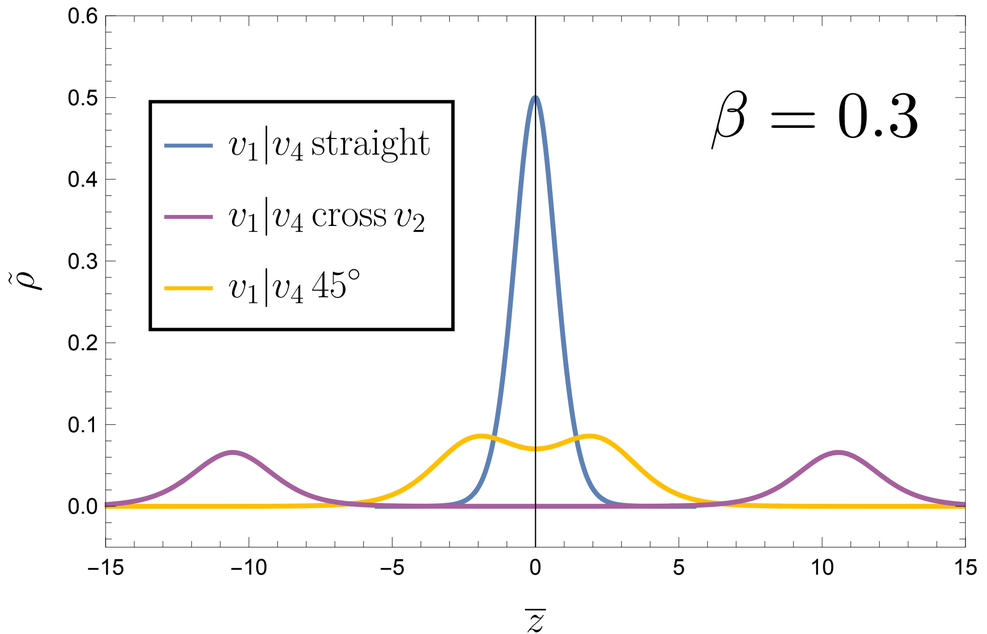
Higher frequencies probe smaller-scale wall structure and motion.
Simulations determine the falling tail.

Causality fixes the IR slope. The UV exponent depends on wall dynamics.

Stability of non-Abelian Domain Walls: $\mathbb{Z}_2$ case





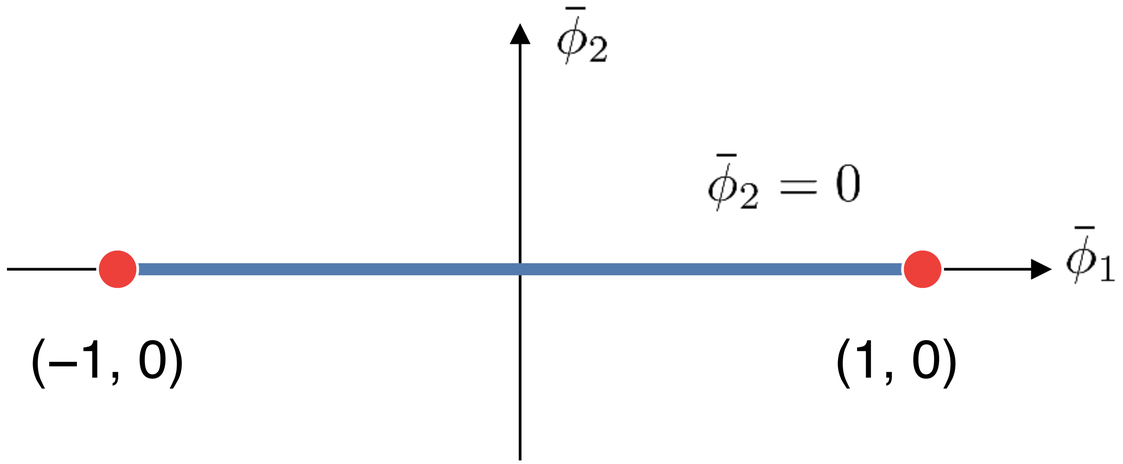


SI: a straight path in field space

Only one field varies; the other two remain zero.

$$\bar{\phi}_3 = 0$$

Opposite vacua



Why can $\bar{\Phi}_2$ stay zero?

There is no transverse slope in the potential on this axis:

$$\left.\frac{\partial U}{\partial \bar{\phi}_2}\right|_{\bar{\phi}_2=0} = 0$$

The β-dependent overlap term also vanishes when $\bar{\Phi}_2 = 0$.

