3D Printing Technologies in Spatial Geometry Education

Exploring Cross-Sections of a Cube using 3D pen

Why do constructions in 3D?

Cube cross-sections require students both to interpret 2D projections and to reason in 3D. Physical models make spatial relationships tangible, connecting spatial reasoning with embodied making [1, 2].

Our aim: explore classroom feasibility of 3D pens incorporation, changes in cross section construction performance, facilitating spatial imagination improvement also in individuals with self-reported issues in this area.

The three cross-section construction rules

1. **Same face:** join two section points lying on the same cube face.
2. **Parallel faces:** section edges on parallel cube faces are parallel.
3. **Concurrency:** for three pairwise non-parallel planes sharing a point, their pairwise intersection lines meet at that point.

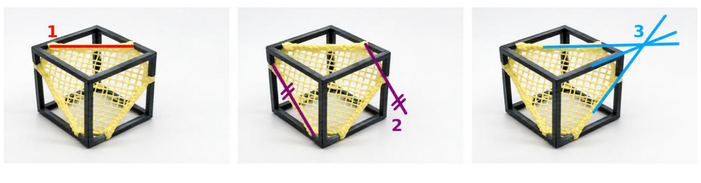


Figure 1. The three rules made visible on a physical model.

An auxiliary plane makes the most demanding constructions possible, but adds further spatial reasoning challenges.

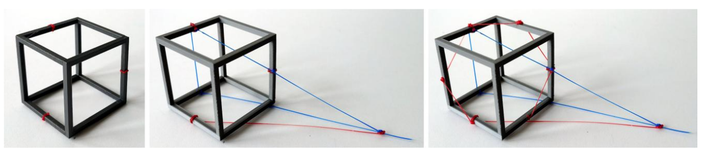


Figure 2. From three marked points to a cross-section using an auxiliary plane (blue).

Classroom study

Participants: 19 fourth-year secondary school students, one year after conventional cross-section instruction and without repetition.

- **Pre-test:** five increasingly difficult paper-based tasks; no manipulatives.
- **Intervention:** two 45-minute lessons in pairs using 3D pens with short prior instruction; four instructors and the mathematics teacher present.
- **Post-test:** analogous tasks the following day; a perception questionnaire immediately after the intervention, followed by a short discussion.

Three independent coders agreed on final task scores. The primary outcome was grading of Tasks 1–4 (0–30 points). Task 5 covered content new to some students and was reported separately.

From paper to model and back

Materials: pre-printed PLA wireframe cubes (60 mm edges, 4 mm edge thickness), 3D pens with ABS filament, markers, paper and rulers.

1. **Locate and construct.** Match the three points in the drawing to the cube, mark them, and build the section edges using the geometric rules.

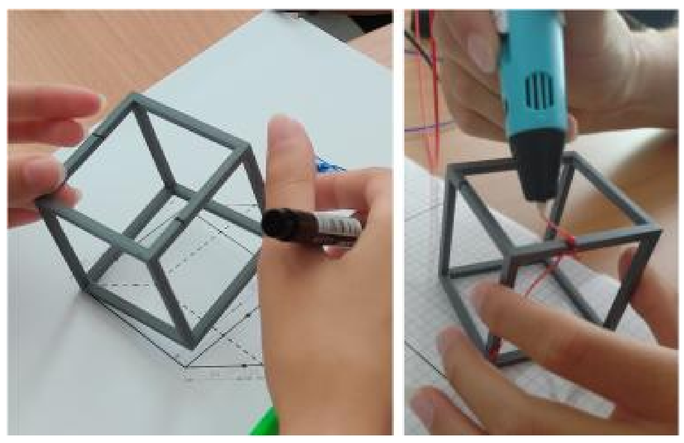


Figure 3. Transferring points to the model and drawing in space.

2. **Rotate and inspect.** Examine the model from different angles; use touch and changes of viewpoint to check parallelism and distinguish skew from intersecting lines.

