Strong CP Revisited

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arXiv:2608.28253

Review: F. Sannino, arXiv:2601.19735

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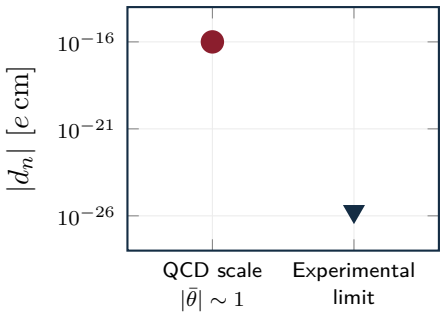
A very small angle with large implications

- QCD permits CP violation.
- d_n: neutron electric dipole moment.
- $\bar{\theta}$: physical QCD angle.

$$|d_n| \sim 10^{-16} |\bar{\theta}| \; e\,\mathrm{cm}$$

$$|\bar{\theta}| \lesssim 10^{-10}$$

Why is strong CP so small?



$|d_n| < 1.8 \times 10^{-26} \; e\,\mathrm{cm}$ (90% CL)

See J. de Vries (theory)
T. Langen (experiment)

QCD notation

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{q}\, i\gamma^\mu D_\mu q - (\bar{q}_R m q_L + \text{h.c.}) + \theta Q$$

$$\widetilde{G}^{a\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} G^a_{\rho\sigma} \qquad\qquad Q = \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu}$$

- $q_{L,R}$: chiral quarks.
- G: gluon field strength; $\widetilde{G}$: dual.
- Q: topological density.

Strong CP enters through θQ.

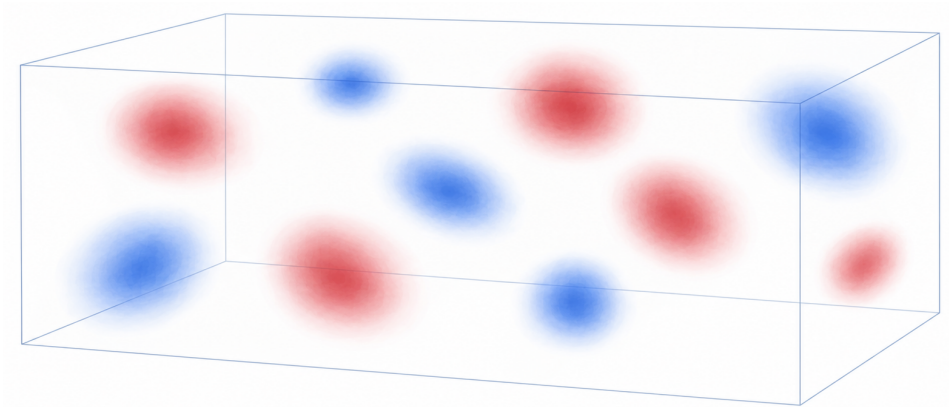
Topology makes a total derivative matter

$$\nu = \int d^4x\, Q_E \in \mathbb{Z} \qquad Z(\theta) = \sum_\nu e^{i\nu\theta} Z_\nu$$

- ν: winding number; Q_E: Euclidean topological density.
- Z_ν: fixed-sector partition function.
- θ: relative phase between topological sectors.

Topology makes θ physical.

Topological structure of the Yang–Mills vacuum



$Q(x) > 0$ $Q(x) < 0$

Schematic illustration: spatial slice of topological-charge density

The physical angle

$$\mathcal{L} \supset -(\bar{q}_R m q_L + \text{h.c.}) + \theta Q$$

- Axial rotations exchange mass and gluonic phases.

$$\bar{\theta} = \theta - \arg\det m$$

- Nonzero quark masses leave physical CP phase.
- Real positive m: $\theta = \bar{\theta}$
- In QCD alone, $\mu\, d\bar{\theta}/d\mu = 0$. M. Lüscher and P. Weisz, EPJ C **81**, 519 (2021)

Phases shift; $\bar{\theta}$ stays.

