Evolution of superhorizon perturbations in early Universe with anisotropic solid remnant

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Motivation

ΛCDM model with inflation fits the data, however anomalies remain: the **Hubble tension** and **low-multipole CMB anomalies** [1], which may be addressed by **anisotropic (Bianchi type-I) expansion**.

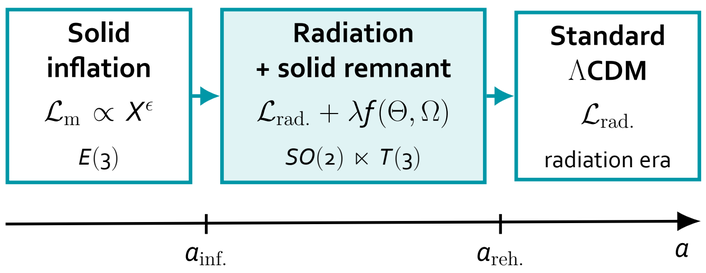
- **Solid inflation** [2] is driven by a triplet of fields Φ^I with global Euclidean symmetry $E(3) = SO(3) \ltimes T(3)$.
- After inflation most energy goes to **radiation**; a small part survives as an **anisotropic solid remnant** with $E(3) \to SO(2) \ltimes T(3)$.

Model

Splitting $X = \mathrm{Tr}\, B = \Theta + 2\Omega$ (where B is body metric [2]),

$$\Theta = g^{\mu\nu}\Phi^1_{,\mu}\Phi^1_{,\nu}, \qquad \Omega = \tfrac{1}{2}\, g^{\mu\nu}\delta_{AB}\Phi^A_{,\mu}\Phi^B_{,\nu}.$$

breaks the full global Euclidean symmetry $E(3)$ to $SO(2) \ltimes T(3)$:



Background metric (Bianchi type-I):

$ds^2 = e^{2\alpha}\left(-d\tau^2 + e^{-4\sigma}(dx^1)^2 + e^{2\sigma}\delta_{AB}dx^A dx^B\right)$,

with small anisotropy, $\sigma = \mathcal{O}(\lambda)$, $\lambda \ll 1$.

Background anisotropy

For $f = K\,\Theta^P\Omega^Q$, the anisotropy parameter $\gamma = \dot{\sigma}/\dot{\alpha}$ in the leading order in λ is

$$\gamma = \lambda\Big(-K_4\,\tau^{-1} + 2(2-P-Q)\,\mathcal{C}_2\,\tau^{2(2-P-Q)}\Big),$$

where K_4 and $\mathcal{C}_2$ are integration constants.

- $P + Q < 2$: anisotropy **grows**, by $e^{50.66(2-P-Q)}$ from GUT to 100 TeV.
- Data allow only small anisotropy [3]: we take $P + Q = 2$ with the strongest effect on perturbations.

Superhorizon perturbations

With $SO(2) \ltimes T(3)$ symmetry there are **no tensor modes**, only 2D scalars and 2D vectors [4]. In the isotropic limit they map onto standard 3D scalars, vectors and tensors [5], which are now **coupled**. In the **superhorizon limit** $\zeta = k\tau \ll 1$ the solid remnant sources modes as

$$\text{sourced modes} \;\propto\; \lambda\,\zeta^{\mathcal{Y}}, \quad \mathcal{Y}(\mathcal{Y}+1)(\mathcal{Y}-3) = 16.$$

Physical root: $\mathcal{Y}_1 \approx 3.85$ (complex roots lead to superluminality).

modes	$\lambda = 0$	metric anisotropy	remnant effect
scalar	$0, -1, -3$	$-2, -4$	**1.85**
vector	-2	-3	**2.85**
tensor	$0, -1$	-3	—

Table 1. Powers of ζ in superhorizon modes: the remnant adds **growing** scalar ($\mathcal{Y}_1 - 2$) and vector ($\mathcal{Y}_1 - 1$) modes.

