3D Pen for Teaching Monge's Projection

Using a 3D Pen for Teaching Monge's Projection in Secondary Schools

Introduction

When learning about Monge's projection – which is based on orthogonal projections on two mutually perpendicular planes – students often encounter difficulties with spatial visualisation. It is particularly challenging to understand the relationships between a three-dimensional object and its two-dimensional representation in a drawing, as well as to interpret the interrelationships between the individual projections.

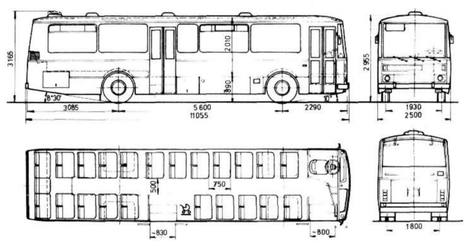


Figure 1. Monge's projection use
Source: https://fpedas.uniza.sk/ vojtek/04oPredn.pdf

A teaching approach using a 3D pen enables constructions to be created directly in three-dimensional space. Psychological research (Hall and Nemirovsky [1]) shows that tactile-kinesthetic learning is highly effective, particularly when it involves manipulating objects. A study in the article by Ng et al.[2] also suggests that visual technologies (such as GeoGebra) promote rapid learning, whilst 3D pens contribute to a deeper and more lasting obtaining of geometric knowledge.

3D pen tool

A 3D pen that supports *learning by making* [3] allows to learn the basics of 3D modelling without the need for a computer. It works by melting a plastic filament, which hardens quickly after flowing out of the tip. The advantage is that this device is affordable (from €15 – 30) and easy to use also for children.



Figure 2. 3D pen and filaments
Source: https://www.tvojsen-mido.sk/3d-pen-magic-pen-3d-refills-za3932-blue

Set of exercises

We have developed transparent projection screens, drawing instruments and a set of exercises on Monge projection using a 3D pen in three-dimensional space. The exercises increase progressively in difficulty, covering the representation of points, lines and planes in both general and special positions. The set also includes an advanced task in which students reconstruct a 3D model of a building from its front, top, and side views, using three projection planes.

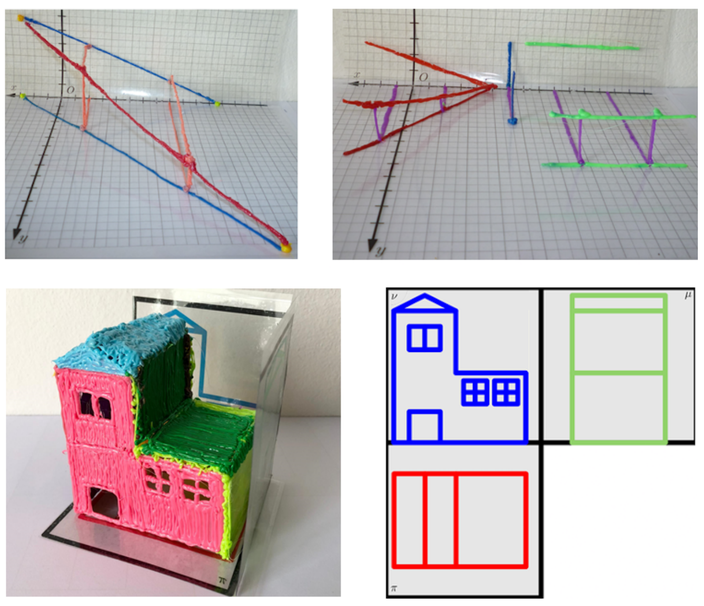


Figure 3. A line in general and special positions, a building view in three projections

A comparison of approaches

3D models allow students to compare both views of spatial constructions with traditional projections, verify the correspondance and recognise that lines appearing to intersect in a drawing may not actually intersect in space.

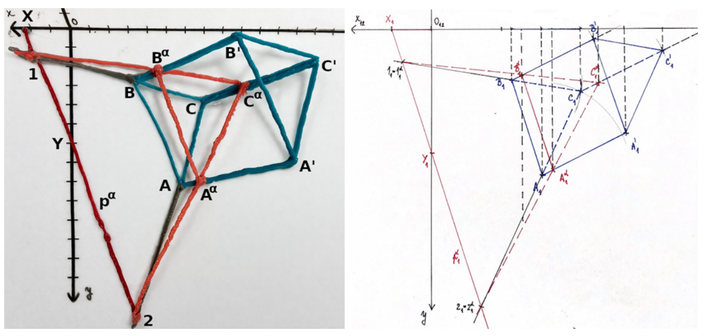


Figure 4. A top-down view of the plane section

Construction of a plane section

In the following exercise, we focus on constructing a plane section of a prism and on illustrating the principle of perspective affinity between two planes in space, which enables a better understanding of axial affinity as applied in a plane. 3D construction allows students to actively interact with the model by rotating it, changing viewing angles and exploring spatial relationships from different perspectives. Moreover, the involvement of movement, gestures and touch during construction strengthens the connection between motor activity and geometric understanding [1], reflecting a constructivist approach to learning [3].

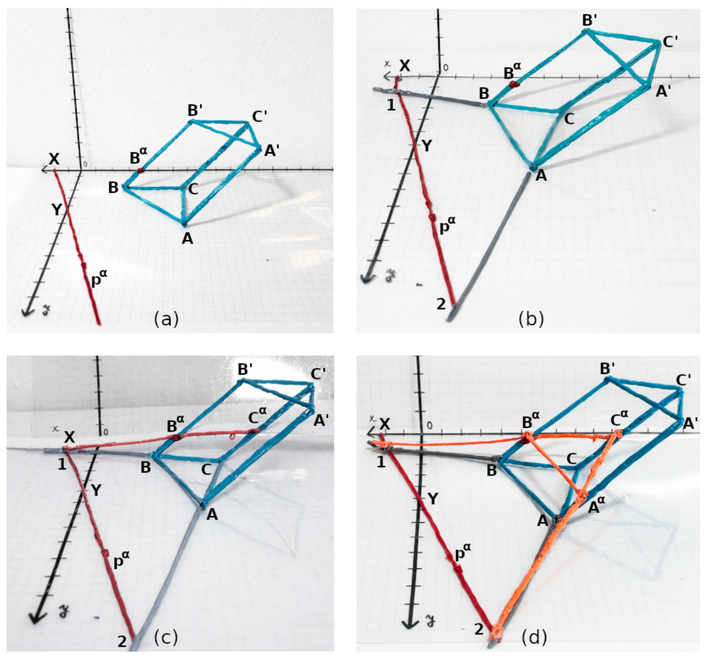


Figure 5. Construction step by step

References

[1] Rogers Hall and Ricardo Nemirovsky.
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