From θ to the $U(1)_A$ problem

A common axial rotation of all N_f flavours:

$$q_R \to e^{i\alpha} q_R, \qquad q_L \to e^{-i\alpha} q_L.$$

For the mass term $\bar{q}_R m q_L$, the compensating shifts are

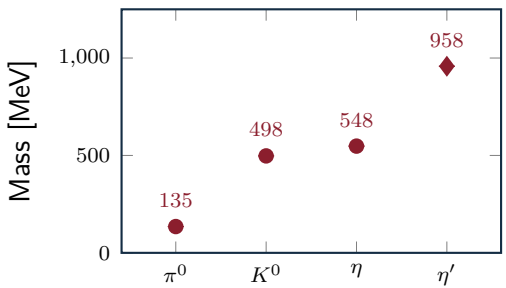
$$m \to e^{2i\alpha} m, \qquad \theta \to \theta + 2N_f\alpha.$$

$$\bar{\theta} \to (\theta + 2N_f\alpha) - (\arg\det m + 2N_f\alpha) = \bar{\theta}.$$

Massless quarks $\Rightarrow$ SSB of $U(1)_A$ $\Rightarrow$ a ninth GB?

The $U(1)_A$ problem is visible in the spectrum

- A light pseudoscalar octet.
- η': much heavier than the octet.

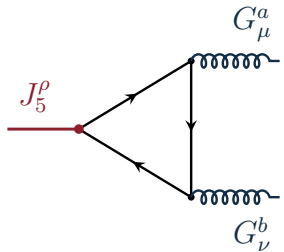


Even accounting for quark masses: no ninth pseudo-Goldstone boson.

The axial anomaly

$$J_5^\rho = \sum_i \bar{q}_i \gamma^\rho \gamma_5 q_i$$

$$\partial_\rho J_5^\rho = 2i \sum_i m_i \bar{q}_i \gamma_5 q_i + 2N_f Q$$



- For N_f massless quarks, the singlet axial current still obeys

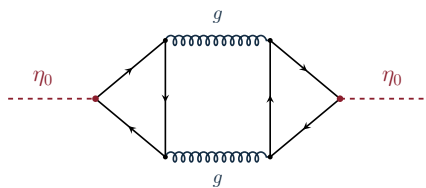
$$\partial_\rho J_5^\rho = 2N_f Q.$$

Massless quarks; anomaly survives.

The anomaly gives the flavor singlet a mass

Three flavor chiral limit

$$m_{\eta_0}^2 = \frac{6A}{F_\pi^2}$$

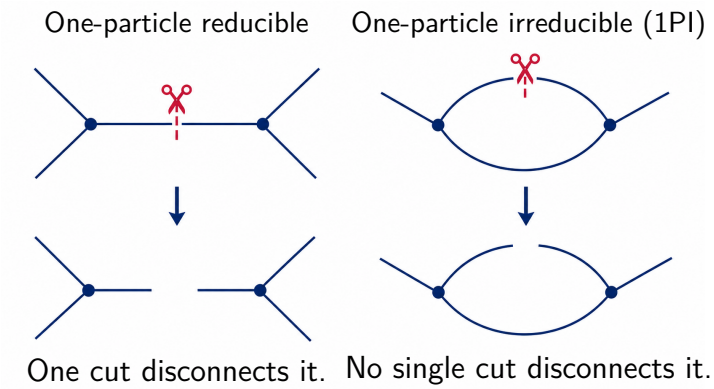


- A: anomaly coefficient.
- At physical quark masses, singlet–octet mixing produces η, η'.

Massless quarks; massive singlet.

Can QCD suppress strong CP and keep the singlet massive?

Full 1PI QCD action, beyond large N_c



One action, vacuum and spectrum.

QCD operators and their sources

Composite operators	Physical source parameters
$\mathcal{M}_{ij}(x) = \bar{q}_{R,i}(x) q_{L,j}(x)$	m
$Q(x) = \dfrac{1}{32\pi^2} G\widetilde{G}(x)$	θ

$$\Gamma[\mathcal{M}^{\rm cl}, Q^{\rm cl}; m, \theta] = \Gamma[\mathcal{M}^{\rm cl}, Q^{\rm cl}; 0, 0] + \int d^4x \left[-\,\mathrm{Tr}(m\mathcal{M}^{\rm cl} + \mathrm{h.c.}) + \theta\, Q^{\rm cl} \right]$$

$$\mathcal{M}^{\rm cl} = \langle \mathcal{M} \rangle_J \quad Q^{\rm cl} = \langle Q \rangle_J \qquad J: \text{ sources at fixed } m \text{ and } \theta.$$

Exact source dependence; controlled approximations.