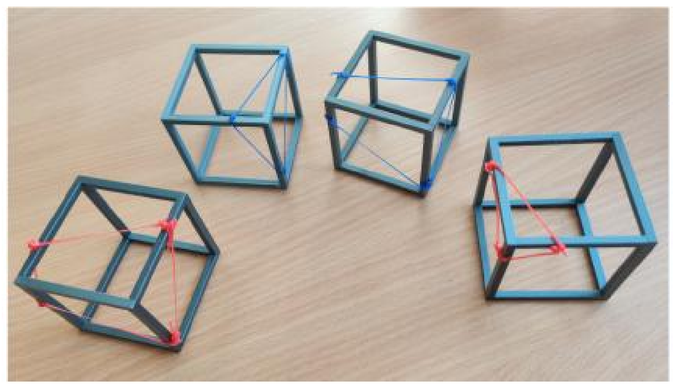


Figure 4. Cross-sections constructed by students.

3. **Return to paper.** Construct the same cross-section in 2D, using the physical model to verify spatial relationships.

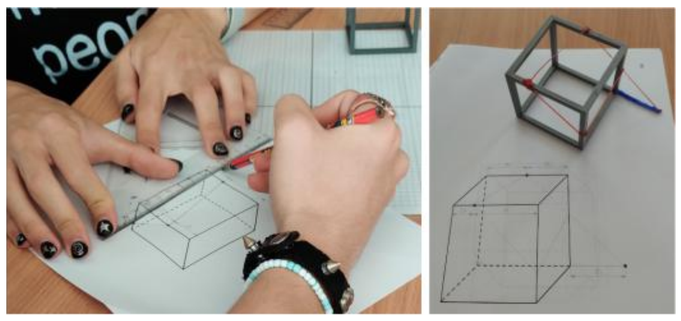


Figure 5. Pair work and the link between a model and its projection.

Pre/post-test results

Primary outcome: Tasks 1–4, n=19.
18.11 / 30 ⟶ 22.00 / 30

Pre-test mean (SD 6.20) Post-test mean (SD 5.96)

Mean paired gain: 3.89 points, 95% CI [1.33, 6.45]; paired $t(18) = 3.20$, $p = .005$; Cohen's $d_z = 0.73$.

Task-level results

	Task 3 (0–9)	Task 4 (0–11)
Pre-test mean	4.84	4.37
Post-test mean	4.95	7.16
Higher / same / lower	7 / 6 / 6	14 / 3 / 2
p_{Holm}	1.000	.008

Exact two-sided sign tests; Holm adjustment across Tasks 3 and 4.

Task 4 showed a clear improvement. Task 3 showed no consistent change, probably due to the post-test construction being particularly sensitive to drawing accuracy; Tasks 1 and 2 showed ceiling effects.

Auxiliary-plane task (Task 5): all students scored zero before the intervention; afterwards, 3 scored the full 15 points and 16 still scored zero.

Feasibility and student experience

- All five activity tasks were completed in two regular 45min lessons. The task involving an auxiliary plane needed a short introduction.
- After a demonstration, pairs worked independently with support available on demand.
- All 3D pen novices reported confidence using the tool independently afterwards.
- Most reported better understanding, describing the activity as "helpful", "interesting" and "active".
- Concerns included time and initial costs.

Next steps and takeaway

- Exploratory study: one class, no control or retention test. Pair work and instructor support may have made gains dependent; pairs were not recorded. No causal inference is possible.
- Next: larger, controlled studies with retention tests.
- **3D construction and rotation may help students connect spatial relationships with 2D drawings.**

Selected references:
[1] Fujita et al. (2020). **Spatial reasoning skills about 2D representations of 3D geometrical shapes in grades 4 to 9.** *Math. Educ. Res. J.*, 32, 235–255.
[2] Ng & Ye (2022). **Mathematics learning as embodied making: Primary students' investigation of 3D geometry with handheld 3D printing technology.** *Asia Pacific Educ. Rev.*, 23, 311–323.

Figures: accompanying technical report.
Problem set: see QR ⟶

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