Numerical Solution of the Two-Dimensional Bates Model
Compact Finite Differences and Finite Element Comparison

MatFyz CONNECTIONS 2026 | Numerical methods for European put option pricing

Why the Bates model?

Equity prices can experience sudden jumps while market volatility varies over time. The Bates model combines a Heston-type stochastic variance process with compound Poisson price jumps. This produces a two-dimensional partial integro-differential equation (PIDE), motivating accurate and efficient numerical solvers.

Risk-neutral dynamics

$$\begin{aligned} dS_t &= (r-\lambda\xi)S_t\,dt + \sqrt{v_t}S_t\,dW_t^S \\ &\quad + S_{t^-}(e^Z-1)\,dN_t, \\ dv_t &= \kappa(\theta - v_t)\,dt + \sigma\sqrt{v_t}\,dW_t^v, \\ d\langle W^S, W^v\rangle_t &= \rho\,dt. \end{aligned}$$

Here $Z \sim \mathcal{N}(\mu_J, \sigma_J^2)$, N_t has intensity λ, and $\xi = \mathbb{E}[e^Z - 1]$. The parameters κ, θ, σ describe variance mean reversion and vol-of-vol.

Transformed option-pricing PIDE

Set $\tau = T - t$, $x = \log(S/K)$, $y = \sigma/v$, and

$$u(x, y, \tau) = \frac{e^{r\tau}}{K} V(S, v, t).$$

The transformed European put problem reads

$$u_\tau = \mathcal{L}u + \mathcal{I}u, \qquad u(x, y, 0) = (1 - e^x)^+,$$

where the local operator is

$$\begin{aligned} \mathcal{L}u &= \frac{\sigma}{2y}u_{xx} + \frac{\sigma y^3}{2}u_{yy} - \rho\sigma y\, u_{xy} \\ &\quad + \left(r - \lambda\xi - \frac{\sigma}{2y}\right)u_x \\ &\quad + \left(\sigma y^2 + \kappa y - \frac{\kappa\theta}{\sigma}y^2\right)u_y, \end{aligned}$$

with nonlocal jump operator

$$\mathcal{I}u = \lambda \int_{\mathbb{R}} [u(x + z, y, \tau) - u(x, y, \tau)] f(z)\,dz.$$

Representative test parameters

K	110	T	1
r	0.03	κ	1.8
θ	0.02	ρ	-0.4
σ	0.15	λ	0.25

Jump parameters μ_J and σ_J depend on the experiment.

Compact finite differences (HOC-FD)

The local operator is discretized using compact fourth-order derivative relations at interior grid points. For example,

$$\frac{1}{4}(u_x)_{i-1,j} + (u_x)_{i,j} + \frac{1}{4}(u_x)_{i+1,j} = \frac{3}{4h_x}(u_{i+1,j} - u_{i-1,j}),$$

$$\frac{1}{12}(u_{xx})_{i-1,j} + \frac{10}{12}(u_{xx})_{i,j} + \frac{1}{12}(u_{xx})_{i+1,j} = \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h_x^2}.$$

Analogous relations apply in y. Mixed derivatives are obtained by successive compact differentiation. The local scheme has interior truncation error $O(h_x^4 + h_y^4)$ for smooth solutions.

Jump quadrature and time stepping

The jump integral uses Simpson quadrature on a truncated domain:

$$(\mathcal{I}_h u)_{ij} = \lambda \sum_{m=-M}^{M} w_m f(z_m)[u(x_i + z_m, y_j) - u_{ij}].$$

Crank–Nicolson treats the local operator implicitly, while Adams–Bashforth treats jumps explicitly (IMEX-CNAB):

$$\boxed{\begin{aligned}(I - \tfrac{k}{2}L_h)U^{n+1} &= (I + \tfrac{k}{2}L_h)U^n \\ &\quad + k\big(\tfrac{3}{2}I_h U^n - \tfrac{1}{2}I_h U^{n-1}\big).\end{aligned}}$$

This separates the local stiff dynamics from the nonlocal jump evaluation.