Straight in field space

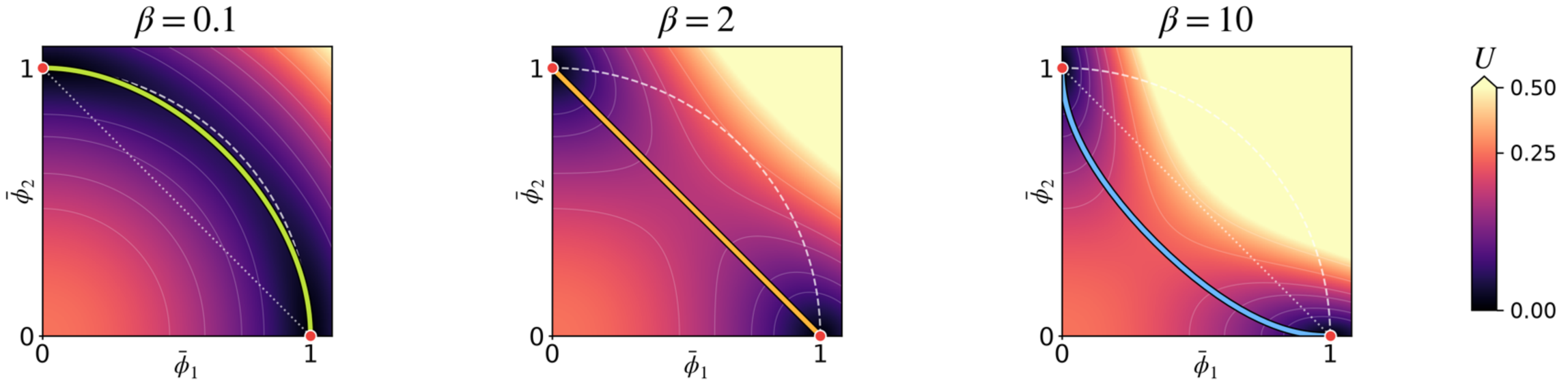
The spatial profile is still a tanh kink.

$$\bar{\phi}_1(\bar{z}) = \tanh(\bar{z}/\sqrt{2})$$

SII: why β bends the path

Both fields must change: (1, 0) → (0, 1).

$$U(\bar{\phi}_1, \bar{\phi}_2) = \underbrace{\frac{1}{4}(\bar{\phi}_1^2 + \bar{\phi}_2^2 - 1)^2}_{\text{favours } \bar{\phi}_1^2+\bar{\phi}_2^2=1} + \underbrace{\frac{\beta}{2}\bar{\phi}_1^2\bar{\phi}_2^2}_{\text{penalises field overlap}} \qquad \bar{\phi}_3 = 0$$



Overlap is cheap

Stays near the circle

Special case: β = 2

Exactly straight path

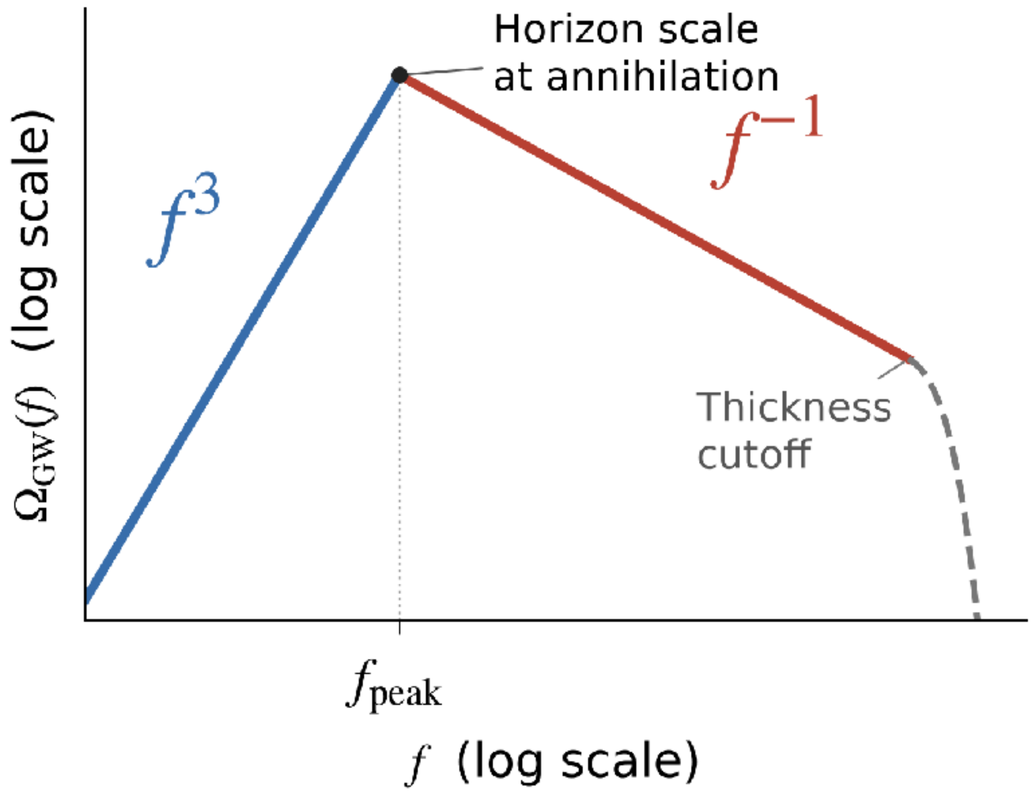
Overlap is costly

Bends towards the origin

Brighter = higher potential. Coloured paths solve the EoM, balancing gradient and potential energy.

Why does the GW spectrum rise and fall?

The two slopes have different physical origins.



Schematic; cutoff position is illustrative.

IR: f^3 rise

Long wavelength samples many independently moving wall regions. The larger the sampled volume, the smaller the squared average stress. In three dimensions, this suppression scales as $1/\lambda^3 \propto f^3$

UV: approximately f^{-1}

Higher frequencies probe smaller-scale wall structure and motion. Less power. Simulations determine the falling tail.

Causality fixes the IR slope. The UV exponent depends on wall dynamics.