1PI action constrained by the Anomalous Ward identity

From here on, $\mathcal{M}$ and Q denote the mean fields $\mathcal{M}^{\rm cl}$ and $Q^{\rm cl}$.

$$\Gamma[\mathcal{M},Q;0,0]=\Gamma_{\rm inv}[\mathcal{M},Q]-\frac{i}{2}\int d^4x\,Q\ \underbrace{\mathrm{Tr}\log(\mathcal{M}/\mathcal{M}^\dagger)}_{\text{Anomalous Ward identity}}\ .$$

Restoring the physical sources

$$\Gamma[\mathcal{M},Q;m,\theta]=\Gamma_{\rm inv}[\mathcal{M},Q]+\int d^4x\left\{-\mathrm{Tr}(m\mathcal{M}+\mathrm{h.c.})+\left[\theta-\frac{i}{2}\mathrm{Tr}\log(\mathcal{M}/\mathcal{M}^\dagger)\right]Q\right\}.$$

- $\Gamma_{\rm inv}$: general invariant action.

The anomaly constrains the action.

Derivative and potential terms

$$\Gamma = \Gamma_{\text{kin}} - V$$

- Γ_{kin}: derivative terms; V: total zero-momentum potential.

$$V(\mathcal{M}, Q) = V_{\text{inv}}(\mathcal{M}, Q) + \text{Tr}[m(\mathcal{M} + \mathcal{M}^\dagger)] - \left[\theta - \frac{i}{2}\text{Tr}\log(\mathcal{M}/\mathcal{M}^\dagger)\right] Q$$

- Small masses and small θ.
- Low energy.
- Derivative terms vanish for a constant vacuum.

The potential determines the vacuum.

Vacuum equations

Write $\langle \mathcal{M}_{ii} \rangle = H_i e^{i\langle \phi_i \rangle}$ and take $m_i > 0$.

$$\left.\frac{\partial V}{\partial \phi_i}\right|_{\text{vac}} = 0 \quad \Longrightarrow \quad 2H_i m_i \sin\langle \phi_i \rangle + \langle Q \rangle = 0$$

$$\left.\frac{\partial V}{\partial Q}\right|_{\text{vac}} = 0 \quad \Longrightarrow \quad \theta + \sum_i \langle \phi_i \rangle = \left.\frac{\partial V_{\text{inv}}(H_i, Q)}{\partial Q}\right|_{Q=\langle Q \rangle}$$

- Leading small-mass order: $H_i \equiv \langle H_{ii} \rangle = -|H|$

Phases and topology adjust together.

From vacuum curvature to topological susceptibility

Expand about $Q = 0$ at fixed CP-conserving values of H_i.

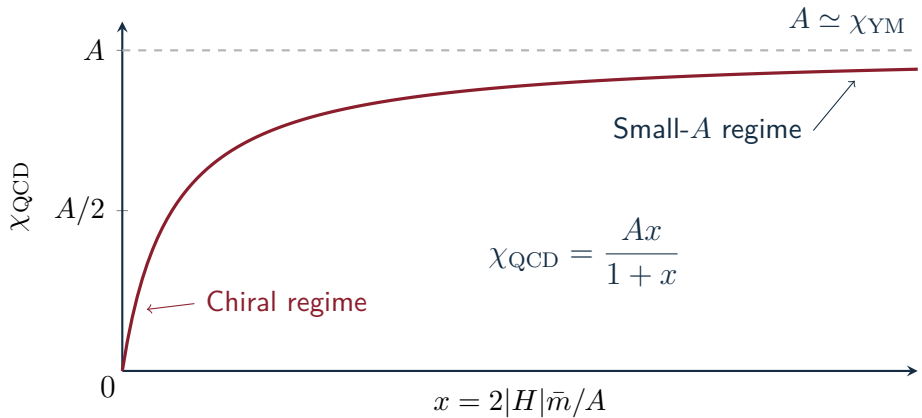
$$\left.\frac{\partial^2 V_{\rm inv}}{\partial Q^2}\right|_{Q=0} = -\frac{1}{A} \qquad \Longrightarrow \qquad \frac{\partial V_{\rm inv}}{\partial Q} = -\frac{Q}{A} + \mathcal{O}(Q^3).$$