Finite element benchmark

A continuous quadratic FEM (P_2 triangles) provides an independent comparison. The Crank–Nicolson/Adams–Bashforth discretization produces

$$\begin{aligned} \mathcal{A}_h U^{n+1} &= \mathcal{B}_h U^n + k\big(\tfrac{3}{2}F^n - \tfrac{1}{2}F^{n-1}\big), \\ \mathcal{A}_h &= M + \tfrac{k}{2}(K + C), \\ \mathcal{B}_h &= M - \tfrac{k}{2}(K + C), \end{aligned}$$

where M, K, and C are the mass, stiffness, and convection matrices.

Observed convergence

- HOC-FD: near-fourth-order observed spatial convergence under the reported coupled refinements.
- FEM(P_2) and standard FD: approximately second-order spatial behavior in the reported tests.
- Time-stepping methods: approximately second-order temporal convergence.

European put prices

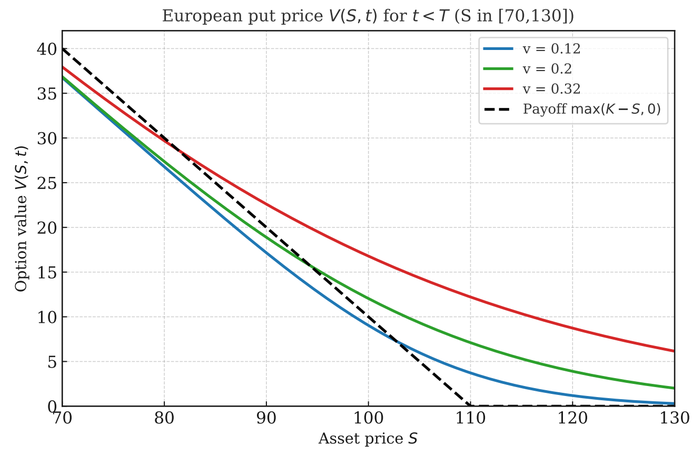


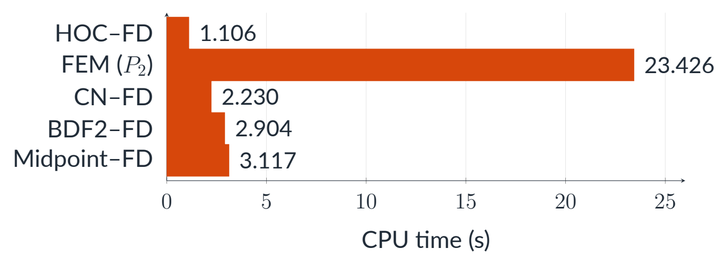
Figure 1: European put prices for three variance levels; dashed line shows the intrinsic payoff. Figure from the supplied ICCF slides.

Error and runtime comparison

Method	DOF	CPU (s)	L^2 error
HOC-FD	1,681	**1.106**	**0.0230**
FEM (P_2)	6,561	23.426	0.1522
CN-FD	1,681	2.230	0.0341
BDF2-FD	1,681	2.904	0.0417
Midpoint-FD	1,681	3.117	0.0425

Under the reported settings, HOC-FD achieves the smallest error and shortest CPU time among these schemes. The comparison uses the model and numerical settings in the source slides.

CPU time by method



Conclusions

- A compact spatial scheme and an IMEX-CNAB integrator solve the transformed Bates PIDE.
- Simpson quadrature evaluates the nonlocal jump contribution.
- Numerical refinements show near-fourth-order observed spatial behavior for HOC-FD and approximately second-order temporal behavior.
- In the reported test, HOC-FD offers the strongest accuracy–runtime tradeoff.

Selected references

[1] D. S. Bates (1996), *Review of Financial Studies*, 9, 69–107.

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[4] R. Cont and E. Voltchkova (2005), *SIAM J. Numer. Anal.*, 43, 1596–1626.

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Content and numerical values adapted from the authors' ICCF 2026 research presentation.