Stability limit

Growing modes do not break perturbative regime if

$$a_\mathrm{reh.}/a_\mathrm{inf.} < \lambda^{-1/\mathcal{Y}_1},$$

e.g. $\lambda = 10^{-3}$: $t_\mathrm{reh.} < 36\, t_\mathrm{inf.}$; then the remnant dissipates, possibly via its own instability.

Tensor- and vector-to-scalar ratio

Tensor-to-scalar $\tilde{r}$ and vector-to-scalar ratio $\tilde{s}$:

$$\tilde{r} = \frac{r_\mathrm{reh.}}{r_\mathrm{inf.}} \sim \frac{1}{\left[1 + \lambda(\tilde{a}^{\mathcal{Y}_1 - 2} - 1)\right]^2},$$

$$\tilde{s} = \frac{s_\mathrm{reh.}}{s_\mathrm{inf.}} \sim \left[\frac{\tilde{a}^{-2} + \lambda(\tilde{a}^{\mathcal{Y}_1 - 1} - 1)}{1 + \lambda(\tilde{a}^{\mathcal{Y}_1 - 2} - 1)}\right]^2,$$

where $\tilde{a} = a_\mathrm{reh.}/a_\mathrm{inf.}$

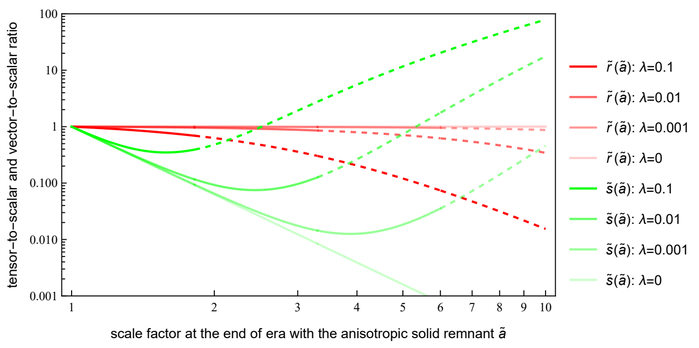


Figure 1. dependence of $\tilde{r}$ (red) and $\tilde{s}$ (green) on $\tilde{a}$; dashed: non-perturbative regime.

Observational consequences

Improving simplified solid inflation

$\mathcal{L}_\mathrm{m} \propto X^\epsilon$ keeps non-Gaussianity small, but

$$n_\mathrm{S} = 1 + \tfrac{2}{3}(1+2\epsilon)\epsilon, \quad r_\mathrm{inf.} = \tfrac{16}{9\sqrt{3}}(1+2\epsilon)^{5/2}\epsilon.$$

$n_\mathrm{S} = 0.965$ [6] yields $r_\mathrm{inf.} = 0.045$, which is above $r < 0.032$ [7]. The effect of the solid remnant:

λ	$r_\mathrm{inf.}$	$r_\mathrm{reh.}$
0.01	0.045	0.038
0.1	0.045	**0.031**

Limit $\lambda \to 0$ and the longest solid remnant era

$\tilde{r} \to 1$, $\tilde{s}/\tilde{a}^{-4} \sim \lambda^{-2/\mathcal{Y}_1} \sim 10^{22}$ (GUT to 100 TeV); and vector perturbations decay as

$$\frac{\mathrm{vector}_\mathrm{reh.}}{\mathrm{vector}_\mathrm{inf.}} \sim \left(\frac{a_\mathrm{reh.}}{a_\mathrm{inf.}}\right)^{-1} \text{ instead of } a^{-2}.$$

Conclusions

- Background anisotropy stays small, perturbations change strongly.
- **Scalar and vector** superhorizon modes **grow**; **tensors** are unaffected.
- **Tensor-to-scalar ratio suppressed** ($0.045 \to 0.031$ for $\lambda = 0.1$).
- **Vector-to-scalar ratio enhanced**; for $\lambda \to 0$ vectors decay as a^{-1}, not a^{-2}.

References

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