Eliminating $\langle \phi_i \rangle$ from the linearized vacuum equations gives

$$\langle Q \rangle = -\chi_{\rm QCD}\, \bar{\theta} + \mathcal{O}(\bar{\theta}^3) \quad \text{with} \quad \chi_{\rm QCD} = \frac{A\, 2|H|\bar{m}}{A + 2|H|\bar{m}}, \qquad \frac{1}{\bar{m}} = \sum_i \frac{1}{m_i}.$$

Susceptibility $\chi_{\rm QCD}$ measures vacuum response.

QCD susceptibility

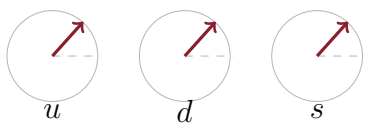


Small masses or small A $\Rightarrow$ small χ_{QCD}.

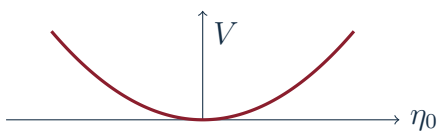
Why does the same A set the η_0 mass?

$$V_{\text{anomaly}} \simeq \frac{A}{2}(\theta+\varphi)^2, \quad \text{where } \varphi = \textstyle\sum_i \phi_i \text{ is the total quark phase.}$$

The η_0 mode



Energy cost at $\theta = 0$



$$\delta\varphi = \sum_i (\phi_i - \langle\phi_i\rangle) \simeq \sqrt{2N_f}\,\eta_0/F_\pi$$

$$m^2_{\eta_0,\text{anomaly}} \simeq \frac{2N_f\,A}{F_\pi^2}$$

Same A, response and mass.

The tension between strong CP and $U(1)_A$

Theorem. At fixed small $\bar{\theta} \neq 0$, CP suppression requires one of two limits.

A vanishingly light quark

$$m_{\text{lightest}} \to 0$$

Current algebra excludes this limit.

A vanishing anomaly mass

$$A \to 0 \;\Rightarrow\; m^2_{\eta_0,\text{anomaly}} \to 0$$

The $U(1)_A$ problem returns.

P. Di Vecchia, F. Sannino, G. M. Shore, G. Veneziano and S. Yankielowicz, arXiv:2608.28253.

One more thing...

At small $\bar{\theta}$, the vacuum responds through the QCD susceptibility:

$$\langle Q \rangle = -\chi_{\mathrm{QCD}}\,\bar{\theta} + \mathcal{O}(\bar{\theta}^3).$$

The same response fixes the universal mixed-flavour CP-odd interaction:

$$V_{\text{CP-odd}} \;\supset\; -\frac{\langle Q \rangle}{6}\,\mathrm{Tr}(\Phi^3) \;\simeq\; \frac{\chi_{\mathrm{QCD}}\,\bar{\theta}}{6}\,\mathrm{Tr}(\Phi^3).$$

Here Φ is the meson field matrix. Its cubic trace contains $\eta_8\,\pi^+\pi^-$.

Vacuum response fixes CP-odd interactions.

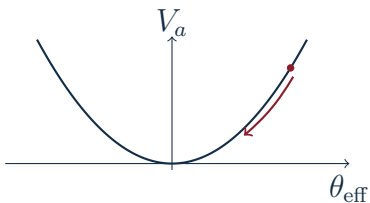
No strong CP: claims and implications

Paper	Claim	Implication
Ai, Cruz, Garbrecht, Tamarit 2001.07152	$V \to \infty$ before topology sum	$\theta - Q$ source identity lost.
Nakamura, Schierholz 2106.11369	IR screening via gradient flow	Compatible with QCD $\bar{\theta}$?
Strocchi 2404.19400	Mass selects $\bar{\theta} = 0$	θ set by vacuum selection.

Each proposal revisits a premise of the QCD analysis.

What if the angle itself can relax to zero?

The Peccei–Quinn axion a makes $\theta_{\text{eff}} = \bar{\theta} + \dfrac{a}{f_a}$ dynamical.



At physical quark masses, $A \neq 0$ still gives the singlet its anomaly mass.

Axion relaxation preserves axial anomaly.

Conclusions

Theorem. At fixed small $\bar{\theta} \neq 0$, strong CP suppression requires a vanishing quark mass or a vanishing singlet anomaly mass.

- Also valid at large N_c and in the 't Hooft one instanton picture.
- Claims of no strong CP require abandoning standard QCD premises.
- The axion relaxes the angle while preserving the anomaly.
- Outlook: finite-θ vacua, spectra and $\mathbb{CP}^N$ theory.


Thank you!

Backup slides

Why a permanent neutron dipole follows spin

- Isolated neutron at rest: no applied field, no external preferred direction.
- Within its spin-$\frac{1}{2}$ ground state, every vector operator is proportional to spin.

$$\mathbf{d}_{\mathrm{perm}} = d_n \boldsymbol{\sigma} \qquad \boldsymbol{\sigma} = \frac{2\mathbf{S}}{\hbar}$$

- d_n: permanent neutron electric dipole moment.

External field: probe a dipole or induce one

$$\mathbf{d}_{\text{perm}} = d_n \boldsymbol{\sigma} \qquad \mathbf{d}_{\text{ind}} = \alpha_E \mathbf{E}$$

- α_E: electric polarizability.
- $\mathbf{E}$: applied electric field.
- Permanent: follows spin; induced: follows the field, vanishes at zero field.

$$H_{\text{electric}} = -d_n \boldsymbol{\sigma} \cdot \mathbf{E} - \frac{1}{2} \alpha_E |\mathbf{E}|^2$$

Why an electric dipole tests CP

$$H_{\mathrm{EDM}} = -d_n \, \boldsymbol{\sigma} \cdot \mathbf{E}$$

Transformation	$\mathbf{E}$	$\boldsymbol{\sigma}$	$\boldsymbol{\sigma} \cdot \mathbf{E}$
Parity P	$-$	$+$	$-$
Time reversal T	$+$	$-$	$-$

- EDM violates P and T; CPT implies CP violation.

1PI action

$$e^{iW[J]} = \int \mathcal{D}\Psi \, e^{\,iS[\Psi] + i\sum_i \int d^4x \, J_i O_i}$$

- Ψ: microscopic fields; O_i: gauge-invariant composites; J_i: sources.

$$O_i^{\rm cl} = \frac{\delta W}{\delta J_i} \qquad \Gamma[O^{\rm cl}] = W[J] - \sum_i \int d^4x \, J_i O_i^{\rm cl}$$

- W: connected generating functional; $O_i^{\rm cl}$: expectation values.

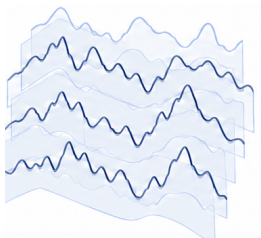
- Γ: full one-particle-irreducible action, including quantum effects.

Sources and expectation values

Microscopic fields Ψ

$W[J]$: response to sources

$J_i(x)$



Quantum expectation



Sources couple to $O_i[\Psi]$.

$$O_i^{\mathrm{cl}}(x) = \langle O_i(x) \rangle_J = \frac{\delta W}{\delta J_i(x)}$$

Quantum fluctuations included.

Given the source, find the expectation value.

$\langle O_i \rangle_J$: expectation value with external sources present.

Legendre transform: expectation values as variables

Legendre transform

$W[J]$ → $\Gamma[O^{\mathrm{cl}}]$

Source as variable

Expectation value as variable



$J_i(x)$

$O_i^{\mathrm{cl}}(x)$

$$\frac{\delta\Gamma[O^{\mathrm{cl}}]}{\delta O_i^{\mathrm{cl}}(x)} = -J_i(x)$$

Given the expectation value,
find the source that sustains it.

For constant expectation values, stable vacua minimize the effective potential.

Auxiliary sources and physical parameters

J: auxiliary probe, varied in the Legendre transform.

λ: fixed physical coupling; in QCD, supplied by m and θ.

$$W[J;\lambda] = W_0[J+\lambda], \qquad K = J + \lambda \quad \Longrightarrow \quad J = K - \lambda$$

$$\Gamma[O^{\rm cl};\lambda] = W_0[K] - \int \Big(K - \lambda\Big) O^{\rm cl} = \underbrace{W_0[K] - \int K O^{\rm cl}}_{\Gamma_0[O^{\rm cl}]} + \int \lambda\, O^{\rm cl}$$

Switching off the probe: $J = 0 \implies K = \lambda$.

Renormalized QCD: impose the Ward identities

- $m\mathcal{M}$: renormalization-group invariant product.
- In QCD alone, $\mu\, d\bar{\theta}/d\mu = 0$.
- The local density $Q(x)$ can mix with $\partial_\mu J_5^\mu$.
- Impose (non)-anomalous chiral Ward identities on Γ.

With weak interactions. CKM violation induces a tiny finite shift in $\bar{\theta}$. Logarithmic running is expected first at seven loops.

Ellis, Gaillard, Nucl. Phys. B 150 (1979) 141. Khriplovich, Vainshtein, hep-ph/9308334.
Shore, Veneziano, Nucl. Phys. B 381 (1992) 3.
Espriu, Tarrach, Z. Phys. C 16 (1982) 77; Lüscher, Weisz, Eur. Phys. J. C 81 (2021) 519.
Adler, Bardeen, Phys. Rev. 182 (1969) 1517; Larin, Phys. Lett. B 303 (1993) 113.

Linearity lemma and assumptions

Linearity lemma

At fixed $\mathcal{M}$ and Q, the 1PI action Γ is linear in quark mass and θ.

Assumptions

- Mass-independent renormalization preserving Ward identities
- Small quark masses and vacuum angle.
- Analyticity near zero topological density.
- Mass gap above the pseudoscalar sector.

Integer topology gives exact periodicity

$$Z(\theta + 2\pi) = \sum_{\nu \in \mathbb{Z}} e^{i\nu\theta} \underbrace{e^{2\pi i \nu}}_{1} Z_\nu = Z(\theta)$$

- ν: integer topological charge; Z_ν: fixed-charge partition function.

$$E(\theta) = -\lim_{V_4 \to \infty} \frac{\log Z(\theta)}{V_4} \quad \Longrightarrow \quad E(\theta + 2\pi) = E(\theta)$$

- E: vacuum energy density; V_4: Euclidean spacetime volume.
- Integer topological charge makes the vacuum energy 2π-periodic.

Pure-glue susceptibility: define the curvature

- Pure Yang–Mills (YM): gluons, no dynamical quarks.

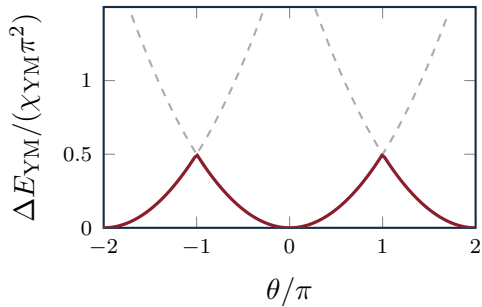
$$\chi_{\rm YM} \equiv \left.\frac{\partial^2 E_{\rm YM}}{\partial\theta^2}\right|_{\theta=0} \qquad [\chi_{\rm YM}] = \text{mass}^4$$

$$\Delta E_{\rm YM}(\theta) \equiv E_{\rm YM}(\theta) - E_{\rm YM}(0) = \frac{1}{2}\chi_{\rm YM}\theta^2 + O(\theta^4)$$

- $\chi_{\rm YM}$: topological susceptibility; $E_{\rm YM}$: YM vacuum energy density.

Vacuum branches: large-N_c illustration

$$\Delta E_{\mathrm{YM}}(\theta) \simeq \min_{k\in\mathbb{Z}} \frac{\chi_{\mathrm{YM}}}{2}(\theta + 2\pi k)^2$$



- k: integer vacuum-branch label.
- Dashed: individual branches; dark red: lowest energy at each θ.
- $\theta \to \theta + 2\pi$ relabels branches; illustrative formula.

Vacuum alignment minimizes the energy

$$E_{\rm vac}(\theta) = \min_{\{\phi_i\}} E(\phi_i;\theta)$$

$$\left.\frac{\partial E}{\partial \phi_i}\right|_{\rm vac} = 0 \quad \Longrightarrow \quad F_\pi^2 B_0 m_i \sin\langle\phi_i\rangle = -A\left(\theta + \sum_j \langle\phi_j\rangle\right)$$

- A: anomaly curvature (mass^4).
- Mass term favors $\phi_i = 0$; anomaly favors $\sum_i \phi_i = -\theta$
- Massive quarks prevent complete screening.

Topological susceptibility measures vacuum curvature

$$E_{\mathrm{vac}}(\theta) = \min_{\phi_i} E(\phi_i;\theta) \qquad \chi_{\mathrm{QCD}} = \left.\frac{d^2 E_{\mathrm{vac}}(\theta)}{d\theta^2}\right|_{\theta=0}$$

- E_{vac}: minimized vacuum energy density.
- χ_{QCD}: QCD topological susceptibility.
- $\langle \nu^2 \rangle_c$: connected variance of the topological charge.

$$\chi_{\mathrm{QCD}} = \lim_{V_4 \to \infty} \frac{\langle \nu^2 \rangle_c}{V_4} \qquad (\theta = 0)$$

Different anomaly potentials, the same local test

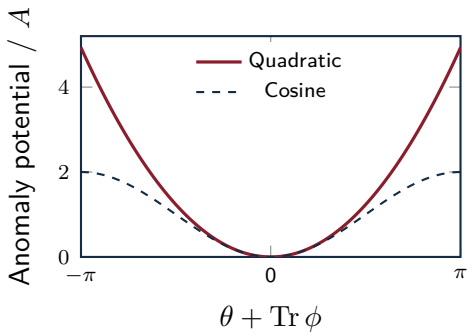
With radial fields frozen; additive constants omitted:

Conventional large N_c

$$\frac{A}{2}(\theta + \operatorname{Tr}\phi)^2 \quad \text{locally}$$

Determinant-type interaction

$$A\Big[1 - \cos(\theta + \operatorname{Tr}\phi)\Big]$$



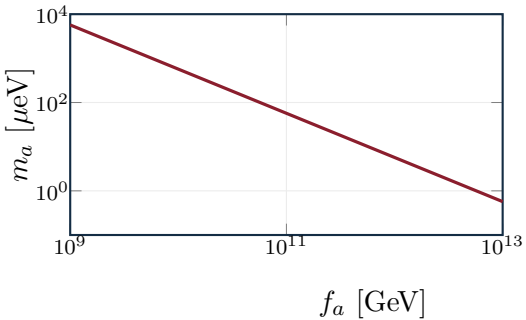
- Same curvature A at $\theta + \operatorname{Tr}\phi = 0$; different higher couplings.

The curvature at the minimum sets the axion mass

$$m_a^2 = \left.\frac{d^2 V_a}{da^2}\right|_{\theta_{\rm eff}=0} = \frac{\chi_{\rm QCD}}{f_a^2} \simeq \frac{m_\pi^2 F_\pi^2}{f_a^2}\frac{m_u m_d}{(m_u+m_d)^2}$$

Physical quark masses

$$m_a \simeq 5.70\,\mu{\rm eV}\left(\frac{10^{12}\,{\rm GeV}}{f_a}\right)$$



What the axion mechanism still has to explain

Extra PQ breaking $\Delta V(a)$ shifts the QCD minimum a_0:

$$\delta\theta_{\text{eff}} \simeq -\frac{f_a \Delta V'(a_0)}{\chi_{\text{QCD}}}$$

- Axion quality: keep this residual angle below the EDM bound.
- Cosmology: abundance, strings, domain walls.